Source Title: Modelling and applications in mathematics education : the 14th ICMI study /

Vol: Issue:

Year/Month: 2007/ Pages: 137-144+title page/verso

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Article/Chapter Title: Emergent modelling as a precursor to mathematical modelling.

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Print session: **4/15/2015 1:01:01 PM**

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Modelling and Applications in Mathematics Education

The 14th ICMI Study

Springer

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Library of Congress Control Number: 2006932713

ISBN -13: 978-0-387-29820-7
ISBN -10: 0-387-29820-7

e-ISBN-13: 978-0-387-29822-1
e-ISBN-10: 0-387-29822-3

Printed on acid-free paper.

9 8 7 6 5 4 3 2 1

springer.com

Chapter 3.1.2

EMERGENT MODELLING AS A PRECURSOR TO MATHEMATICAL MODELLING

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Abstract: This chapter discusses the relation between 'emergent modelling' and 'mathematical modelling'. The former that has its roots in RME theory constitutes the main theme of this chapter. It is argued that mathematical modelling requires a preceding learning process, since it requires abstract mathematical knowledge to construe a mathematical model. The emergent-modelling design heuristic offers a means for shaping a series of modelling tasks that may foster the development of that abstract mathematical knowledge. The emergent-modelling heuristic is illustrated with an instructional sequence on data analysis.

1. INTRODUCTION

Students often seem to have difficulties with applying the mathematics they have learned. This problem may be described in various ways. One may describe it, for instance, in terms of mathematical modelling: The problem solver has to translate the given contextual problem into a mathematical problem to make it assessable for mathematical tools and procedures. In doing so, he or she construes a 'mathematical model' of the situation. In primary-school mathematics, solving word problems, offers a typical example of this type of modelling (Verschaffel, Greer, & De Corte, 2002). This modelling process can also be described as 'abstraction'. It may be useful to note, however, that abstraction, or abstracting, may refer to two very different situations, (a) situations that concern the activity of solving a given problem, and (b) situations that concern the long-term process of developing more abstract mathematical knowledge. In the former case students have to put more formal, abstract knowledge to use by making connections between the problem situation and that abstract knowledge. Here one often speaks of *'reduction'*, or, 'cutting bonds with everyday-life reality'. In the latter case, that of the long-term process, however, the central activity is that of *'construction'*. We may link the latter to the notion of 'emergent modelling' – which will be the topic of this contribution.

2. EMERGENT MODELLING

In contrast with the observed problems of students with mathematical modelling, there are also many reports that students are very inventive and successful when asked to solve novel, engaging, contextual problems. We may mention in this respect, the work of Lesh (Lesh & Harel, 2003) on model-eliciting activities, where the activity of the students is not so much that of applying mathematical ideas but of developing new mathematical ideas. The emergent modelling approach taps into the same potential, but with a focus on long-term learning processes, in which a model develops from an informal, situated model into a more sophisticated model. These emergent models are seen as originating from activity in, and reasoning about situations. From this perspective, the process of constructing models is one of progressively reorganizing situations. The model and the situation being modeled co-evolve and are mutually constituted in the course of modelling activity.

Although emergent modelling is an activity of the students, the term emergent modelling has it roots in the description of an instructional design heuristic within the domain-specific instruction theory for realistic mathematics education (RME). The 'emergent-modelling' design heuristic (Gravemeijer, 1999) was initially developed as an alternative for the common use of what we may call 'didactical models', manipulative materials and visual models that are meant to make abstract mathematics more accessible for the students. Especially at the primary and lower secondary level, manipulative materials and visual models are typically used as embodiments of mathematical concepts and objects in mathematics education. The problem with this kind of models, however, is that external representations do not come with intrinsic meaning. From a constructivist perspective, it may be argued that the meaning of external representations is dependent on the knowledge and understanding of the interpreter. This implies that in order to interpret these models correctly, students should already have at their disposal, the knowledge and understanding that is to be conveyed by the concrete models (Cobb, Yackel, & Wood, 1992).

The emergent-modelling design heuristic tries to circumvent this dilemma, by aiming at a dynamic process of symbolizing and modelling, within which the process of symbolizing and the development of meaning are reflexively related. The idea is that students start with modelling their own informal mathematical activity. Then, in the process that follows, the character of the model should change for the students. The model of their informal mathematical activity is expected to gradually develop into a model for more formal mathematical reasoning. In its latter form, the model may function in a manner as was intended for the didactical models, but now as a model that is rooted in the experiential knowledge of the students.

Mark that the model we are referring to is more an overarching concept than one specific model. In practice, 'the model' in the emergent-modelling heuristic is actually shaped as a series of consecutive *sub-models* that can be described as a cascade of inscriptions or a chain of signification. From a more global perspective, these sub-models can be seen as various manifestations of the same model. So when we speak of a shift in the role of the model in the following, we are talking about *'the model'* on a more general level. On a more detailed level, this transition may encompass various sub-models that gradually take on different roles.

The label 'emergent' refers both to the character of the process by which models emerge within RME, and to the process by which these models support the emergence of formal mathematical ways of knowing. According to the emergent-modelling design heuristic, the model first comes to the fore as a *model of* the students' situated informal strategies. Then, over time the model gradually takes on a life of its own. The model becomes an entity in its own right and starts to serve as a *model for* more formal, yet personally meaningful, mathematical reasoning.

In relation to this, we can discern four different types or levels of activity (Gravemeijer, 1999):

1. *activity in the task setting*, in which interpretations and solutions depend on understanding of how to act in the setting
2. *referential activity*, in which models-of refer to activity in the setting described in instructional activities
3. *general activity*, in which models-for derive their meaning from a framework of mathematical relations
4. *formal mathematical reasoning*, which is no longer dependent on the support of models-for mathematical activity.

These four levels of activity illustrate that models are initially tied to activity in specific settings and involve situation-specific imagery; at the referential level, models are grounded in students' understandings of paradigmatic, experientially real settings. General activity begins to emerge as the students start to reason about the mathematical relations that are involved. As a consequence, the model looses its dependency on situation-specific imagery, and gradually develops into a model that derives its meaning from the framework of mathematical relations that the students construe in the process. The transition from model-of to model-for coincides with a progression from informal to more formal mathematical reasoning that is interwoven with the creation of some new mathematical reality – consisting of mathematical objects (Sfard, 1991) within a framework of mathematical relations. Thus, the model-of/model-for transition is not tied to specific manifestations of the model, instead, it relates to the student's thinking, within which 'model-of' refers to an activity in a specific setting or context, and 'model for' to a framework of mathematical relations.[1]

3. DATA ANALYSIS AS AN EXAMPLE

The emergent-modelling heuristic is elaborated in various research projects on a variety of topics. We will take one of those research projects to illustrate the emergent modelling with a concrete example. This example concerns a teaching experiment on data analysis, carried out by Cobb, Gravemeijer, McClain and Konold in a 7th-grade classroom in Nashville (USA) (see Cobb, 2002). Our point of departure was, that although user-friendly data analysis software packages may seem to be the self-evident accessories for exploratory data analysis, this is only true for experienced data analysts, and not for students who still have to learn about data analysis. In order to be able to use such software in a proficient manner, one has to be able to anticipate what kind information one might be able to deduct from a certain way of representing the data. Working with such data analysis software packages therefore rather signifies an end point of the intended learning process, than a means of supporting it. We therefore turned to designing software tools that can be used for exploratory data analysis on an elementary level. In fact, these so-called 'minitools' are so designed, that they can support a process of progressive mathematization by which conventional statistical concepts and representations are reinvented. What is especially aimed for, is that the activity of structuring data sets with the minitools will foster a process by which the students come to view data sets as entities that are distributed within a space of possible values.

The visualizations offered by the minitools can be seen as manifestations of the same overarching model, which we may be describe as *a graphical representation of the distribution of the data values.*

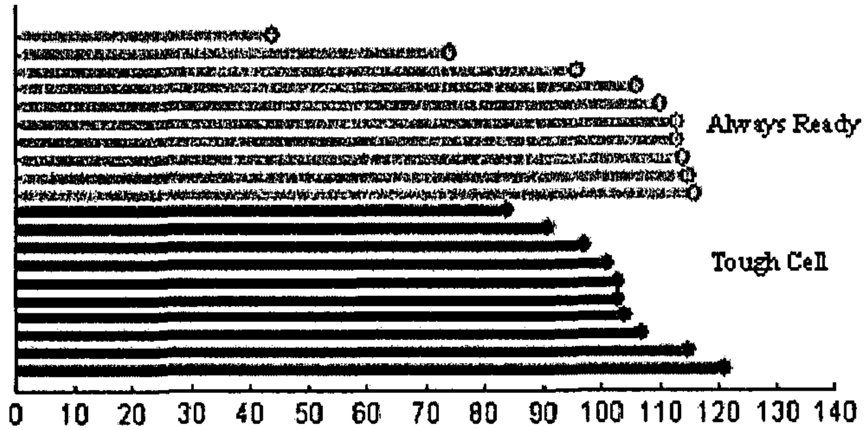

Figure 3.1.2-1. The life span of two brands of batteries.

The starting point is in visualizing the *measures*, or magnitudes, that constitute the data set. With minitool 1, *magnitude-value bars* (Fig. III.1.2-1) are introduced, where each value bar signifies a single measure. This tool has various tool options that can be used when analyzing data sets, such as a vertical value bar to mark certain values, or to split the data set, and various options for sorting the data.

One of the first tasks concerns the comparison to the life spans of two brands of batteries, Though Cell and Always Ready. The lif-span measures of ten batteries of each brand are presented as value bars in the minitool (Fig. 3.1.2-1). When confronted with this problem, the 7th-grade students introduced the term 'consistency' to argue that they 'would rather have a consistent battery (...) than one that you just have to try to guess'.We may interpret this argument as referring to the shape of the distribution, which is visible in the way the endpoints of the value bars are distributed in regard to the axis. In relation to this, we may speak of a graphical representation of the distribution as a *model of* a set of measures.

In the discussions on distributions represented by value bars, the students started to focus on the end points of value bars. As a consequence, these end points came to signify the lengths of the corresponding value bars for them. This allowed for the introduction of a line plot as a more condense (local) model, that leaves out the value bars, and only keeps the end points (Fig. 3.1.2-2 next page).

In Minitool 2 various tool options are made available to help the student structure the distribution of data points on a line plot. One of the tool options partitions a set of data points into four quartiles. The corresponding inscription is in principle similar to the conventional box plot (see Fig. 3.1.2-3 next page).

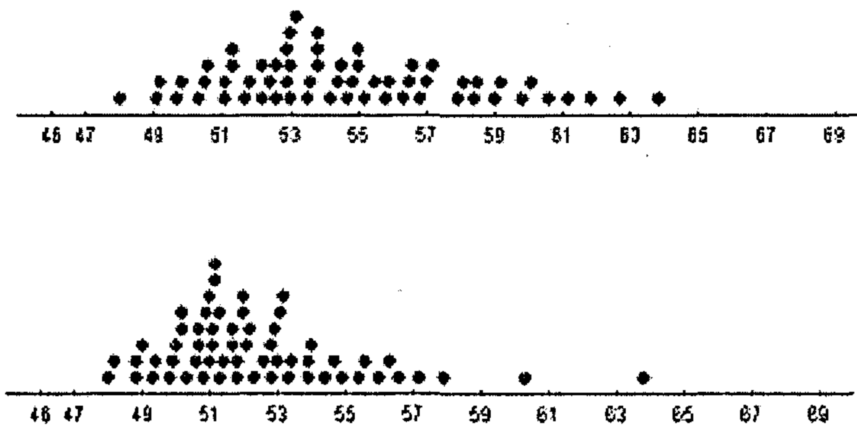

Figure 3.1.2-2. Data on the speeds of cars before and after a speed trap

While working with the second minitool, the students started to use the term 'hill' to denote the shape of the distribution. They did so for the first time when they discussed the effect of a speed trap on the basis of data on the speeds of cars before and after the speed trap (see Fig. 3.1.2-2). One of the students used the following argumentation: 'If you look at the graphs and look at them like hills, then for the before group the speeds are spread out and more than 55, and if you look at the after graph, then more people are bunched up close to the speed limit which means that the majority of the people slowed down close to the speed limit.'

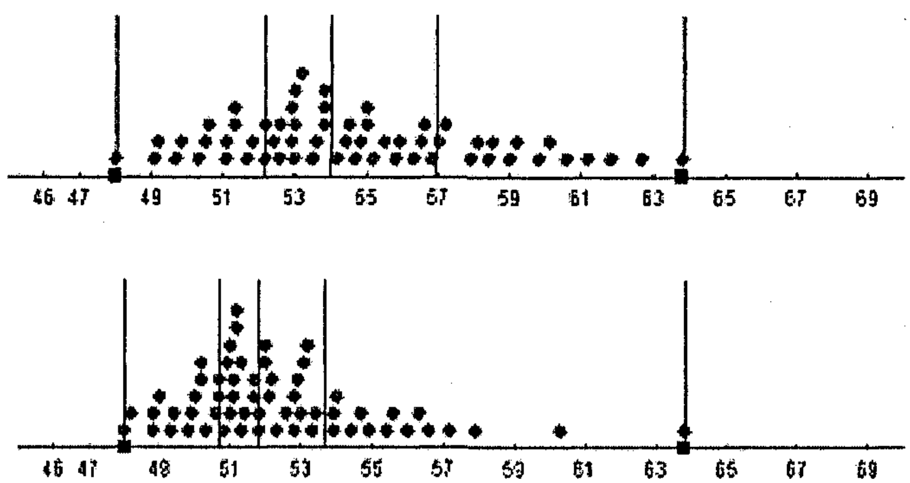

Figure 3.1.2-3. Four equal groups as precursor for the box plot.

Eventually the students started to use the four-equal-groups display of the second mintool to reason about shape and density. The distance between two vertical bars that mark a quartile were interpreted as indicating how much the data are 'bunched up'. Moreover, the median started to function as an indicator of 'where the hill is', for unimodal distributions. Finally, the students started to treat distributions as entities with certain characteristics. In this regard, we may describe the four-equal groups display as a graphical representation of the distribution that started to function as a *model for* reasoning about distributions.

In the sequence, the model initially comes to the fore as a *model of* a set of measures. At first, the density-function aspect is rather implicit, although the shape of a sorted magnitude-value-bar graph of minitool 1 can be interpreted as signifying variation in density. Gradually, however, density comes more to the foreground, and in this manner, the model can become a *model for* reasoning about various types of distributions. Not only does the distribution become an entity with certain characteristics, but the students also begin to see relations between these characteristics. The normal distribution can be taken as a typical example; the students may learn eventually that a normal distribution is symmetrical, and that as a consequence, mean, median, mode, and midrange coincide.

4. CONCLUSION

We started this chapter with the observation that students experience difficulties when they are expected to apply the mathematics they know, but are good at tackling applied problems, if they feel challenged to invent novel solutions. We believe that we can resolve this paradox by using emergent modelling to shape mathematics education that prepares students for mathematical modelling. The emergent-modelling instructional design heuristic is based on the idea of sequencing modelling tasks in order to support a long term process of 'abstraction-as-construction', within which students construe mathematical knowledge that is grounded in their earlier informal experience, and which is meaningful, and applicable. In addition, the implied modelling activity familiarizes them with a mathematical approach to everyday-life situations. In this sense, modelling serves both as an instructional goal and as a means of helping students reinvent mathematics, and preparing them for 'applications' and 'modelling'.

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[1] Within the context of the emergent modelling heuristic, the model-of/model-for terminology is only used when this transition is linked to the constitution of a framework of mathematical relations.