NORTH DAKOTA DEPARTMENT OF
PUBLIC INSTRUCTION

North Dakota
High School State Standards

Algebra II Pathway

July 2017

Overview

The North Dakota State Standards (NDSS) for Mathematics are organized by grade level in Grades K-8. At the high school level, the standards are organized by conceptual category (number and quantity, algebra, functions, geometry, modeling and probability and statistics), showing the body of knowledge students should learn in each category to be college and career ready, and to be prepared to study more advanced mathematics. As North Dakota school districts consider how to implement the high school standards, an important consideration is how the high school NDSS might be organized into courses that provide a strong foundation for post-secondary success. To address this need, the NDSS writing committee has provided a possible pathway to implement the NDSS in the traditional courses of Algebra I, Geometry, Algebra II and Course IV.

In considering this document, it is important to note the following:

1. The pathway is a model, not a mandate. It illustrates a possible approach to organize the content of the NDSS into coherent and rigorous courses that lead to college and career readiness. Districts are not expected to adopt these courses as is; rather, they may use this pathway as a starting point for developing their own.

2. All college and career ready standards have been included in the pathway. Standards with a (+) are included to increase coherence but are not necessarily expected to be addressed on high stakes assessments.

While the focus of this document is on organizing the Standards for Mathematical Content into a pathway to college and career readiness, the content standards must also be connected to the Standards for Mathematical Practice to ensure that the skills needed for alter success are developed. In particular, Modeling (defined by a * in the NDSS) is defined as both a *conceptual category* for high school mathematics and a *mathematical practice* and is an important avenue for motivating students to study mathematics, for building their understanding of mathematics, and for preparing them for future success. Development of the pathway into instructional programs will require careful attention to modeling and the mathematical practices. Assessments based on the pathway should reflect both the content and mathematical practice standards.

Strategic use of technology is expected in all work. This may include employing technological tools to assist students in forming and testing conjectures, creating graphs and data displays, as well as determining and assessing lines of fit for data. Geometric

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*

constructions may also be performed using geometric software, as well as classical tools and technology aiding in three-dimensional visualization.

Mathematical Practices

It should be noted that throughout each course, the following **mathematical practices** from the NDSS are to be emphasized:

1. Make sense of problems and persevere in solving them.

Mathematically proficient students:

- Explain to themselves the meaning of a problem and looking for entry points to its solution.
- Analyze givens, constraints, relationships, and goals.
- Make conjectures about the form and meaning of the solution attempt.
- Consider analogous problems, and try special cases and simpler forms of the original problem.
- Monitor and evaluate their progress and change course if necessary.
- Transform algebraic expressions or change the viewing window on their graphing calculator to get information.
- Explain correspondences between equations, verbal descriptions, tables, and graphs.
- Draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- Use concrete objects or pictures to help conceptualize and solve a problem.
- Check their answers to problems using a different method.
- Ask themselves, “Does this make sense?”
- Understand the approaches of others to solving complex problems.

2. Reason abstractly and quantitatively.

Mathematically proficient students:

- Make sense of quantities and their relationships in problem situations.
  - ✓ *Decontextualize* (abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents) and
  - ✓ *Contextualize* (pause as needed during the manipulation process in order to probe into the referents for the symbols involved).

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
** indicates modeling standards*

- Use quantitative reasoning that entails creating a coherent representation of quantities, not just how to compute them.
- Know and flexibly use different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- Understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- Make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- Analyze situations by breaking them into cases.
- Recognize and use counterexamples.
- Justify their conclusions, communicate them to others, and respond to the arguments of others.
- Reason inductively about data, making plausible arguments that take into account the context.
- Compare the effectiveness of plausible arguments.
- Distinguish correct logic or reasoning from that which is flawed.
  - ✓ Elementary students construct arguments using objects, drawings, diagrams, and actions.
  - ✓ Later students learn to determine domains to which an argument applies.
- Listen or read the arguments of others, decide whether they make sense, and ask useful questions.

4. Model with mathematics.

Mathematically proficient students:

- Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
  - ✓ In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
  - ✓ By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- Simplify a complicated situation, realizing that these may need revision later.
- Identify important quantities in a practical situation.
- Map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- Analyze those relationships mathematically to draw conclusions.

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*

- Interpret their mathematical results in the context of the situation.
- Reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students:
- Consider available tools when solving a mathematical problem.
- Are familiar with tools appropriate for their grade or course to make sound decisions about each of these tools.
- Detect possible errors by using estimations and other mathematical knowledge.
- Know that technology can enable them to visualize the results of varying assumptions, and explore consequences.
- Identify relevant mathematical resources and use them to pose or solve problems.
- Use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Mathematically proficient students:
- Try to communicate precisely to others.
- Use clear definitions in discussion with others and in their own reasoning.
- State the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- Specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the context.
  - ✓ In the elementary grades, students give carefully formulated explanations to each other.
  - ✓ In high school, students have learned to examine claims and make explicit use of definitions.

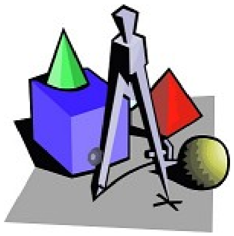
7. Look for and make use of structure.
Mathematically proficient students:
- Look closely to discern a pattern or structure.
  - ✓ Young students might notice that three and seven more is the same amount as seven and three more.
  - ✓ Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for the distributive property.
  - ✓ In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7.
- Step back for an overview and can shift perspective.
- See complicated things, such as some algebraic expressions, as single objects or composed of several objects.

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
** indicates modeling standards*

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- Notice if calculations are repeated.
- Look for both general methods and for shortcuts.
- Maintain oversight of the process, while attending to the details.
- Continually evaluate the reasonableness of intermediate results.



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*

Conceptual Category: Number and Quantity

Domain: **The Real Number System**

Cluster: **Extend the properties of exponents to rational numbers**

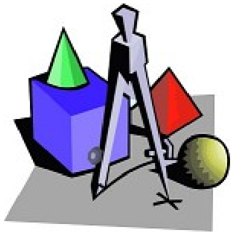
HS.N-RN

Standard	**Students Can**	**Resources**
HS.N-RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	• use exponential properties to explain how rational exponents follow from integer exponents	
Vocabulary	**Annotations**	**Notes**
• rational • exponent	For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.	

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*

* *indicates modeling standards*

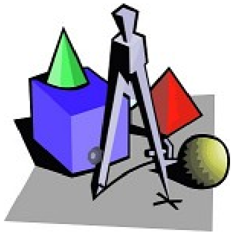


North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Number and Quantity**
Domain: **The Real Number System**
Cluster: **Extend the properties of exponents to rational numbers**

HS.N-RN

Standard HS.N-RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	**Students Can** • understand that the denominator of a rational exponent is the root index and the numerator is the exponent of the radicand • convert an expression in radical form to rational exponents and vice-versa	**Resources**
Vocabulary • root index • radicand • radical	**Annotations** Example: $\sqrt{x^3} = x^{\frac{3}{2}}$ Example: $(\sqrt{4})^3 = ((4)^{\frac{1}{2}})^3 = 2^3 = 8$	**Notes**

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



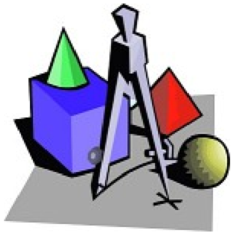
North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Number and Quantity**
Domain: **The Real Number System**
Cluster: **Use properties of rational and irrational numbers**

HS.N-RN

Standard	**Students Can**	**Resources**
HS.N-RN.3 Demonstrate that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.	• use the closure property or show by example the sum or product of two rational numbers is rational • show by example the sum of a rational number and an irrational number is irrational • show by example the product of a nonzero rational number and an irrational number is irrational	Rational and Irrational Numbers 1 Rational and Irrational Numbers 2
Vocabulary	**Annotations**	**Notes**
• closure property • irrational number • rational number	Example: Evaluate $\sqrt{2} \cdot \sqrt{4}$ and identify which subset of the real number system the solution is in. Solution: $\sqrt{8} = 2\sqrt{2}$, which is irrational.	

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



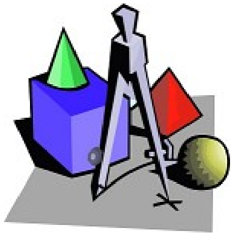
North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Number and Quantity**
Domain: **The Real Number System**
Cluster: **Use properties of rational and irrational numbers**

HS.N-RN

Standard	**Students Can**	**Resources**
HS.N-RN.4 Perform basic operations on radicals and simplify radicals to write equivalent expressions.	• perform basic operations on radicals • simplify radicals to write equivalent expressions	
Vocabulary • radicals • rationalizing the denominator	**Annotations** Basic operations include addition, subtraction, multiplication and division (e.g., rationalizing the denominator).	**Notes**

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



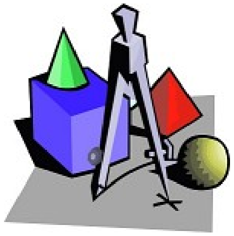
North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Number and Quantity**
Domain: **Quantities* (Mathematical Practices 1, 4, and 6)**
Cluster: **Reason quantitatively and use units to solve problems**

HS.N-Q

Standard	**Students Can**	**Resources**
HS.N-Q.1* Use units as a way to understand problems and to guide the solution of multi-step problems (e.g., unit analysis). Choose and interpret units consistently in formulas. Choose and interpret the scale and the origin in graphs and data displays.	• interpret units in the context of the problem • use unit analysis to check the reasonability of your solution • choose and interpret an appropriate scale given data to be represented on a graph or display	Estimations and Approximations: The Money Munchers Leaky Faucet Yogurt
Vocabulary	**Annotations**	**Notes**
• unit analysis	Example: While driving in the United Kingdom (UK), a U.S. tourist puts 60 liters of gas in his car. The gas cost is £1.28 per liter. The exchange rate is £ 0.62978 for each US $1.00. The price for a gallon of a gasoline in the United States is US $3.05. The driver wants to compare costs for the same amount and the same type of gasoline in UK and in the United States if he pays in UK Pounds.	

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



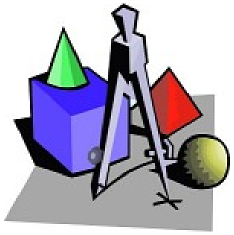
North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Number and Quantity**
Domain: **Quantities* (Mathematical Practices 1, 4, and 6)**
Cluster: **Reason quantitatively and use units to solve problems**

HS.N-Q

Standard	**Students Can**	**Resources**
HS.N-Q.2* Define appropriate quantities for the purpose of descriptive modeling.	• determine an appropriate quantity to model a situation	Estimations and Approximations: The Money Munchers Yogurt Leaky Faucet
Vocabulary	**Annotations**	**Notes**
• modeling	Example: When carpeting a room, students might consider whether it is best to use square feet or square yards. When considering a remodeling project, they might choose such units as cost per room, cost per month of the project, or cost per contractor.	

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



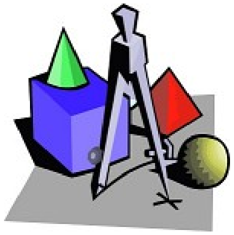
North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Number and Quantity**
Domain: **Quantities* (Mathematical Practices 1, 4, and 6)**
Cluster: **Reason quantitatively and use units to solve problems**

HS.N-Q

Standard	**Students Can**	**Resources**
HS.N-Q.3* Choose a level of accuracy or precision appropriate to limitations on measurement when reporting quantities.	• choose a level of accuracy or precision appropriate to the measuring tool or situation	Estimations and Approximations: The Money Munchers Leaky Faucet Yogurt
Vocabulary	**Annotations**	**Notes**
• significant digits • precision: refers to how much information is conveyed by a number (in terms of the number of digits) • accuracy: the degree to which a measurement conforms to the correct value or a standard	Example: When using a ruler, students choose to report their measurements based on the precision of the ruler (e.g., to the nearest 1/16 or the nearest 1/32). Example: If you are playing soccer and you always hit the left goal post instead of scoring, then you are not accurate; you are precise. Example: When using a ruler, students are able to measure accurately. Example: When calculating the cost of a road trip, students are given the cost of gasoline to the thousandths place. When reporting the cost of the trip, students determine what level of precision—to the hundredths place or to the thousandths place—is appropriate and why.	

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
** indicates modeling standards*

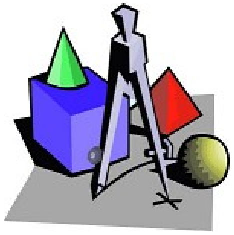


North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Number and Quantity**
Domain: **The Complex Number System**
Cluster: **Perform arithmetic operations with complex numbers**

HS.N-CN

Standard	**Students Can**	**Resources**
HS.N-CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. Understand the hierarchal relationships among subsets of the complex number system.	• recognize every number as a complex number of the form a+bi where a and b are real numbers • recognize i as complex and $i^2 = -1$ • classify a number into the correct subset	
Vocabulary • real number • imaginary number • complex number • whole number • natural number • integer • rational number • irrational number	**Annotations** Knowledge of complex numbers extends and reinforces student knowledge of the real number system. Example: $\sqrt{8}$ is a complex number because it can be written in the form $\sqrt{8} + 0i$. $\sqrt{8}$ is also a real number since its imaginary coefficient is 0. $\sqrt{8}$ is also an irrational number because it cannot be written as a ratio of two integers.	**Notes**

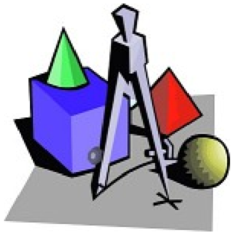
Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Number and Quantity**
Domain: **The Complex Number System**
Cluster**: Perform arithmetic operations with complex numbers**

Standard	**Students Can**	**Resources**
HS.N-CN.2 Use the definition $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	• apply the fact that the complex number $i^2 = -1$ to perform operations on complex numbers	
Vocabulary • distributive property • commutative property • associative property	**Annotations** Knowledge of complex numbers extends and reinforces student knowledge of basic operations and properties of the real number system. Example: $(2+3i)+(4-5i)=6-2i$ $(2+3i)-(4-5i)=-2+8i$ $(2+3i)(4-5i)=8-10i+12i-15i^2$ $=8+2i+15$ $=23+2i$	**Notes**

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



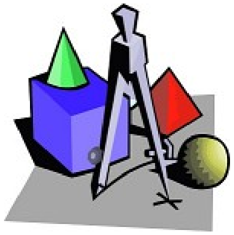
North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Number and Quantity**
Domain: **The Complex Number System**
Cluster**: Perform arithmetic operations with complex numbers**

HS.N-CN

Standard	**Students Can**	**Resources**
HS.N-CN.3 Use conjugates to find quotients of complex numbers.	• find the conjugate of a complex number • use the conjugate to find the quotient of a complex number	
Vocabulary	**Annotations**	**Notes**
• conjugate		

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*

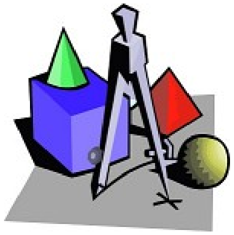


North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Number and Quantity**
Domain: **The Complex Number System**
Cluster**: Use complex numbers in polynomial identities and equations**

HS.N-CN

Standard	**Students Can**	**Resources**
HS.N-CN.7 Solve quadratic equations with real coefficients that have complex solutions.	• solve quadratic equations with real coefficients that have solutions of the form a+b*i* and a-b*i*	
Vocabulary • quadratic equation • complex solution	**Annotations** This topic is also addressed in HS.A-REI.4. Example: $x^2 + 2x + 5 = 0$ $x = \frac{-2 \pm \sqrt{(2)^2 - (4)(1)(5)}}{2 \cdot 1}$ $x = \frac{-2 \pm \sqrt{-16}}{2}$ $x = \frac{-2 \pm 4i}{2}$ $x = -1 \pm 2i$	**Notes**

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
** indicates modeling standards*



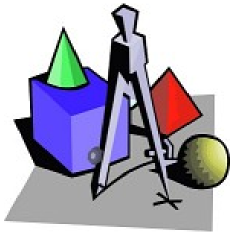
North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Number and Quantity**
Domain: **The Complex Number System**
Cluster**: Use complex numbers in polynomial identities and equations**

HS.N-CN

Standard	**Students Can**	**Resources**
(+) HS.N-CN.8 Extend polynomial identities to the complex numbers.	• use polynomial identities to write equivalent expressions in the form of complex numbers	
Vocabulary • polynomial identities	**Annotations** Example: Rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$. Polynomial identities include but are not limited to: $(a+b)^2 = a^2 + 2ab + b^2$ $(a+b)(c+d) = ac + ad + bc + bd$ $a^2 - b^2 = (a+b)(a-b)$ (*Differences of Squares*) $a^3 \pm b^3 = (a+b)(a^2 \mp ab + b^2)$ (*Sum and Differences of Cubes*) $x^2 + (a+b)x + ab = (x+a)(x+b)$	**Notes**

Note:

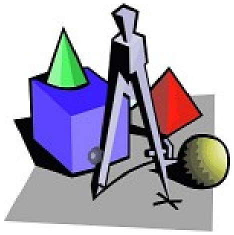
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



Standard (+) HS.N-CN.9 Apply the Fundamental Theorem of Algebra to determine the number of zeros for polynomial functions. Find all solutions to a polynomial equation.	**Students Can** • recognize that the Fundamental Theorem of Algebra states that the number of complex solutions to a polynomial equation is equal to the degree of the polynomial • find all the solutions to a polynomial equation	**Resources**
Vocabulary • Fundamental Theorem of Algebra: The number of complex solutions to a polynomial equation is equal to the degree of the polynomial. • zeros • degree	**Annotations**	**Notes**

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*

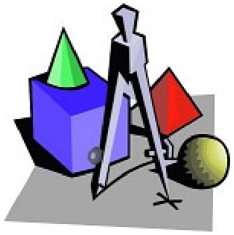
Conceptual Category: **Algebra**
Domain: **Seeing Structure in Expressions**
Cluster**: Interpret the structure of expressions**

HS.A-SSE

Standard	**Students Can**	**Resources**
HS.A-SSE.1* Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	• interpret expressions that represent a quantity in terms of its context • identify the different parts of an expression and explain their meaning within the context of a problem • interpret expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts	Forming Quadratics
Vocabulary	**Annotations**	**Notes**
• expression • term • factor • coefficient	Example: Interpret $\frac{1}{2}h(b_1 + b_2)$ as the product of the height of a trapezoid and the average of its base lengths. Algebra II students extend their focus to polynomial and rational expressions.	

Note:

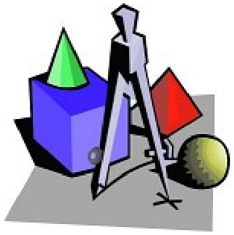
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Algebra**
Domain: **Seeing Structure in Expressions**
Cluster**: Interpret the structure of expressions**

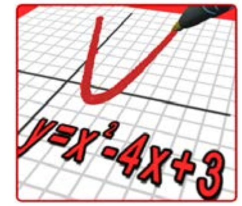
Standard	**Students Can**	**Resources**
HS.A-SSE.2 Use the structure of an expression to identify ways to rewrite it.	• rewrite algebraic expressions in equivalent forms such as factoring or combining like terms • use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor an expression completely • simplify expressions by combining like terms, using the distributive property and using other operations with polynomials • recognize patterns and structures in expressions	[Forming Quadratics]
Vocabulary	**Annotations** Example: See $9a^2 - 4b^2$ as $(3a)^2 - (2b)^2$ and recognize it as a difference of squares that can be factored as $(3a - 2b)(3a + 2b)$. Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$, and further to $(x-y)(x+y)(x^2+y^2)$. Algebra II students extend their focus to polynomial and rational expressions.	**Notes**

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
** indicates modeling standards*



HS.A-SSE

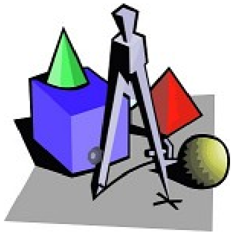
North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Algebra**
Domain: **Seeing Structure in Expressions**
Cluster: **Write expressions in equivalent forms to solve problems**



Standard	**Students Can**	**Resources**
HS.A-SSE.3* Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to produce and equivalent expression. c. Use the properties of exponents to transform exponential expressions.	• choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression • write expressions in equivalent forms by factoring to find the zeros of a quadratic function and explain the meaning of the zeros • complete the square in a quadratic expression to convey the vertex form and determine the maximum or minimum value of the quadratic function, and to explain the meaning of the vertex • use properties of exponents (such as power of a power, product of powers, power of a product, power of a quotient) to write an equivalent form of an exponential function to reveal and explain specific information about its approximate rate of growth or decay	Forming Quadratics
Vocabulary • equivalent form • quadratic function • zero of a function • complete the square • maximum • minimum • vertex • exponent • exponential • rate of growth or decay	**Annotations** Example: Given a quadratic function explain the meaning of the zeros of the function. That is if $f(x) = x^2-7x+12 =(x – 4) (x – 3)$ then $f(4) = 0$ and $f(3) = 0$. Example: A toy rocket is shot up at 128 ft/sec from a height of 5 ft. What is the maximum height of the rocket and when does it reach that height? Solution: After 4 seconds the rocket is 261 ft high Example: Write the equation of a circle in standard form to find the center and radius. Example: $8^t = 2^{3t}$ Example: The expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.	**Notes**

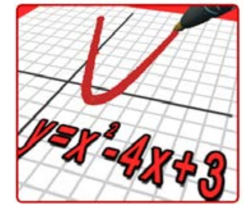
Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Algebra**
Domain: **Arithmetic with Polynomials and Rational Expressions**
Cluster**: Perform arithmetic operations on polynomials**

HS.A-APR



Standard	**Students Can**	**Resources**
HS.A-APR.1 Add, subtract, and multiply polynomials. Understand that polynomials form a system comparable to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.	• identify polynomials • add, subtract, and multiply polynomials • recognize how closure applies under these operations • relate the addition, subtraction, and multiplication of polynomials to the same operations with integers	
Vocabulary	**Annotations**	**Notes**
• polynomial • closure	Algebra II students extend their understanding beyond the quadratic polynomials.	

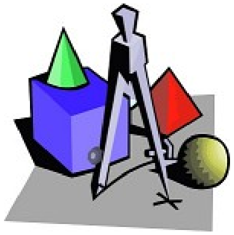
Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
** indicates modeling standards*

North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Algebra**
Domain: **Arithmetic with Polynomials and Rational Expressions**
Cluster**: Understand the relationship between zeros and factors of polynomials**

HS.A-APR

Standard	**Students Can**	**Resources**
HS.A-APR.2 Apply the Remainder Theorem.	• apply the Remainder Theorem to a polynomial	
Vocabulary	**Annotations**	**Notes**
• Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). • zero of a polynomial	Pre-requisite knowledge for this standard includes understanding of polynomial division and factoring.	

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



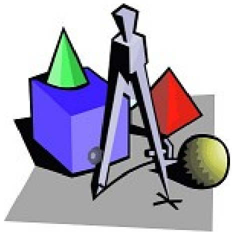
Conceptual Category: **Algebra**
Domain: **Arithmetic with Polynomials and Rational Expressions**
Cluster**: Understand the relationship between zeros and factors of polynomials**

HS.A-APR

Standard	**Students Can**	**Resources**
HS.A-APR.3 Identify zeros of polynomials when suitable factorizations are available. Use the zeros to construct a rough graph of the function defined by the polynomial.	• factor a polynomial • find the zeros of a factored polynomial function • use the zeros (x-intercepts) of a polynomial function to sketch a graph of the function	
Vocabulary	**Annotations**	**Notes**

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



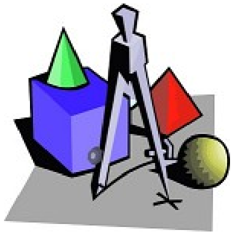
North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Algebra**
Domain: **Arithmetic with Polynomials and Rational Expressions**
Cluster**: Use polynomial identities to solve problems**

HS.A-APR

Standard (+) HS.A-APR.5 Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n.	**Students Can** • use Pascal's Triangle to determine the coefficients of the binomial expansion $(x+y)^n$ • use the Binomial Theorem to find the nth term in the expansion of a binomial to a positive integer power	**Resources**
Vocabulary • Pascal's Triangle • Binomial Theorem	**Annotations**	**Notes**

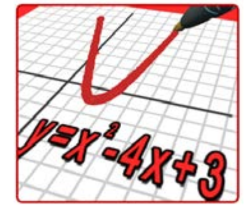
Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
** indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Algebra**
Domain: **Arithmetic with Polynomials and Rational Expressions**
Cluster**: Rewrite rational expressions**

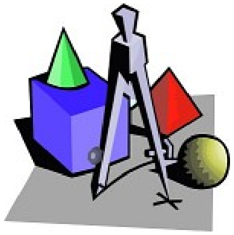
HS.A-APR



Standard	**Students Can**	**Resources**
HS.A-APR.6 Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x)* + *r(x)/b(x)*, where *a(x)*, *b(x)*, *q(x)*, and *r(x)* are polynomials with the degree of r(x) less than the degree of *b(x)*, using inspection, long division, or, for the more complicated examples, a computer algebra system.	• rewrite rational expressions, $\frac{a(x)}{b(x)}$, in the form $q(x)+\frac{r(x)}{b(x)}$ using inspection or long division (or a computer algebra system for complicated examples)	Simplifying Rational Expressions Resource
Vocabulary	**Annotations**	**Notes**
• rational expression	Example: Use long division to rewrite: $$\frac{3x^3-2x^2+4x-3}{x^2+3x+3}$$ in the form: $$(3x-11)+\frac{28x+30}{x^2+3x+3}$$	

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



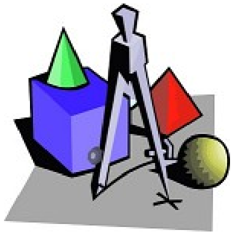
North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Algebra**
Domain: **Arithmetic with Polynomials and Rational Expressions**
Cluster**: Rewrite rational expressions**

HS.A-APR

Standard	**Students Can**	**Resources**
HS.A-APR.7 Add, subtract, multiply, and divide rational expressions. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.	• add, subtract, multiply, and divide rational expressions • recognize that rational expressions are closed under addition, subtraction, multiplication, and division by a nonzero expression • relate the rational expressions system to the rational numbers system	
Vocabulary • closure	**Annotations**	**Notes**

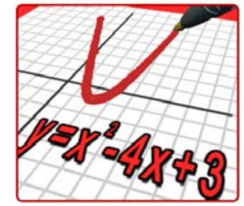
Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Algebra**
Domain: **Creating Equations and Inequalities***
Cluster**: Create equations that describe numbers or relationships**

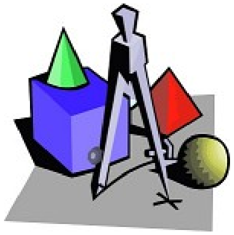
HS.A-CED



Standard HS.A-CED.1* Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	**Students Can** • create linear, quadratic, rational and exponential equations and inequalities in one variable and use them in a contextual situation to solve problems	**Resources**
Vocabulary • linear equations • quadratic equations • rational equations • exponential equations • inequalities	**Annotations** Algebra II students use all available types of functions to create such equations, including root functions, but constrain to simple cases.	**Notes**

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



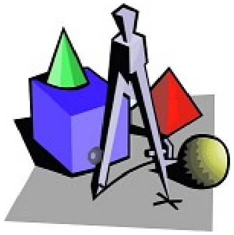
North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Algebra**
Domain: **Creating Equations and Inequalities***
Cluster**: Create equations that describe numbers or relationships**

HS.A-CED

Standard	**Students Can**	**Resources**
HS.A-CED.2* Create equations in two or more variables to represent relationships between quantities. Graph equations on coordinate axes with appropriate labels and scales.	• create equations in two or more variables to represent relationships between quantities • graph equations in two variables on a coordinate plane and label the axes and scales	
Vocabulary	**Annotations**	**Notes**
• coordinate plane • scale	Example: The cost to rent a car is $50 plus $0.25 per mile driven. Write and graph an equation to represent the situation. In Algebra II, the linear, exponential, or quadratic types of problems should draw from more complex situations than those addressed in Algebra I.	

Note:

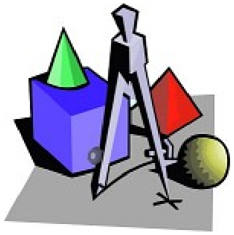
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
** indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Algebra**
Domain: **Creating Equations and Inequalities***
Cluster**: Create equations that describe numbers or relationships**

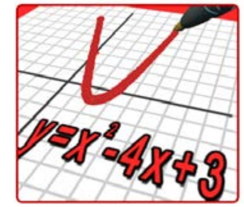
Standard	**Students Can**	**Resources**
HS.A-CED.3* Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.	• write and use a system of equations and/or inequalities to solve a real world problem • use equations and inequalities to represent problem constraints and objectives (linear programming) • interpret solutions to problems as viable or non-viable in a problem context	
Vocabulary	**Annotations**	**Notes**
• linear programming • constraint • feasible region	Example: Willy Wonka's Chocolate Factory makes *Wonka Bars* and *The Everlasting Gobstopper*, among other amazing treats. Oompa Loompas and Fuzzy Fizzies work on each item. The Oompa Loompas spend 6 minutes making a *Wonka Bar* and 4 minutes mixing the ingredients for an *Everlasting Gobstopper*. There are enough Oompa Loompas for up to 6000 worker-minutes per day. The Fuzzy Fizzies spend about 1 minute wrapping each Wonka Bar and 2 minutes wrapping each *Everlasting Gobstopper*. There are enough Fuzzy Fizzies for a maximum of 1200 worker-minutes per day. Given the above constraints, find the feasible region for the number of *Wonka Bars* and *Everlasting Gobstoppers* that can be made per day. **Solution:** Oompa Loompas: (6 min/bar)(x bars) + (4 min/gob) (y gob) $\leq$6000 min. Fuzzy Fizzies: (1min/bar)(x bars) + (2 min/gob)(y gob) $\leq$1200 min. Using substitution, y = 150, x = 900 if the maximum number of hours are worked. Therefore, the feasible region for the number of *Wonka Bars* made in a day is $0\leq x\leq 900$, and the feasible region for the number of *Everlasting Gobstoppers* is $0\leq y\leq 150$. In Algebra II, the linear, exponential, or quadratic types of problems should draw from more complex situations than those addressed in Algebra I.	

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Algebra**
Domain: **Creating Equations and Inequalities***
Cluster**: Create equations that describe numbers or relationships**

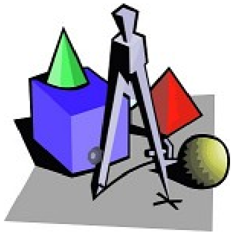
HS.A-CED



Standard	Students Can	Resources
HS.A-CED.4* Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	• solve multi-variable formulas or literal equations for a specific variable	
Vocabulary • literal equation	**Annotations** Example: Rearrange Ohm's law V = IR to highlight resistance R. In Algebra II, the linear, exponential, or quadratic types of problems should draw from more complex situations than those addressed in Algebra I.	**Notes**

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Algebra**
Domain**: Reasoning with Equations and Inequalities**
Cluster**: Understand solving equations as a process of reasoning and explain the reasoning**

HS.A-REI

Standard HS.A-REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	**Students Can** • solve simple rational equations in one variable • solve simple radical equations in one variable • provide examples of how extraneous solutions arise from solving simple rational and radical equations	**Resources**
Vocabulary • extraneous solutions	**Annotations** Example: Solve $\sqrt{6-x}=x$ $\left(\sqrt{6-x}\right)^2=x^2$ $6-x=x^2$ $x^2+x-6=0$ $(x-2)(x+3)=0$ $x=2\ or\ x=-3$ Check 2 and -3 in the original equation: $\sqrt{6-2}=\sqrt{4}=2$ $\sqrt{6{-}{-}3}=\sqrt{9}\neq-3$ Because -3 does not satisfy the original equation, -3 is an extraneous solution. 2 is the only solution to the equation.	**Notes**

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*

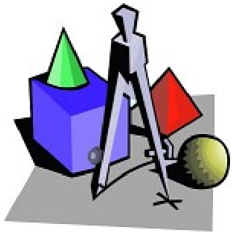
HS.A-REI

North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Algebra**
Domain: **Reasoning with Equations and Inequalities**
Cluster: **Solve equations and inequalities in one variable**

Standard	**Students Can**	**Resources**
HS.A-REI.4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. b. Solve quadratic equations by inspection (e.g., for $x^2=49$) taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a±bi for real numbers a and b.	• solve quadratic equations in one variable • transform a quadratic equation to an equation in the form $(x\text{-}p)^2=q$ by completing the square • solve quadratic equations in one variable by simple inspection, taking the square root, factoring, and completing the square • recognize which method of solving a quadratic equation is appropriate for a given equation • explain why taking the square root of both sides of an equation can yield two solutions • use the quadratic formula to solve any quadratic equation, recognizing the formula produces all complex solutions and write the solutions in the form a ± bi , where *a* and *b* are real numbers	
Vocabulary	**Annotations**	**Notes**
• completing the square • quadratic formula	This standard is related to HS.N-CN.7.	

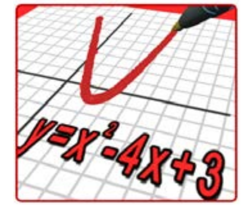
Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Algebra**
Domain: **Reasoning with Equations and Inequalities**
Cluster: **Solve systems of equations**

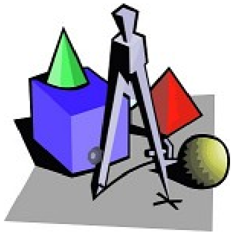
HS.A-REI



Standard HS.A-REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	**Students Can** • solve a system containing a linear equation and a quadratic equation in two variables algebraically and graphically	**Resources**
Vocabulary	**Annotations** Example: Find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$.	**Notes**

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
** indicates modeling standards*

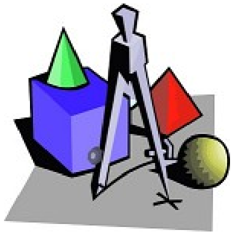


North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Algebra**
Domain: **Reasoning with Equations and Inequalities**
Cluster**: Represent and solve equations and inequalities graphically**

Standard	**Students Can**	**Resources**
HS.A-REI.11 Using graphs, technology, tables, or successive approximations, show that the solution(s) to the equation f(x) = g(x) are the x-value(s) that result in the y-values of f(x) and g(x) being the same.	• explain why the intersection of y = f(x) and y = g(x) is the solution of f(x) = g(x) for any combination of linear, polynomial, rational, absolute value, exponential, and logarithmic functions • use technology to graph the equations and find their points of intersection • use tables of values or successive approximations to find solutions	
Vocabulary • intersection	**Annotations** Algebra II will include combinations of linear, polynomial, rational, radical, absolute value, exponential and logarithmic functions. Example: Use a graphing calculator to find and justify the approximate solution(s) to the system below. $\begin{cases} f(x) = x + 4 \\ g(x) = 4 - x^2 \end{cases}$ Solutions*:* x= 0 and x= -1. f(0) = 4, g (0) = 4; f(-1)=3, g(-1)=3	**Notes**

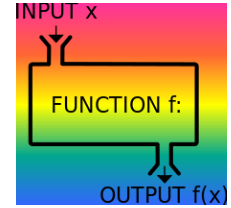
Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
** indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*

Conceptual Category: **Functions**
Domain: **Interpreting Functions**
Cluster: **Interpret functions that arise in applications in terms of the context**



Standard

HS.F-IF.4*
Use tables, graphs, verbal descriptions, and equations to interpret and sketch the key features of a function modeling the relationship between two quantities.

Students Can

- identify key features in graphs and tables to include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior for a given function
- use appropriate notation to indicate where functions are increasing, decreasing, positive, and negative
- sketch the graph of a function given its key features

Resources

[Interpreting Distance-Time Graphs]

[Forming Quadratics]

Vocabulary

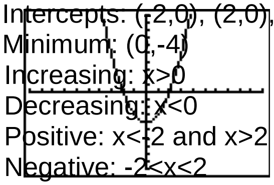
- intercepts
- relative maximum
- relative minimum
- end behavior
- periodicity
- symmetry

Annotations

Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Example: Given $f(x) = x^2 - 4$. Graph the function and identify the intercepts, intervals where the function is increasing, decreasing, positive, and negative, the relative maximum and minimum, and any symmetry.

Solution: Intercepts: (-2,0), (2,0), (0,-4)
Relative Minimum: (0,-4)
Increasing: x>0
Decreasing: x<0
Positive: x<-2 and x>2
Negative: -2<x<2
Symmetric to the y axis

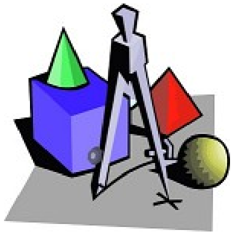


Notes

Algebra II emphasizes the selection of a model function based on behavior of data and context.

Note:

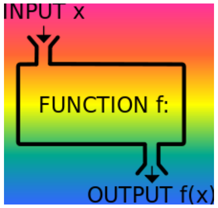
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*

Conceptual Category: **Functions**
Domain: **Interpreting Functions**
Cluster**: Interpret functions that arise in applications in terms of the context**

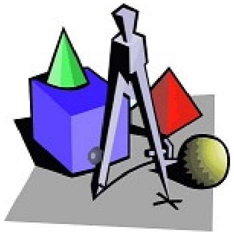
HS.F-IF



Standard	**Students Can**	**Resources**
HS.F-IF.5* Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	• interpret a graph to determine the appropriate numerical domain being described	Interpreting Distance-Time Graphs Forming Quadratics
Vocabulary	**Annotations**	**Notes**
	Example: If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Algebra II emphasizes the selection of a model function based on behavior of data and content.	

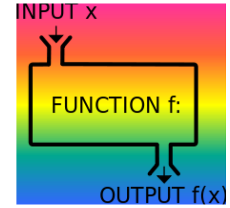
Note:

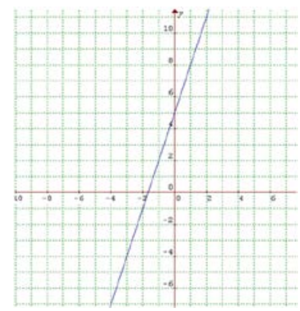
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Functions**
Domain: **Interpreting Functions**
Cluster**: Interpret functions that arise in applications in terms of the context**

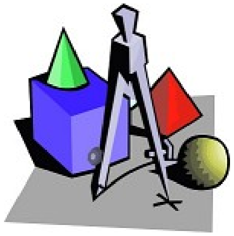
HS.F-IF



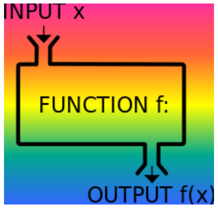
Standard HS.F-IF.6* Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	**Students Can** • calculate and interpret the average rate of change of a function presented symbolically or as a table • estimate the average rate of change over a specified interval of a function from its graph	**Resources** Interpreting Distance-Time Graphs The High School Gym
Vocabulary • rate of change • interval	**Annotations** Example: Estimate the rate of change given the graph below:  Solution: The average rate of change of a function $y = f(x)$ over an interval $[a, b]$ is $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$. Therefore, the estimated average rate of change for the function graphed above is: $\frac{(5-0)}{(0-(-1.75))} \approx 2.86$	**Notes** Algebra II emphasizes the selection of a model function based on behavior of data and context.

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



HS.F-IF

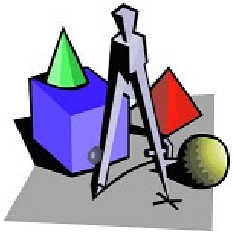


North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Functions**
Domain: **Interpreting Functions**
Cluster**: Analyze functions using different representations**

Standard	**Students Can**	**Resources**
HS.F-IF.7* Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior. f. Graph f(x) = sin x and f(x) = cos x as representations of periodic phenomena.	• graph functions expressed symbolically and show key features of the graph (graph simple cases by hand and use technology to show more complicated cases) • graph square root, cube root and piecewise-defined functions, including step functions and absolute value functions • graph polynomial functions, identifying zeros when factorable, and show end behavior • graph exponential and logarithmic functions, showing intercepts and end behavior • graph f(x)=sin x and f(x)= cos x as representations of periodic phenomena	Interpreting Distance-Time Graphs Forming Quadratics Functions and Everyday Situations
Vocabulary • square root function • cube root function • piecewise-defined function • step function • absolute value function • polynomial function • exponential function • logarithmic function • asymptote • period • midline • amplitude	**Annotations** Example: Solve the annual compound interest formula $A = P(1 + r)^t$ for t and draw a graph of time vs. amount for a given rate and principle amount, showing intercepts and end behavior. Compare this graph to the graph of amount vs. time.	**Notes** Algebra II focuses on applications, how key features relate to characteristics of situations, and making selections of a particular type of function model.

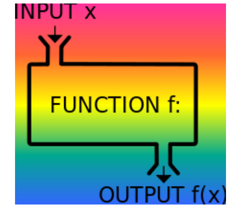
Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Functions**
Domain: **Interpreting Functions**
Cluster**: Analyze functions using different representations**

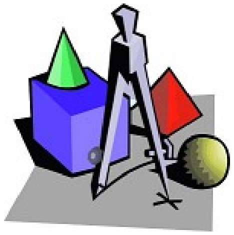
HS.F-IF



Standard	**Students Can**	**Resources**
HS.F-IF.8* Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions.	• write a function in equivalent forms (e.g., factored vs. general form) to show different properties of the function • explain different properties of a function that are revealed by writing a function in equivalent forms • use the process of factoring and completing the square in a quadratic function to show zeros, a maximum or minimum, and symmetry of the graph, and interpret these in terms of a real-world situation • use the properties of exponents to interpret exponential functions as growth or decay	Interpreting Distance-Time Graphs Forming Quadratics Drug Filtering Which Equation
Vocabulary	**Annotations**	**Notes**
• exponential growth • exponential decay • extreme values	Example: Identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.	Algebra II focuses on using key features to guide selection of appropriate type of function model.

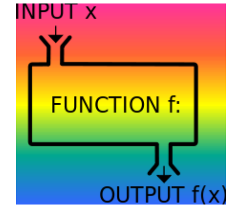
Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Functions**
Domain: **Interpreting Functions**
Cluster**: Analyze functions using different representations**

HS.F-IF

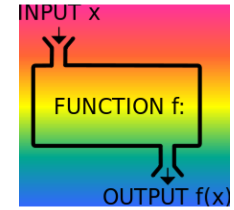


Standard HS.F-IF.9* Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	**Students Can** • compare the key features of two functions that are represented in different ways	**Resources** Interpreting Distance-Time Graphs Forming Quadratics
Vocabulary	**Annotations** Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. Example: Compare the intercepts of two functions, one of which is represented graphically and the other is represented symbolically. Algebra II focuses on using key features to guide selection of the appropriate type of model function.	**Notes**

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*

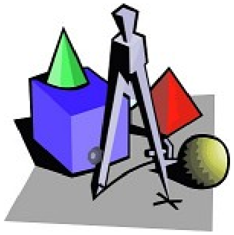
North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Functions**
Domain: **Building Functions***
Cluster**: Build a function that models a relationship between two quantifiers**

HS.F-BF

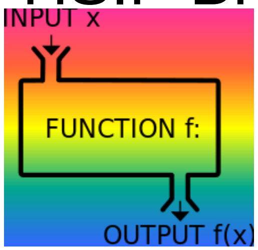


Standard HS.F-BF.1* Write a function that describes a relationship between two quantities. a. Combine standard function types using arithmetic operations. b. Compose functions.	**Students Can** • write a functions that describes a relationship between two quantities • combine function types, such as linear and exponential, using arithmetic operations • compose functions and interpret the result in a real-world situation	**Resources** Generalizing Patterns: Table Tiles Printing Tickets
Vocabulary • composition of functions	**Annotations** Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Example: If T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.	**Notes** Algebra II will develop models for more complex or sophisticated situations than in previous courses.

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
** indicates modeling standards*



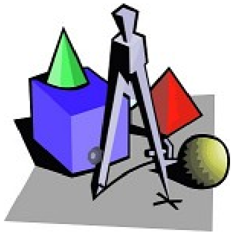
HS.F-BF



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Functions**
Domain: **Building Functions***
Cluster: **Build a function that models a relationship between two quantities**

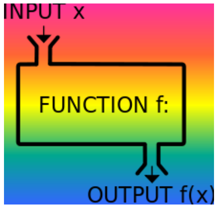
Standard	**Students Can**	**Resources**
HS.F-BF.2* Write arithmetic and geometric sequences both recursively and with an explicit formula and convert between the two forms. Use sequences to model situations.	• write and translate between the recursive and explicit formula for a arithmetic sequence and use the formulas to model a situation • write and translate between the recursive and explicit formula for a geometric sequence and use the formulas to model a situation • make connections between linear functions and arithmetic sequences, and exponential functions and geometric sequences	Generalizing Patterns: Table Tiles
Vocabulary • sequence • recursive • explicit • arithmetic sequences • geometric sequences	**Annotations** Example: Allen is training for a biking race and begins his workout regimen by biking 10 miles on day one and increasing his mileage by 2 miles per day for the next 15 days. Express the situation with a recursive and explicit formula. Solution: $a_n=2n+8$ (explicit) $a_1=10$; $a_n=a_{n-1}+2$ with $n>1$(recursive)	**Notes**

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Functions**
Domain: **Building Functions***
Cluster**: Build new functions from existing functions**

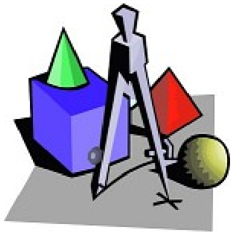
HS.F-BF



Standard	**Students Can**	**Resources**
HS.F-BF.3* Identify the effect on the graph of replacing f(x) by f(x) + k, f(x + k), k f(x), and f(kx), for specific values of k (both positive and negative); find the value of k given the graphs. Recognize even and odd functions from their graphs.	• identify, the transformational effects on the graph of a function f(x) when f(x) is replaced by f(x)+k, k·f(x), f(kx), and f(x+k) for specific values of k, both positive and negative • find the value of k given the graph of a transformed function • recognize even and odd functions from their graphs and equations	
Vocabulary • transformation • even functions • odd functions	**Annotations** Technology may be used to experiment with the effects of transformations on a graph. Algebra II will use transformations of functions to find models as students consider increasingly more complex situations; note the effect of multiple transformations on a single graph and the common effect of each transformation across function types.	**Notes**

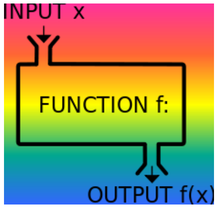
Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Functions**
Domain: **Building Functions***
Cluster**: Build new functions from existing functions**

HS.F-BF



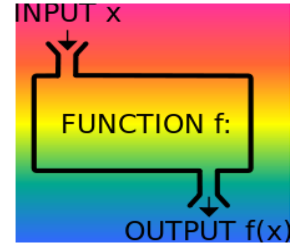
Standard	**Students Can**	**Resources**
HS.F-BF.4* Find inverse functions. a. Write an equation for the inverse given a function has an inverse. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or a table, given that the function has an inverse. d. Produce an invertible function from a non-invertible function by restricting the domain.	• find the inverse of a given function • verify that one function is the inverse of the other by composition • read values of an inverse function from a graph or table • restrict the domain on a function that is not one-to-one so that its inverse is a function	
Vocabulary • inverse function • independent variable • dependent variable • one-to-one function • invertible function	**Annotations** Example: Find the inverse for each function: $f(x) = 2x^3$ or $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$. Algebra II will extend to simple rational, simple radical, and simple exponential functions.	**Notes**

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
** indicates modeling standards*

North Dakota *HIGH SCHOOL* State Standards: *Algebra II*
Conceptual Category: **Functions**
Domain: **Building Functions***
Cluster: **Build new functions from existing functions**

HS.F-BF



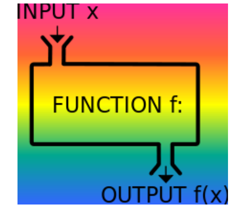
Standard	**Students Can**	**Resources**
HS.F-BF.5* Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	• write an exponential function in logarithmic form and vice-versa • solve problems involving logarithms and exponents	
Vocabulary	**Annotations**	**Notes**
• logarithm		

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*

North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Functions**
Domain: **Linear, Quadratic and Exponential Models***
Cluster**: Construct and compare linear and exponential models and solve problems**

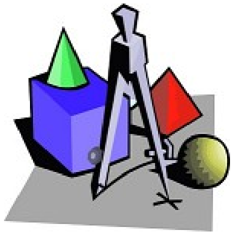
HS.F-LE



Standard	**Students Can**	**Resources**
HS.F-LE.4* Use logarithms to express the solution to $ab^{ct} = d$ where a, c, and d are real numbers and b is a positive real number. Evaluate the logarithm using technology when appropriate.	• express the solution to an exponential function as a logarithm • use properties of logarithms to simplify logarithmic expressions • evaluate logarithms using technology	Basketball Rebounds (exponential /logarithms utilized)
Vocabulary • natural logarithm • common logarithm	**Annotations** Example: $3e^{2t} = 317$ $e^{2t} = \frac{317}{3}$ $\ln e^{3t} = \ln\left(\frac{317}{3}\right)$ $3t = \ln\left(\frac{317}{3}\right)$ Using a calculator and rounding t to the nearest hundredth: $t \approx 1.55$.	**Notes**

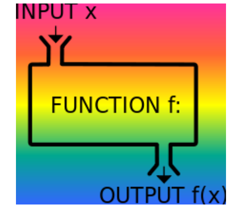
Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Functions**
Domain: **Trigonometric Functions**
Cluster**: Extend the domain of trigonometric functions using the unit circle**

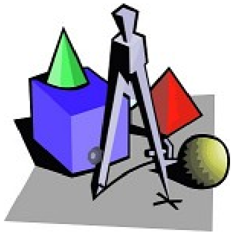
HS.F-TF



Standard	**Students Can**	**Resources**
HS.F-TF.2 Extend right triangle trigonometry to the four quadrants.	• use a reference triangle to extend the domain of the trigonometric functions beyond quadrant I and interpret the positive angles as counterclockwise rotations in radian form and negative angles as clockwise rotations in radian form	
Vocabulary • reference triangle • radian	**Annotations** Example: Find sin 210°	**Notes**

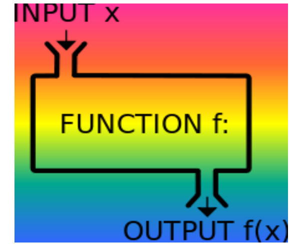
Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



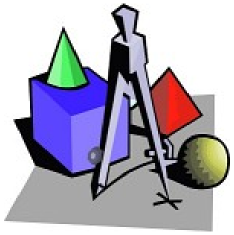
North Dakota *HIGH SCHOOL* State Standards: *Algebra II*
Conceptual Category: **Functions**
Domain: **Trigonometric Functions**
Cluster: **Extend the domain of trigonometric functions using the unit circle**

HS.F-TF



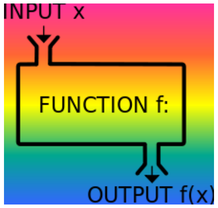
Standard HS.F-TF.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for π /3, π/4 and π/6.	**Students Can** • use special triangles (45°-45°-90° and 30°-60°-90°) to determine the values of sine, cosine and tangent for π/3, π/4 and π/6	**Resources**
Vocabulary • special triangles • reference triangle	**Annotations**	**Notes**

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Functions**
Domain: **Trigonometric Functions**
Cluster**: Prove and apply trigonometric identities**

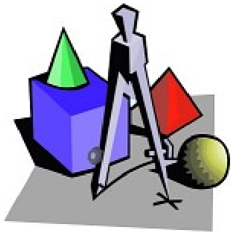
HS.F-TF



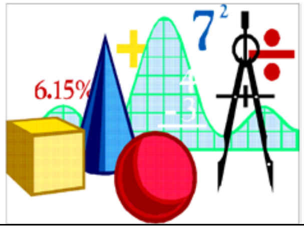
Standard HS.F-TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	**Students Can** • use the unit circle to prove the Pythagorean Identity $\sin^2(\theta) + \cos^2(\theta) = 1$ • use the Pythagorean Identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle	**Resources**
Vocabulary • Pythagorean Identity	**Annotations** Students might "prove" by providing a formal proof, demonstrating, or justifying. Example: Given θ is a Quadrant II angle and $\sin\theta = 4/5$, find $\cos\theta$ using the Pythagorean Identity.	**Notes**

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Geometry**
Domain: **Expressing Geometric Properties with Equations**
Cluster: **Understand and use conic sections**

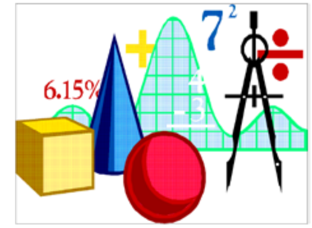


Standard	**Students Can**	**Resources**
HS.G-GPE.1 Derive the equation of a circle of given center and radius. Derive the equation of a parabola given a focus and directrix.	• use the Pythagorean Theorem to derive the equation of a circle, given the center and radius • derive the equation of a parabola given the focus and directrix	
Vocabulary	**Annotations**	**Notes**
• complete the square • center • radius • focus • directrix		

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*

North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Geometry**
Domain: **Expressing Geometric Properties with Equations**
Cluster: **Understand and use conic sections**

HS.G-GPE



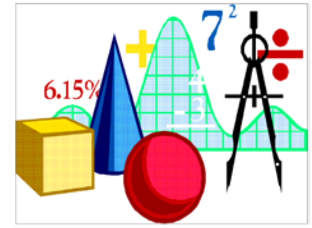
Standard HS.G-GPE.2 Convert between the standard form and general form of conic sections.	**Students Can** • complete the square to create an equivalent expression • convert between standard form and general form of conic section equations	**Resources**
Vocabulary	**Annotations** Conic sections include the circle, ellipse, parabola, and hyperbola.	**Notes**

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*

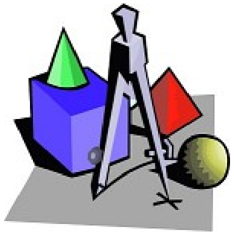
HS.G-GPE

North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Geometry**
Domain: **Expressing Geometric Properties with Equations**
Cluster: **Understand and use conic sections**



Standard	**Students Can**	**Resources**
HS.G-GPE.3 Identify key features of conic sections given their equations. Apply properties of conic sections in real-world situations.*	• complete the square to create an equivalent expression • convert between standard form and general form of conic sections	
Vocabulary • center • radius • vertex • focus • directrix • major axis • minor axis • asymptotes	**Annotations** Conic sections include the circle, ellipse, parabola, and hyperbola. Key features include: Circle – center, radius Parabola – vertex, focus, directrix Ellipse – center, foci, vertices, length of major and minor axis Hyperbola – center, foci, asymptotes	**Notes**

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



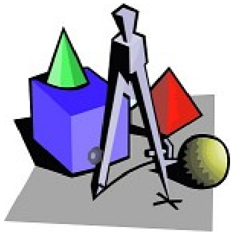
North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Statistics and Probability**
Domain: **Interpreting Categorical and Quantitative Data***
Cluster**: Summarize, represent, and interpret data on a single count or measurement variable**

HS.S-ID

Standard	**Students Can**	**Resources**
HS.S-ID.4* Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	• identify whether data sets are approximately normal • use the mean and standard deviation to fit the data set to a normal distribution where appropriate • use calculators, spreadsheets, and tables to estimate areas under the normal curve • interpret areas under a normal curve in context	Resource: Progressions for the Common Core State Standards HS Statistics & Probability
Vocabulary • normal distribution	**Annotations** Example: An example of a data set that does not fit to a normal distribution is age at retirement. Most people retire in their mid 60s or older, with increasingly fewer retiring at increasingly earlier ages. This results in a skewed-left distribution.	**Notes**

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*

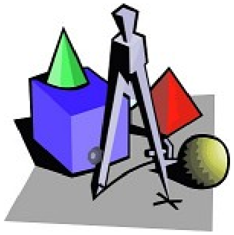


North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Statistics and Probability**
Domain: **Interpreting Categorical and Quantitative Data***
Cluster: **Summarize, represent, and interpret data on two categorical and quantitative variables**

HS.S-ID

Standard	**Students Can**	**Resources**
HS.S-ID.6* Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data (with or without technology). Use functions fitted to data to solve problems in the context of the data. b. (+) Informally assess the fit of a function by plotting and analyzing residuals.	• create a scatter plot from two quantitative variables • describe the form, strength and direction of the relationship • categorize data as linear, exponential, quadratic or neither • use algebraic methods or technology to fit the data to a linear, exponential or quadratic function • use the function to predict values • explain the meaning of slope and y-intercept (linear model) or the meaning of the growth rate and y-intercept (exponential model) or the meaning of the coefficients (quadratic model) in context • calculate a residual • create and analyze a residual plot	Interpreting Statistics: A Case of Muddying the Waters Resource: Progressions for the Common Core State Standards HS Statistics & Probability
Vocabulary	**Annotations**	**Notes**
• scatter plot • residual: the observed value minus the predicted value. It is the difference of the results obtained by observation, and by computation from a formula. • residual plot	Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.	

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Statistics and Probability**
Domain: **Making Inferences and Justifying Conclusions***
Cluster**: Understand and evaluate random processes underlying statistical experiments**

HS.S-IC

Standard HS.S-IC.1* Understand the process of making inferences about population parameters based on a random sample from that population.	**Students Can** • explain how to make inferences about a population based on a sample of that population	**Resources** Interpreting Statistics: A Case of Muddying the Waters Resource: Progressions for the Common Core State Standards HS Statistics & Probability
Vocabulary • sample • population	**Annotations** Example: Suppose 50 fish are tagged in a pond. A fisherman catches 5 fish from the pond and one has a tag. What conclusion can you draw about the fish population?	**Notes**

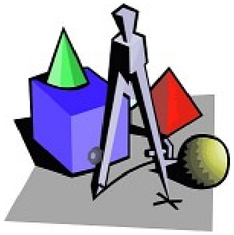
Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*

North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Statistics and Probability**
Domain: **Making Inferences and Justifying Conclusions***
Cluster**: Understand and evaluate random processes underlying statistical experiments**

HS.S-IC

Standard	**Students Can**	**Resources**
HS.S-IC.2* Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.	• demonstrate understanding of the different kinds of sampling methods • design simulations of random sampling: assign digits in appropriate proportions for events, carry out the simulation using random number generators and random number tables and explain the outcomes in context of the population and the known proportions	Interpreting Statistics: A Case of Muddying the Waters The Hermit's Epidemic Resource: Progressions for the Common Core State Standards HS Statistics & Probability
Vocabulary	**Annotations** Example: A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?	**Notes**

Note:
(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



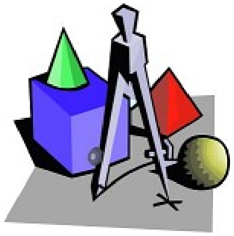
North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Statistics and Probability**
Domain: **Making Inferences and Justifying Conclusions***
Cluster**: Make inferences and justify conclusions from sample surveys, experiments, and observational studies**

HS.S-IC

Standard	**Students Can**	**Resources**
HS.S-IC.3* Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	• identify situations as sample survey, experiment, or observational study and discuss the appropriateness of each one's use in contexts with limiting factors • design or evaluate sample surveys, experiments and observational studies with randomization and discuss the importance of randomization in these processes	Interpreting Statistics: A Case of Muddying the Waters Resource: Progressions for the Common Core State Standards HS Statistics & Probability
Vocabulary • sample survey • experiment • observational study	**Annotations** Example: Design a simple study and explain the impact of sampling methods, bias and the phrasing of questions asked during data collection.	**Notes**

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*



North Dakota *HIGH SCHOOL* State Standards: *ALGEBRA II*
Conceptual Category: **Statistics and Probability**
Domain: **Making Inferences and Justifying Conclusions***
Cluster**: Make inferences and justify conclusions from sample surveys, experiments, and observational studies**

HS.S-IC

Standard	**Students Can**	**Resources**
HS.S-IC.6* Evaluate reports based on data. a. Evaluate articles, reports or websites based on data published in the media by identifying the source of the data, the design of the study, and the way the data are analyzed and displayed. b. Identify and explain misleading use of data; recognize when claims based on data confuse correlation and causation. c. Recognize and describe how graphs and data can be distorted to support different points of view.	• evaluate reports based on data • evaluate articles, reports or websites • identify and explain the misleading use of data • recognize when claims based on data confuse correlation and causation • recognize and describe how graphs can be distorted	Interpreting Statistics: A Case of Muddying the Waters Resource: Progressions for the Common Core State Standards HS Statistics & Probability
Vocabulary	**Annotations**	**Notes**

Note:

(+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics*
* *indicates modeling standards*