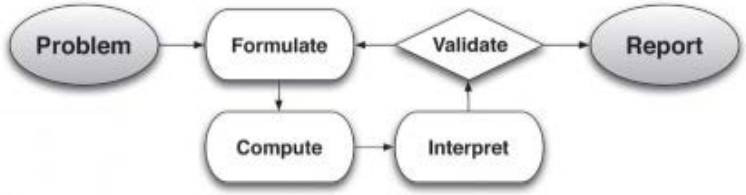
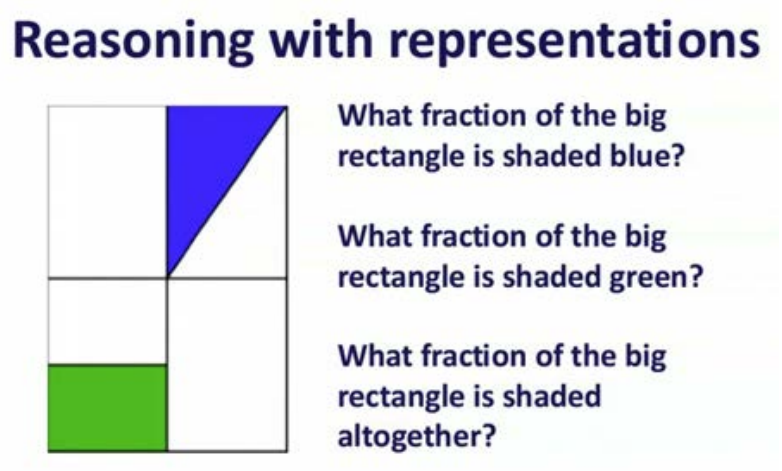
Level Descriptor	Supporting Research
Formalizing Mathematics	...the idea is not to motivate students with everyday-life contexts but to look for contexts that are experientially real for the students and can be used as starting points for progressive mathematization. (Gravemeijer, 1999, p.158) ...formal mathematics is not something "out there" with which the student has to connect. Instead, formal mathematics is seen as something that grows out of the students' activity. The students are expected to develop formal mathematics by way of mathematizing their own informal mathematical activities (Gravemeijer, 1999, p.160) Sometimes, the informal ways by which students solve applied problems "anticipate" the more formal procedures we are aiming for (Streefland, 1999 in Gravemeijer, 1999, p.158). I believe, along with curriculum developers (e.g., Clarke, Breed, and Fraser 2004; Hirsch et al.1995) and prominent mathematics educators (e.g., Freudenthal 1991), that the real world can be a source of rich contexts in which students engage in mathematical processes and from which mathematical ideas can grow. (Otten, 2011)
Modeling Mathematics	***Modeling*** is different from application in that modeling involves standing outside mathematics and looking into mathematics to find things that conceivably might help to resolve the ***driving question*** whereas ***applications*** come from standing inside mathematics and noting that particular pieces of mathematics can be used (i.e., applied) to better understand or highlight objects outside of mathematics (Dossey, 2010, p.88). ...But this analysis suggests that textbooks offer students limited opportunities to model, tending to complete the first two actions for students (identifying variables and formulating models) while ignoring the last action (validating conclusions). Teachers must make up the difference, offering opportunities for students to engage in those five actions and also to implement tasks that occasionally require all five in concert. (See, for example, Three Act Tasks [threeacts.mrmeyer.com] and 101 Questions [101qs.com]) (Meyer, 2015, p. 583) Basic modeling cycle according to CCSSM:

	 (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle. (CCSSI, 2010, p.72)
Applying Mathematics	***Applications*** come from standing inside mathematics and noting that particular pieces of mathematics can be used (i.e., applied) to better understand or highlight objects outside of mathematics (Dossey, 2010, p.88). Most commonly, problems involving contexts are presented as direct applications of mathematical techniques. In these cases the students merely need to follow the procedures developed in recent lessons (Llinares & Roig, 2008) rather than have to grapple with the realities of the context (Harvey & Averill, 2012, p. 42)
Pseudocontextualizing Mathematics	Jo Boaler writes about the idea of "pseudocontext." She states that "Instead of giving students realistic situations that they could analyze, textbook authors began to fill books with make-believe contexts - situations that students were meant to believe but for which they should not use any of their real-world knowledge." According to Boaler, pseudocontexts "should only be used when they are realistic and when the contexts offer something to the students, such as increasing their interest or modeling a mathematical concept. A realistic use of context is one where students are given real situations that need mathematical analysis, for which they do need to

	consider (rather than ignore) the variables" (Boaler, 2008, p. 53). Dan Meyer's (2013) Pseudocontext Saturdays: Introduction  It's interesting because these problems — speaking of context — aren't really contextualized. We're not making them into fake cakes or breads or anything. They really are just shaded rectangles. And the kids are finding them very challenging and interesting. So I do think, on the question of context, it's worth remembering that mathematics itself is a context and that puzzle-like problems are often both very engaging for kids and good equalizers because kids looking at those diagrams aren't shaped by some of those same inequities about kids' experiences. There are certainly differences in their experiences but they're not the same as problems about how cakes get shared or other kinds of real-world things. (Ball, in Dan Meyer, 2010)