

1.1.1 How can I work with my team to figure it out?

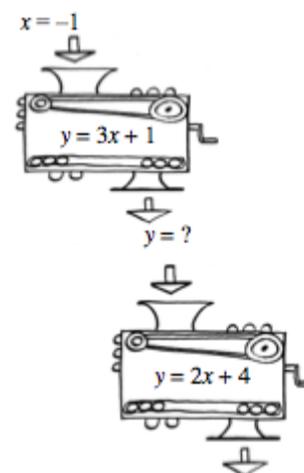


Solving Puzzles in Teams

In previous courses, you have looked at patterns in tables, graphs, equations, and situations. In this course, you will not only continue your study of linear functions (which you have previously called “linear equations”), but will extend these patterns to new kinds of functions. Working with patterns will be the key to many of these functions. Note that we will define a function formally in Section 2 of this chapter. In today’s lesson, you and your team will use clues in order to find patterns and solve puzzles.

1-1. TEAM SORT

Your teacher will give you a card with a representation of a line (a table, graph, equation, or situation). Consider what you know about the line represented on your card. Then find the other students in your class who have a representation of the same line. These students will be your teammates, so you should sit together as instructed by your teacher. Be prepared to justify how you know your representation matches those of your teammates.



1-2. In this problem you will work with “function machines” like those pictured.

- When a value for x is put into the machine, a value for y comes out. That output then becomes the input for another machine. An example is shown above.

What is the output from the second function machine? Explain.

1-3. Your team’s job is to use a specified input to get a particular output by putting those machines in order so that one machine’s output becomes the next machine’s input.

As you work, discuss what you know about the kind of output each machine produces to help you arrange the machines in an appropriate order.

The four relations are reprinted below:

$$y = -2x + 34$$

$$y = \frac{-x}{3} - 10$$

$$y = -|3x|$$

$$y = (x - 2)^2$$

- In what order should you stack the machines so that when 15 is dropped into the first machine, and all four machines have had their effect, the last machine’s output is -6 ?
- What order will result in a final output of 2 when the first input is 8?

5.1.1 How does the pattern grow?



Representing Exponential Growth

So far in this course, you have been investigating the family of linear functions using multiple representations (especially tables, graphs, and equations). In this chapter, you will learn about a new family of functions and the type of growth it models.

5-1. MULTIPLYING LIKE BUNNIES

In the book *Of Mice and Men* by John Steinbeck, two good friends named Lenny and George dream of raising rabbits and living off the land. What if their dream came true?

Suppose Lenny and George started with two rabbits and that in each month following those rabbits have two babies. Also suppose that every month thereafter, each pair of rabbits has two babies.

Your Task: With your team, determine how many rabbits Lenny and George would have after one year (12 months). Represent this situation with a written description of the pattern of growth, a diagram, and a table. What patterns can you find and how do they compare to other patterns that you have investigated previously?

Discussion Points

What strategies could help us keep track of the total number of rabbits?

What patterns can we see in the growth of the rabbit population?

How can we predict the total number of rabbits after many months have passed?

Further Guidance

5-2. How can you determine the number of rabbits that will exist at the end of one year? Consider this as you answer the questions below.

- Draw a diagram to represent how the total number of rabbits is growing each month. How many rabbits will Lenny and George have after three months?
- As the number of rabbits becomes larger, a diagram becomes too cumbersome to be useful. A table might work better. Organize your information in a table showing the total number of rabbits for the first several months (at least 6 months). What patterns can you find in your table? Describe the pattern of growth in words.
- If you have not done so already, use your pattern to determine the number of rabbits that Lenny and George would have after one year (12 months) have passed.
- How does the growth in the table that you created compare to the growth patterns that you have investigated previously? How is it similar and how is it different?

8.2.4 What is the connection?

Completing the Quadratic Web



In just three lessons you have almost completed the quadratic web. Revisit the web posted in your classroom. What connections, if any, still need to be made?

Today you will focus on how to get a quadratic equation from a table, graph, and a situation. As you work, ask yourself the following questions:

Which representation am I given?

Which representation am I looking for?

How can I reverse this process?

Is there another way?

8-89. TABLE TO RULE

You know how to make a table for a quadratic rule, but how can you write an equation when given the table? Examine this new connection that requires you to reverse your understanding of the Zero Product Property as you find a rule for each table below. What clues in the tables helped you find the rule? How can you check your equation using your graphing calculator?

a.

| | | | | | | | | | |
|-----|----|----|----|----|----|----|---|---|----|
| x | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| y | 6 | 0 | -4 | -6 | -6 | -4 | 0 | 6 | 14 |

b.

| | | | | | | | | | | | |
|-----|----|----|----|----|----|----|----|---|---|----|----|
| x | -6 | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| y | 7 | 0 | -5 | -8 | -9 | -8 | -5 | 0 | 7 | 16 | 27 |



8-90. QUALITY CONTROL, Part One

Congratulations! With your promotion, you are now the Quality Assurance Representative of the Function Factory. Your job is to make sure your clients are happy. Whenever a client writes to the company, you must reply with clear directions that will solve his or her problem.

Your boss has provided graphing technology and a team of fellow employees to help you fulfill your job description.

Your Task:



- a. Carefully read the complaints below. Use your graphing calculator to study each situation. Work with your team to resolve each situation.
- b. Write each customer a friendly response that offers a solution to his or her problem. Remember that the customers are not parabola experts! Do not assume that they know anything about parabolas.

Dear Ms. Quadratic,

A

I followed all of the directions given in your brochure about how to order a parabola. I tried to order a parabola that passed through the points $(1, 0)$ and $(-6, 0)$, only to have you send me the wrong one!

Please tell me how to order the correct parabola. Your immediate reply is appreciated.

Perturbed in Pennsylvania

Dear Ms. Quadratic,

B

*I am a very dissatisfied customer. I want a parabola that hits the x-axis only once at $(5, 0)$, yet I see **NO** mention of this type of parabola in your pamphlet. Your company mission statement assures me that "my needs will be met no matter what." How should I order my special parabola?*

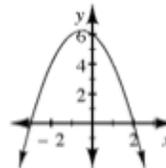
Sincerely,

Troubled in Texas

Dear Ms. Quadratic,

C

Please help! I have searched through your entire brochure and did not see a parabola that would fit my needs. All I want is a parabola that looks like this:



Every time I order an equation to give me this parabola you always send me a different one! I refuse to pay for any parabola but the one shown above. Please tell me how I should find the equation of this parabola or I will take my business elsewhere!

Thank you,

Agitated in Alaska

A possible student explanation for problem part (a) of problem 1-3 might start as follows: “We knew that the squared equation would create a number that was very large, and none of the other equations would likely undo that. So the squared equation could not be first. Whatever was before the squared equation would have to have a small output, so we looked for equations that would make 16 much smaller. Also, one of the two “negative” equations had to be last.”

A possible explanation for part (b) might start as follows: “The final answer was not a perfect square, so the squared equation could not be last. Since the answer was positive, the absolute value equation could not be last, and probably the equation with all negatives was not last.”

If students are simply using guess-and-check, call for a **Huddle** with the Task Managers. (Huddle is a **Study Team Strategy** explained in the Teacher Tab: Team Support of this eBook.) Remind the Task Managers that their teams should be considering the kind of output each relation produces, and using the outputs to logically arrange the machines.

1-2. See below:

- a. The output from the first machine is -2 ; the output from the second machine is 0 .

1-3. See Suggested Lesson Activity and the Closure for possible student discussions.

- a. The correct order is $y = -2x + 34$, $y = (x - 2)^2$, $y = -|3x|$, $y = \frac{-x}{3} - 10$.
- b. The correct order is $y = -|3x|$, $y = \frac{-x}{3} - 10$, $y = (x - 2)^2$, $y = -2x + 34$.
-

Problem 5-1, “Multiplying Like Bunnies,” asks students to represent a growing population of bunnies first as a diagram, then in a table, and finally as a written description. Before students delve into the problem too deeply, make sure they correctly interpret how the population is growing. At the same time, it is important for students to try developing their own organizational strategies so they can successfully build on them. One way to accomplish both of these goals is by using a **Teammates Consult**.

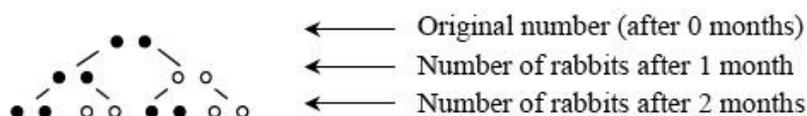
Discuss this strategy and its purpose briefly with the class and then have a member from each team read the problem. Remind students that during this time they are only allowed to read and discuss their ideas for organizing and solving the problem, but they may not write anything yet. The purpose of this activity is to get students to think thoroughly about the problem before they start working on it. As the discussions are coming to a close, direct teams to work on the problem while one team member records ideas on a piece of paper in the middle of the workspace. As you circulate, observe the organizational strategies of each team.

Teams that are struggling during the investigation can be directed to problem 5-2 for further guidance.

After most teams have come up with a strategy for drawing the diagram and filling in the table, give each team a blank sheet of paper and markers and have teams summarize their work to present briefly to the class. It is not necessary to have all teams report, so carefully choose the teams that do and the order in which you want them to present. Try to select teams in an order that causes the

ideas to connect and build on each other. Throughout the reporting, students should ask teams brief questions and comments for clarification instead of waiting until after all of the presentations are completed. At the end of each presentation, you could then ask, “As a class, do we agree that...?” and then make sure that all students understand the problem: that each month, *each pair* of rabbits has two babies, so in the second month, both the original pair of rabbits and the new pair each have two babies, for a total of eight rabbits. Ask each team to decide which organizational strategy makes it easiest to see what is happening with the population of the rabbits. They do not have to agree on a particular strategy but should choose one that works for their team.

A possible diagram that students may come up with is shown below.



| Month # | # of Rabbits |
|---------|--------------|
| 0 | 2 |
| 1 | 4 |
| 2 | 8 |
| 3 | 16 |
| 4 | 32 |
| 5 | 64 |
| 6 | 128 |
| 7 | 256 |
| 8 | 512 |
| 9 | 1,024 |
| 10 | 2,048 |
| 11 | 4,096 |
| 12 | 8,192 |

In this example, students chose to shade each pair of rabbits that are parents and leave each month’s babies not shaded. For the first three or four months, the total number of rabbits can be found by using the diagram, but diagrams get big and difficult to follow very quickly. This inherently follows the nature of an exponential function; ideally, students will observe this visually.

After teams share, prompt them to return to work with questions such as, “How can you continue the table without having to draw it?” As you circulate, check to see if teams are starting their tables with the initial number of rabbits, that is, the number of rabbits after zero months have passed, as shown at right. As students consider how to extend the table to 12 months, there are different patterns they might use and different ways to show those patterns. You could encourage them

to add a third column to show the pattern they see developing, as shown in the table below.

Encourage students to share what they see. Looking at the pattern in different ways will be useful for writing an equation in Lesson 5.3.2. Students are likely to see the entries in the table as doubling or multiplying by 2.

| Month # | # of Rabbits | Patterns |
|---------|--------------|---------------|
| 0 | 2 | 2 |
| 1 | 4 | 2(2) |
| 2 | 8 | 2(2)(2) |
| 3 | 16 | 2(2)(2)(2) |
| 4 | 32 | 2(2)(2)(2)(2) |
| 5 | 64 | |
| 6 | 128 | |
| 7 | 256 | |
| 8 | 512 | |
| 9 | 1,024 | |
| 10 | 2,048 | |
| 11 | 4,096 | |
| 12 | 8,192 | |

As most teams finish with problem 5-1 (or 5-2 if using Further Guidance) and have found the number of bunnies after a year, lead a whole-class discussion about the growth pattern in this situation. This discussion provides an opportunity for students to think explicitly about how this growth compares to growth patterns that they have looked at before and to begin considering what the possible equation is and why some types of functions are not good possibilities. Ask questions such as, “How is this growth the same or different from what we have seen before?”, “Is this equation linear? How can you tell?”

At this point, if you feel your students are ready, you could add a third column to the table to show the pattern a different way. Month 1 is double Month 0, that is, $2(\text{Month } 0)$, or $2(2)$. Then in Month 2, there are $2(\text{Month } 1)$, $2(2(2))$, and so on. You can then guide students to proposing an expression for the n th month, such as $2 \cdot 2^n$. However, this pattern of multiplying and writing the pattern as an exponential equation ($2 \cdot 2^x$) is further developed in Lesson 5.3.2 and does not need to be elaborated on here unless your students bring it up.

5-1. They would have 8192 rabbits after one year.

5-2.

- Diagrams vary. At the end of one month, there are four rabbits (the original two and their two offspring), so there will be 16 rabbits after three months.
 - The number of rabbits begins with 2 and doubles every month.
 - At the end of 12 months there will be 8192 rabbits.
 - Students will likely notice that this growth pattern is not linear or that it does not grow by a constant amount.
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Launch problem 8-89 by asking, “*We know how to make a table for a rule. How can we make a rule from a table?*” If students are not sure how to start, ask, “*What pieces of information in the table can help us?*” and “*How can that help us?*” If students are still stuck, ask, “*What are the roots of this equation? How can they help us?*” Students should focus on building an equation from its roots. For example, in part (a) of problem 8- 89, once students notice that the x -intercepts are $(-3, 0)$ and $(2, 0)$, then the quadratic equation $(x + 3)(x - 2) = 0$ is needed to get those intercepts. The function must be $y = (x + 3)(x - 2)$. This could be done as a [Think-Ink-Pair-Share](#).

Unless students bring it up, there is no need to point out that there could be a GCF, “ a ,” multiplying the quadratic. Students will graduate to the more general form $y = a(x + b)(x + c)$ in problem 8-91; the value of “ a ” will be considered in that problem.

Note: This course does not expect students to find a comprehensive method for finding the rule for all quadratics from tables. This would require an analysis of finite differences or would require students to solve systems of three equations with three unknowns. Those methods will be learned in a later course. By having students build equations from the x -intercepts, their conceptual understanding of the [Zero Product Property](#) will be strengthened.

Read problem 8-90 together as a class and make sure that students understand the goals of the task. Clearly state what type of product you are expecting. For example, are you expecting students to write a formal letter back to each customer? Are you expecting teams to design a poster with the information for the client? You can use a [Teammates Consult](#) that could be followed by a [Traveling Salesman](#), who reads the letters to another team.

Then have the teams begin addressing the concern stated in the first letter. After 10 minutes or so, ask one or two teams to share their findings, thus helping any teams that are struggling. Ask teams questions that help them generalize the process. For example, ask, “*If you have the graph of the parabola, how can you build its equation?*” then “*Is that always true if a parabola has only one x -intercept?*” or “*How can you verify that your equation is correct?*”

8-89. Students can build the equation using the x -intercepts. Check by entering the equation and verifying that the table in the calculator matches the one in the problem statement.

a. $y = (x + 3)(x - 2) = x^2 + x - 6$

b. $y = (x + 5)(x - 1) = x^2 + 4x - 5$

8-90. Solutions: Letter A: The client should order the parabola $y = (x - 1)(x + 6)$; Letter B: Students should recommend the parabola $y = (x - 5)^2$; Letter C: Students should recommend the parabola $y = -(x + 3)(x - 2)$.