Levels of Demands

Lower-level demands (memorization):

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced

Lower-level demands (procedures without connections):

- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used
- Are focused on producing correct answers instead of on developing mathematical understanding
- Require no explanations or explanations that focus solely on describing the procedure that was used

Higher-level demands (procedures with connections):

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Higher-level demands (doing mathematics):

- Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships
- Demand self-monitoring or self-regulation of one's own cognitive processes
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

These characteristics are derived from the work of Doyle on academic tasks (1988) and Resnick on high-level-thinking skills (1987), the *Professional Standards for Teaching Mathematics* (NCTM 1991), and the examination and categorization of hundreds of tasks used in QUASAR classrooms (Stein, Grover, and Henningsen 1996; Stein, Lane, and Silver 1996).

Fig. 2 Characteristics of mathematical instructional tasks

Lower-Level Demands

Memorization

What is the rule for multiplying fractions?

Expected student response:

You multiply the numerator times the numerator and the denominator times the denominator.

or

You multiply the two top numbers and then the two bottom numbers.

Procedures without Connections

Multiply:

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{5}{6} \times \frac{7}{8}$$

$$\frac{4}{9} \times \frac{3}{5}$$

Expected student response:

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12}$$

$$\frac{5}{6} \times \frac{7}{8} = \frac{5 \times 7}{6 \times 8} = \frac{35}{48}$$

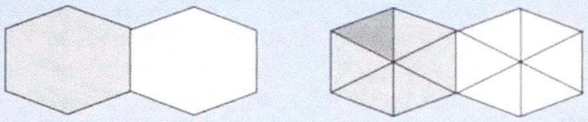
$$\frac{4}{9} \times \frac{3}{5} = \frac{4 \times 3}{9 \times 5} = \frac{12}{45}$$

Higher-Level Demands

Procedures with Connections

Find 1/6 of 1/2. Use pattern blocks. Draw your answer and explain your solution.

Expected student response:



First you take half of the whole, which would be one hexagon. Then you take one-sixth of that half. So I divided the hexagon into six pieces, which would be six triangles. I only needed one-sixth, so that would be one triangle. Then I needed to figure out what part of the two hexagons one triangle was, and it was 1 out of 12. So 1/6 of 1/2 is 1/12.

Doing Mathematics

Create a real-world situation for the following problem:

$$\frac{2}{3} \times \frac{3}{4}.$$

Solve the problem you have created without using the rule, and explain your solution.

One possible student response:

For lunch Mom gave me three-fourths of a pizza that we ordered. I could only finish two-thirds of what she gave me. How much of the whole pizza did I eat?

I drew a rectangle to show the whole pizza. Then I cut it into fourths and shaded three of them to show the part Mom gave me. Since I only ate two-thirds of what she gave me, that would be only two of the shaded sections.

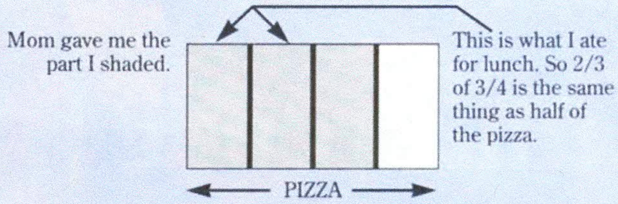


Fig. 3 Examples of tasks at each of the four levels of cognitive demand

References

Bennett, Albert B., and Linda Foreman. *Visual Mathematics Course Guide: Integrated Math Topics and Teaching Strategies for Developing Insights and Concepts*, vol. 1. Salem, Ore.: Math Learning Center, 1989/1991.

Doyle, Walter. "Work in Mathematics Classes: The Context of Students' Thinking during Instruction." *Educational Psychologist* 23 (February 1988): 167–80.

National Council of Teachers of Mathematics (NCTM). *Professional Standards for Teaching Mathematics*. Reston, Va.: NCTM, 1991.

Lower-Level Demands

Memorization

What are the decimal and percent equivalents for the fractions $\frac{1}{2}$ and $\frac{1}{4}$?

Expected Student Response:

$\frac{1}{2} = .5 = 50\%$

$\frac{1}{4} = .25 = 25\%$

Procedures without connections

Convert the fraction $\frac{3}{8}$ to a decimal and a percent.

Expected Student Response:

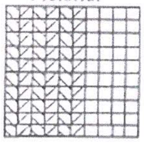
Fraction	Decimal	Percent
$\frac{3}{8}$	$8 \overline{)3.000}$ = .375 (24, 60, 56, 40, 40)	.375 = 37.5%

Higher-Level Demands

Procedures With Connections

Using a 10 x 10 grid, identify the decimal and percent equivalents of $\frac{3}{5}$.

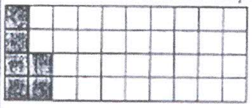
Expected Student Response:

Pictorial	Fraction	Decimal	Percent
	$\frac{60}{100} = \frac{3}{5}$	$\frac{60}{100} = .60$	$.60 = 60\%$

Doing Mathematics

Shade 6 small squares in a 4 x 10 rectangle. Using the rectangle, explain how to determine each of the following: a) the percent of area that is shaded, b) the decimal part of area that is shaded, and c) the fractional part of area that is shaded.

One Possible Student Response:



a) One column will be 10% since there are 10 columns. So four squares is 10%. Then 2 squares is half a column and half of 10% which is 5%. So the 6 shaded blocks equal 10% plus 5% or 15%.

b) One column will be .10 since there are 10 columns. The second column has only 2 squares shaded so that would be one half of .10 which is .05. So the 6 shaded blocks equal .1 plus .05 which equals .15.

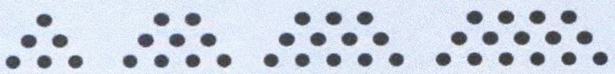
c) Six shaded squares out of 40 squares is $\frac{6}{40}$ which reduces to $\frac{3}{20}$.

FIGURE 1.1. Lower-level vs. higher-level approaches to the task of determining the relationships among different representations of fractional quantities (Stein & Smith, 1998). (Reprinted with permission from *Mathematics Teaching in the Middle School,* copyright 1998 by the National Council of Teachers of Mathematics. All rights reserved.)

TASK A

Manipulatives/Tools: Counters

For homework Mark's teacher asked him to look at the pattern below and draw the figure that should come next.



Mark does not know how to find the next figure.

A. Draw the next figure for Mark.

B. Write a description for Mark telling him how you knew which figure comes next.

(QUASAR Project—QUASAR Cognitive Assessment Instrument—Release Task)

TASK B

Manipulatives/Tools: None

Part A: After the first two games of the season, the best player on the girls' basketball team had made 12 out of 20 free throws. The best player on the boys' basketball team had made 14 out of 25 free throws. Which player had made the greater percent of free throws?

Part B: The "better" player had to sit out the third game because of an injury. How many baskets, out of an additional 10 free-throw "tries," would the other player need to make to take the lead in terms of greatest percentage of free throws?

(Adapted from *Investigating Mathematics* [New York: Glencoe Macmillan/McGraw-Hill, 1994])

TASK C

Manipulatives/Tools: Calculator

Your school's science club has decided to do a special project on nature photography. They decided to take a few more than 300 outdoor photos in a variety of natural settings and in all different types of weather. They want to choose some of the best photographs and enter the state nature photography contest. The club was thinking of buying a 35 mm camera, but one member suggested that it might be better to buy disposable cameras instead. The regular camera with autofocus and automatic light meter would cost about $40.00, and film would cost $3.98 for 24 exposures and $5.95 for 36 exposures. The disposable cameras could be purchased in packs of three for $20.00, with two of the three taking 24 pictures and the third one taking 27 pictures. Single disposables could be purchased for $8.95. The club officers have to decide which would be the better option and justify their decisions to the club advisor. Do you think that they should purchase the regular camera or the disposable cameras? Write a justification that clearly explains your reasoning.

TASK D

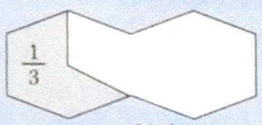
Manipulatives/Tools: None

The cost of a sweater at a department store was $45. At the store's "day and night" sale it was marked 30 percent off the original price. What was the price of the sweater during the sale? Explain the process you used to find the sale price.

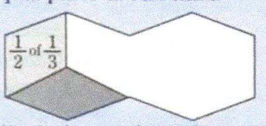
TASK E

Manipulatives/Tools: Pattern blocks

1/2 of 1/3 means one of two equal parts of one-third

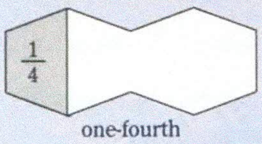


one-third

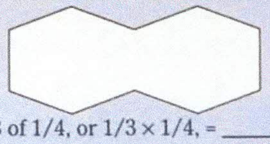


1/2 of 1/3, or 1/2 × 1/3, = 1/6

Find 1/3 of 1/4. Use pattern blocks. Draw your answer.

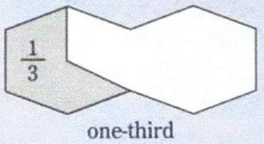


one-fourth

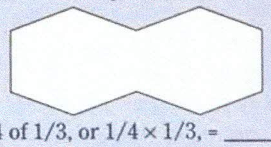


1/3 of 1/4, or 1/3 × 1/4, = ______.

Find 1/4 of 1/3. Use pattern blocks. Draw your answer.



one-third

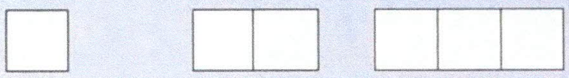


1/4 of 1/3, or 1/4 × 1/3, = ______.

TASK F

Manipulatives/Tools: Square pattern tiles

Using the side of a square pattern tile as a measure, find the perimeter of, or distance around, each train in the pattern-block figure shown.

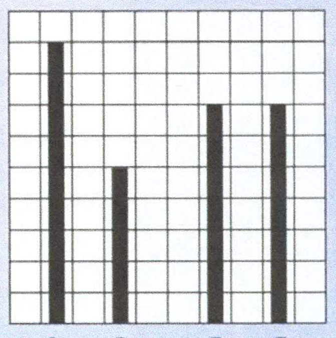


Train 1 Train 2 Train 3

TASK G

Manipulatives/Tools: Grid paper

The pairs of numbers in (a)–(d) represent the heights of stacks of cubes to be leveled off. On grid paper, sketch the front views of the columns of cubes with these heights before and after they are leveled off. Write a statement under the sketches that explains how your method of leveling off is related to finding the average of the two numbers.



9 5 7 7

(a) 14 and 8 (b) 16 and 7 (c) 7 and 12 (d) 13 and 15

By taking two blocks off the first stack and giving them to the second stack, I've made the two stacks the same. So the total number of cubes is now distributed into two columns of equal height. And that is what average means.

(Taken from Bennett and Foreman [1989/1991])

TASK H

Manipulatives/Tools: None

Give the fraction and percent for each decimal.

0.20 = ______ = ______.
0.25 = ______ = ______.
0.33 = ______ = ______.
0.50 = ______ = ______.
0.66 = ______ = ______.
0.75 = ______ = ______.