

Review

Mathematical Modeling on the Catwalk: A Review of *Modelling and Applications in Mathematics Education: The 14th ICMI Study*

Modelling and Applications in Mathematics Education: The 14th ICMI Study.
Werner Blum, Peter Galbraith, Hans-Wolfgang Henn, and Mogens Niss (Eds)
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A frequently heard query concerns why students receive relatively high grades in their mathematical school work and yet fail to be able to apply their mathematical knowledge and skills in contextual settings that they encounter in their other studies and their daily lives, not to mention meeting the mathematical challenges presented in subsequent mathematical courses. Many reform documents call for students to have opportunities to apply their mathematical knowledge in modeling settings and to reason and make sense of quantitative, spatial, or data-intensive situations (NCTM, 2000, 2009). The acquisition of such modeling expertise was a central focus of the 2004 ICMI conference at the University of Dortmund and this ICMI volume that documents the plenary presentations and issue papers related to themes discussed at or emerging from that meeting. What are the barriers that stand in the way of developing the adaptive expertise called for by modeling challenges?

Mogens Niss, Werner Blum, and Peter Galbraith introduce the discussion with a look at what constitutes modeling and what differentiates models from modeling and modeling from applications. They posit that models are, in and of themselves, a byproduct of the process of modeling. As modeling takes place, the modeler creates new conceptual tools and structures that later provide frames of reference for those wrestling with similar situations. Modeling, on the other hand, is the recursive process of cycling between the contextualized extra-mathematical setting and mathematics, trying to link bits of the problem situation with known mathematics and then working to link these pieces together to resolve the question at hand. Models are products; modeling is a process.

Further, modeling involves standing outside mathematics and looking into mathematics to find things that conceivably might help resolve the driving question. Applications, on the other hand, come from standing inside mathematics and noting that particular pieces of mathematics can be used to better understand or highlight objects outside of mathematics. Using these distinctions, they set a

framework for considering the plenary papers, issue-specific papers, education-level consideration, and a group of case studies on modeling activities in the classroom.

PLENARY PAPERS

The second portion of the report of the ICMI study contains plenary papers addressing the major focal points of the study. Claudi Alsina introduces the seven plenary papers by addressing some of the barriers slowing the adoption of modeling activities into the curriculum and the classroom. Central to these barriers was the discussion of the use of real objects as a starting point for examination and understanding, real places as a source of motivation and authentic contexts, informed and experienced teachers as guides, and the need for additional time to adequately engage in modeling activities. Morten Blomhøj and Tomas Jensen provide a basis for beginning to think about the cognitive side of the modeling process in terms of competencies, building on the work of Niss and Jensen (2002). Jere Confrey and Alan Maloney examine the role of technology as a supporting and expanding factor in student learning in modeling settings. Helen Doerr addresses a very central issue, that of teacher knowledge and competence concerning modeling. This issue is perhaps one of two central themes to the volume, the other being bringing modeling to a more central role in countries' curricula. Doerr builds on the theme that what one knows is directly linked to how one learned it oneself. She illustrates this point with four students' experiences in confronting a problem and their interaction with their teacher. Doerr notes the substantial demands that modeling places on a teacher's capacity to handle diverse comments and reflect on their own knowledge of both mathematics and pedagogy.

Peter Galbraith's plenary paper focuses on issues associated with moving the discussion and integration of modeling beyond application word problems and fitting functions to data on a hand calculator. Galbraith raises the bar for mathematics educators by questioning why we are not using modeling in university courses, for diagnosing students' understanding of modeling, and for designing and driving high-stakes assessments at a level commensurate with standards for mathematics education. Brian Greer, Lieven Verschaffel, and Swapna Mukhopadhyay argue that modeling experiences provide children with a natural pathway to motivation and understanding the role that mathematics plays in life. Authentic targeted modeling problems could provide a central component of sense making and promote adaptive expertise growth in students. Such a move could counter the effects that research suggests mindless word problems add to the suspension of sense making in students. Mathematics educators know that application and modeling can be developed in children, but the road will require focus, the development of quality materials for educating both teachers and students, and an increased focus on mathematics accompanied by a reduction in the long lists of menial objectives students are expected to master each year.

Gabriele Kaiser and Katja Maaß provide an analysis of possibilities for modeling in the lower secondary school through the discussion of a research study involving



Outgrowth of modeling

modeling in a gymnasium in Germany. They discuss the roles of beliefs and attitudes of students and the interactions these have with modeling. A parallel study of teachers and their beliefs highlighted the strong role played by teachers' conceptions of mathematics and how it should be taught. These beliefs present one of the strongest barriers to effectively implementing modeling activities successfully in classrooms, especially at the secondary and tertiary levels. The final plenary paper is one by Henry Pollak. He comments as follows:

The feeling has been, well, what we need to do is to teach some pure mathematics and let the other people apply it. And the great difficulty with this is that you lose your students. You are asking them—and it is one of the many different ways in which we ask them to do something like this—we are asking them for delayed clarification. We do this over and over again. (Pollak, 2007, p. 111)

We lose students when we only teach them "pure" mathematics.

He goes on to liken the development of understanding of the modeling process and the application of models to the development of computer programs. One has to consider the creative development of processes and creative analytic capabilities that undergird program development, as well as learn the skills that allow for debugging them when they do not produce the desired results. The question is, how does one balance the emphasis on process goals with the mastery and application of known knowledge? Pollack provides examples of problem recognition, formulation, and solution with respect to modeling situations that he experienced working at Bell Labs. A special treat is an interview with Pollak, which is contained on a CD that is included with the book.

Modeling process and the application of models can be likened to the creative processes that undergird programming.

ISSUE PAPERS

The third portion of the ICMI study report, which encompasses two thirds of the volume, is devoted to the consideration of issue papers gathered under seven broad themes: epistemology and modeling, authenticity and goals, modeling competencies, applications and modeling for mathematics, modeling pedagogy, implementation and practice, and assessment and evaluation. Each of these sections opens with an introductory paper overviewing the remaining papers in the section. Although it is safe, from a reviewer's standpoint, to discuss each of the papers, this reviewer has decided to follow a more dangerous path and note only a few from each section to provide a flavor of the issues discussed in Dortmund or papers that received their genesis there.

The first of these is a reconsideration of what constitutes "modeling" in a mathematics class. The question of modeling for developing processes vital to mathematical literacy versus using extant models to show the power of mathematics is discussed at length. Papers in the first section illustrate the many nuances of this continuum. A number provide excellent exemplars of how modeling as a method of developing mathematics can even include justification and proof among their objectives. Koeno Gravemeijer's paper examines an emergent modeling sequence in which students move from engaging in activities in a task setting, to developing models-of for activities from instruction, to using models to foster deeper under-



standing of mathematical relationships, to formal mathematical reasoning. Focusing on these stages and using carefully designed experiences, students can be brought to a deeper level of understanding of mathematics while developing process skills that will serve them well in modeling and application settings. Gila Hanna and Hans Niels Jahnke illustrate how work with physical objects, thought experiments, and modeling serve as valuable adjuncts to developing proof-related processes.

Richard Lesh and Caroline Yoon speak to the issue with which we opened this review: students with high grades who yet have difficulty in modeling situations. They note that *mathematizing reality* is quite different from *realizing mathematics* and continue to discuss another group of misconceptions that need to be dismissed about the acts of modeling in order to bring it to prominence as a valued curriculum outcome. They note that:

... mathematics learners and problem solvers are model developers at least as much as they are information processors; and, because models are the tools that mathematicians use to interpret experience, powerful, sharable, and reusable models are among the most important cognitive objectives of mathematics instruction. (Lesh & Yoon, 2007, p. 164)

The second set of issue papers discusses authenticity and goals associated with modeling. How is it that mathematics educators can speak of modeling the real world when, in reality, the classroom activities are transformed and often truncated to fit the particular knowledge and skills possessed by the learners, and models are only considered to illustrate the applications of extant models? Eva Jablonka argues that authenticity exists when students and teachers are engaged in a modeling activity about an issue that is relevant to them or to their location. Some discuss the issue of how problems and models can be altered to fit the students' backgrounds and present knowledge levels, whereas others discuss using models as a mode of coming to know new knowledge.

The third set of issue papers focuses on modeling competencies. There was agreement that modeling as a process and curricular goal was too complex to be itself modeled through some staged model of development. Herbert Henning and Mike Keune present two models for describing modeling competence. The level-oriented model consists of three levels: recognizing and understanding modeling, independent modeling, and meta-reflection on modeling. They then continue to describe characteristic capabilities within each of these levels. Mihaela Singer argues that perhaps the most important outcome of learning in a modeling environment is itself the development of a structured set of cognitive competencies related to mathematics and the domains on which the modeling activities were focused. In doing so, the complexity involved in the modeling interacts with the grouping activities associated with abstracting, which is associated with concept, principle, and procedural development, which are associated with the mathematics program. The nature of such interactions is often disordinal and perhaps the source of difficulty in the growth of both or one of the two goals areas.

Dirk De Bock, Wim Van Dooren, and Dirk Janssens relate a particular example of this aspect in reporting a study of students' nonadaptive modeling behavior as they progress from elementary grades to early secondary levels. While they develop routine expertise with linear models during this period, they fail to develop the adaptive expertise that signals when such models are not appropriate. The focus on modeling in a supportive atmosphere may provide a means of negating the development of such inappropriate applications of learned concepts such as linear models when students have the opportunity, on a regular basis, to consider features focusing on the appropriateness of particular classes of models. They even posit that an early elementary introduction to the process of modeling might prevent the development of the forcing of routine behavior in inappropriate settings.

The interplay between modeling and applications framed the fourth issue. Eric Muller and Hugh Burkhardt provide a sound overview of the role of modeling in both generating competency in students and helping students to draw on their extant competencies in creating practice and powerful models addressing real problems. They argue that modeling in authentic contexts requires holistic approaches. However, the result strengthens the students' cognitive connections of mathematical concepts, principles, and procedures; engenders new competencies and understandings; and builds both motivation and positive beliefs toward mathematics.

Malcolm Swan, Ross Turner, and Caroline Yoon, with Eric Muller detail modeling experiences in which students have to construct a pop-up card with various features, analyze a horse race game, develop a tournament schedule under constraints, and structure a train schedule for three trains using one rail line. These modeling experiences are used to illustrate the mathematizing of reality and realization of how mathematics fits into solutions. As such, they develop cognitive competencies related to the connections between known mathematics and the structuring of realistic settings through simplification and elaboration. These examples are followed by others in which students are forced to discuss alternative approaches, work in multivariate environments, reflect, and change directions.

Soeren Antonius, Chris Haines, Tomas Højgaard, and Mogens Niss examine the varied classroom activities related to modeling: tasks, investigations, projects, dissertations, and lecture/demonstrations. Each is examined from the standpoint of time and resources needed. These analyses support the authors' contention that modeling can only be taught through an investigative, student-centered approach, in which the teacher is a consultant—not the leader. This paper carefully examines the varied roles of the teacher in terms of verbal prompts and feedback, getting students ready, observing their actions, and debriefing them afterward.

The fifth issue targets modeling pedagogy directly. Thomas Lingefjärd recounts the results of a 2003 survey about modeling given to relevant mathematics, education, and administrative staff at the 26 different tertiary institutions in Sweden. He reports that the results showed a lack of knowledge about mathematical modeling with a particular bent toward considering it not "real mathematics" because of its interdisciplinary aspects. Other mathematics faculty dismissed it because it

involved applications of technology or because it dealt with ill-defined settings. These themes were repeated in a number of other papers and provide prima facie evidence of one of the major barriers to providing preservice or in-service teachers with modeling experiences as part of their professional development. Where these attitudes exist, courses will either not develop or not flourish.

Djordje Kadijevich notes that moves to standardize technology-based mathematics courses and the idiosyncratic nature of most modeling courses act against the growth of modeling courses in upper secondary and tertiary settings. Kadijevich offers some standards for computer-based modeling and suggests some ways in which standards supportive of computer-based modeling might be developed and instituted.

The sixth issue focuses on implementation and practice of modeling and applications. Michèle Artaud elaborates Yves Chevallard's (1992) six moments in the mathematizing of a situation:

- First moment focuses on at least one of the situation's features.
- Second moment explores one feature and elaborates a technique related to this feature.
- Third moment involves the constitution of the technological-theoretical environment relative to the technique identified.
- Fourth moment concerns the technical work, which may involve a further elaboration of the technique and an extension of its mastery.
- Fifth moment works to institutionalize, through elaboration, the mathematized organization.
- Sixth moment reflects and evaluates the produced object in terms of its generality and value.

The practice, or method, outlined by these moments provides a map to the consideration of both partial models and entire models. The noting of these moments provides a basis for the development of pedagogical actions that may further the construction of effective modeling courses and learning situations.

Additional papers in this section on practice and method provide suggestions for dealing with imperfectly stated contextual situations, integrating probability and statistics into modeling situations, and modeling workplace problems. Each of these papers provides rich examples for methods of developing authentic problems for classroom modeling investigations.

The seventh, and final, issue section deals with the topic of assessment and evaluation. Given the interaction of high-stakes assessments, the push to either add more content or reduce current time allocations, and the growth of societal demands on teachers, how can those interested in modeling in the curriculum chart students' progress toward its goals and document contributions of modeling to students' capabilities? The paper by Søren Antonius on a model for project-based examinations for the upper secondary level in Denmark and that of Jerry Legé detailing a U.S. study illustrate methods of developing both kinds of evidence. Ross Turner and

Pauline Vos, respectively, detail methods employed by the PISA and TIMSS assessments to address modeling and application activities in large-scale assessments.

EDUCATIONAL LEVELS

The fourth portion of the ICMI study report examines the interaction of modeling with various levels of the educational process. The five papers in this section consider the elementary, middle-grade/lower secondary, upper secondary, tertiary, and teacher education levels. These papers document the increasing difficulty of introducing mathematical modeling content into the curriculum as the level within the education system increases. Although modeling fits well within the confines of the elementary program, as students' age increases, so do the curricular demands. Time becomes an issue, but, perhaps more important, so do the beliefs of those in positions of responsibility. The educational system and standards often outline a framework of fact-based content and procedures. This, in many cases, matches well with the persistent belief of many teachers that mathematics is a logic-based study in which modeling plays a minor role at best. This belief, with increasing years of exposure, becomes the belief of many students as well, making the initial institution of modeling in the upper levels a difficult, almost impossible, task.

Eight countries' case studies note the stress on intra-mathematical knowledge and the belief that one must master skills before they can be applied or used to create models. When one adds to this perception the impact of high-stakes assessments and the lack of consistent professional development experiences focused on modeling activities and pedagogy, the challenge appears insurmountable. However, these papers also provide ideas that might bear fruit for those involved in modeling and the development of modeling experiences for students and the curriculum. Particular attention is once again given to the fact that modeling can be used to develop competencies, not just practice them.

The picture at tertiary institutions is not much different from that at the lower levels. The Mathematical and Interdisciplinary Contests in Modeling provided by COMAP and the existence of applied mathematics groups have helped with the acceptance of modeling activities in some corners of mathematics offerings at this level, but the same prejudices mentioned previously still remain within many of the pure mathematics groups. There is research at this level, as well as at the earlier levels, that suggests that the use of modeling to introduce and study topics has positive effects. Further, a careful analysis of the majority of students in mathematics classes at this level shows that they are taking mathematics classes because they provide background in modeling and applications to their disciplines.

The final paper in the section makes the case for teacher education. Like other areas of tertiary level mathematics, there are special examples of solid implementation of modeling experiences, but they are rare. Such examples will have to become the norm if substantial progress is to be made toward the curricular goals of modeling and toward reaping the rewards that such activities might hold for students.

CASES IN APPLICATIONS AND MODELING

The final section of the ICMI study report provides four case studies on the implementation of modeling activities. The first deals with a study in which students in Texas were engaged in studying the topic of equity in testing. The second discusses the experiences associated with the increased focus on modeling and applications in the provincial guidelines for mathematics in Ontario. The third paper details the implementation experiences in the Australian state of Victoria. The findings of this evaluative study reflect the evolution and modification of goals that emerge in such curricular initiatives. Both of these papers provide suggestions and potential pitfalls for those working through the implementation of modeling in the curriculum and present potentially useful policy “levers.” The final paper details the incorporation of mathematical modeling and applications in the last 3 years of the secondary school curriculum. It provides a look into a classroom in which students are engaged in working on a project dealing with providing safe water supply for villages.

The student comments from this classroom reveal the “learned helplessness” that many students proffer when challenged to confront a real-world context and develop a model to deal with it. These and the other issues of beliefs of mathematics teachers about modeling and the paucity of their own experiences with it remain substantive challenges to all urging that more modeling be included in mathematics classrooms around the world. The ICMI study report provides a solid basis for continued discussion and planning, action, and modes for evaluating the continued efforts to develop student and teacher competencies in this important area.

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