NORTH DAKOTA MATHEMATICS CONTENT STANDARDS

Grades K–12

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DRAFT ONE

NORTH DAKOTA DEPARTMENT OF
PUBLIC INSTRUCTION

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How to read the grade level standards

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

Cluster **Domain** **Code** (grade, domain, standard) **Standard**

Domain: Counting and Cardinality

Cluster: Know number names and the count sequence.

Code	Standard	Annotation
K.CC.1	Count to 100 by ones and tens.	Pennies and dimes may be used to model ones and tens.
K.CC.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	Number range for this skill should be up to 100. Student is given a number within the range of 0 to 100. Example: Use 56. Student must count forward in sequence from that number “56, 57, 58, 59” on so on.
K.CC.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).	

The characters of the code are separated by periods. The first characters represent the grade; the second characters represent the Domain and the last characters represent the number of the standard.

These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, “Students who already know ... should next come to learn ....” But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematics | Kindergarten

In Kindergarten, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Grade K Overview

Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

- Work with numbers 11-19 to gain foundations for place value.

Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in categories.

Geometry

- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

Mathematical Practices

1. Make sense of problems and persevere in solvi ng them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Kindergarten

Domain: Counting and Cardinality — K.CC

Cluster: Know number names and the count sequence.

Code	Standards	Annotation
K.CC.1	Count to 100 by ones and by tens. Count backward from 20 by ones.	
K.CC.2	Count forward beginning from a given number within 100. Count backward from a given number within 10.	
K.CC.3	Write numbers sequentially from 0 to 20. Write a given number from 0 to 20.	

Cluster: Count to tell the number of objects.

Code	Standards	Annotation
K.CC.4	Understand the relationship between numbers and quantities up to 20; connect counting to cardinality. a. Use one to one correspondence when counting objects. b. Understand that the last number name said tells the number of objects counted, regardless of their arrangement or order in which they were counted. c. Understand that each successive number name refers to a quantity that is one more.	
K.CC.5	Count to answer “how many?” questions. a. Tell how many objects up to 20 are in an arranged pattern (e.g., a line or an array) or up to 10 objects in a scattered configuration. b. Represent a number of objects up to 20 with a written numeral. c. Given a number from 1-20, count out that many objects.	

Cluster: Compare numbers.

Code	Standards	Annotation
K.CC.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, using groups of up to 10 objects.	Students may use matching and counting strategies.
K.CC.7	Compare two numbers between 1 and 10 presented as written numerals.	

Domain: Operations and Algebraic Thinking		K.OA
Cluster: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.		
Code	**Standards**	**Annotation**
K.OA.1	Represent addition and subtraction in a variety of ways.	Drawings need not show details, but should show the mathematics in the problem. Students may use objects, fingers, mental images, drawings, acting out situations, verbal explanations, expressions, or equations.
K.OA.2	Use an appropriate strategy to solve word problems that involve adding and subtracting within 10.	Students may use mental math, objects or drawings to represent the problem.
K.OA.3	Decompose numbers less than or equal to 10 into multiple combinations of two parts. Record each decomposition with a drawing or equation.	Example: The number 8 could be broken into 5 and 3, 6 and 2, 7 and 1, etc. Students may use objects or drawings. Example: 8 = 5 + 3, 8 = 6 + 2, 8 = 7 + 1, etc.
K.OA.4	Find the number that makes 10 when added to a given number from 1 to 9. Record the answer with a drawing or equation.	Students may use objects or drawings.
K.OA.5	Fluently add and subtract within 5.	

Domain: Number and Operations in Base Ten		K.NB
Cluster: Work with numbers 11-19 to gain foundations for place value.		
Code	**Standards**	**Annotation**
K.NBT.1	Compose and decompose numbers from 11 to 19 using a group of ten ones and additional ones. Record each composition or decomposition with a drawing or equation.	Students may use objects or drawings. Example: 18 = 10 + 8.

Domain: Measurement and Data		K.MD
Cluster: Describe and compare measurable attributes.		
Code	**Standards**	**Annotation**
K.MD.1	Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	Students are not measuring but rather describing what attributes could be measured.
K.MD.2	Compare two objects with a common measurable attribute and describe the difference.	Example: Compare the heights of two children and describe one child as taller/shorter.
Cluster: Classify objects and count the number of objects in each category.		
Code	**Standards**	**Annotation**
K.MD.3	Classify objects into given categories limiting the number in each category to 10 or less. Count the numbers of objects in each category and sort the categories by count.	

Domain: Geometry		K.G
Cluster: Identify and describe shapes (squares, circles, triangles, rectangles, cubes, and spheres).		
Code	**Standards**	**Annotation**
K.G.1	Describe objects in the environment using names of shapes and solids (squares, circles, triangles, rectangles, cubes, and spheres).	
K.G.2	Correctly name shapes and solids (squares, circles, triangles, rectangles, cubes, and spheres) regardless of their orientations or overall size.	
K.G.3	Identify shapes and solids (squares, circles, triangles, rectangles, cubes, and spheres) as two-dimensional or three-dimensional.	Two-dimensional: may use the terms “shape” or “flat”. Three-dimensional: may use the term “solid”.
Cluster: Compare, classify, and compose shapes.		
Code	**Standards**	**Annotation**
K.G.4	Compare and classify two-dimensional shapes (squares, circles, triangles, rectangles) of different sizes and orientations, using informal language to describe their similarities, differences, and attributes.	
K.G.5	*This standard has been moved/removed by the committee*	
K.G.6	Compose a new shape by combining two or more simple shapes.	Example: Use two triangles to make a square.

Mathematics | Grade 1

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

(2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

(3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[1]

(4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

[1] Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

Grade 1 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Understand and apply properties of operations and the relationship between addition and subtraction.
- Add and subtract within 20.
- Work with addition and subtraction equations.

Number and Operations in Base Ten

- Extend the counting sequence.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure lengths indirectly and by iterating length units.
- Tell and write time.
- Represent and interpret data.

Geometry

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Domain: Operations and Algebraic Thinking		1.OA
Cluster: Represent and solve problems involving addition and subtraction.		
Code	**Standards**	**Annotation**
1.OA.1	Use strategies to add and subtract within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.	See Glossary, Table 1. Strategies may include using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
1.OA.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20.	Students may use objects, drawings, and equations with a symbol for the unknown number to represent the problem.
Cluster: Understand and apply properties of operations and the relationship between addition and subtraction.		
Code	**Standards**	**Annotation**
1.OA.3	Apply properties of operations as strategies to add and subtract.	Students do not need to use formal terms for these properties. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)
1.OA.4	Demonstrate understanding of subtraction as an unknown-addend problem.	Example: Subtract 10 – 8 by finding the number that makes 10 when added to 8.
Cluster: Add and subtract within 20.		
Code	**Standards**	**Annotation**
1.OA.5	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	Example: Count on 2 to add 2, count back 4 to subtract 4.
1.OA.6	Use strategies to add and subtract within 20. Fluently add and subtract within 10.	Strategies may include counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
Cluster: Work with addition and subtraction equations.		
Code	**Standards**	**Annotation**
1.OA.7	Demonstrate understanding of the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.	The equal sign represents that one side of an equation is the same as or has the same value as the other side. Example: Which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.
1.OA.8	Determine the unknown whole number in an addition or subtraction equation that uses three whole numbers.	Example: Determine the unknown number that makes the equation true in each of the equations 8 +? = 11, 5 = □ - 3, 6 + 6 = □.

Domain: Number and Operations in Base Ten		1.NBT
Cluster: Extend the counting sequence.		
Code	**Standards**	**Annotation**
1.NBT.1	Count forward and backward within 120, starting at any given number. Read and write numerals within 120. Represent a number of objects up to 120 with a written numeral.	
Cluster: Understand place value.		
Code	**Standards**	**Annotation**
1.NBT.2	Demonstrate understanding that the two digits of a two-digit number represent amounts of tens and ones, including: a. 10 can be thought of as a bundle of ten ones — called a “ten.” b. The numbers from 11 to 19 are composed of a ten and additional ones. c. Multiples of 10 up to 90 represent a number of tens and 0 ones.	
1.NBT.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	
Cluster: Use place value understanding and properties of operations to add and subtract.		
Code	**Standards**	**Annotation**
1.NBT.4	Demonstrate understanding of place value when adding two-digit numbers within 100. a. Add a two-digit number and a one-digit number. b. Add a two-digit number and a multiple of 10. c. Use concrete models or drawing strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	Note: This does not need to be introduced through standard algorithm. The standard addition algorithm is introduced in third grade (3.NBT.2).
1.NBT.5	Mentally add or subtract 10 to or from a given two-digit number. Explain the reasoning used.	
1.NBT.6	Use concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction to subtract multiples of 10 in the range of 10-90 from multiples of 10 in the same range resulting in a positive or zero difference. Use a written method to explain the strategy.	A written method is an informal recording of a process or observation and may include narrative writing, numbers and symbols, pictures, equations, etc.

Domain: Measurement and Data 1.MD

Cluster: Measure lengths indirectly and by iterating length units.

Code	Standards	Annotation
1.MD.1	Order three objects by length. Compare the lengths of two objects indirectly by using a third object.	Example: If A is longer than B, and B is longer than C, then A is longer than C.
1.MD.2	Demonstrate understanding that the length measurement of an object is the number of same-size length units that span the object with no gaps or overlaps. Measure and express the length of an object using whole non-standards units.	

Cluster: Work with time and money.

Code	Standards	Annotation
1.MD.3	Tell and write time to the hour and half-hour (including o’clock and half past) using analog and digital clocks.	
1.MD.4a	Identify and tell the value of a dollar bill, quarter, dime, nickel, and penny.	
1.MD.4b	Count and tell the value of combinations of dimes and pennies up to one dollar.	

Cluster: Represent and interpret data.

Code	Standards	Annotation
1.MD.5	Organize, represent, and interpret data with up to three categories. Ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	

Domain: Geometry		1.G
Cluster: Reason with shapes and their attributes (squares, circles, triangles, rectangles, trapezoids, rhombuses, pentagons, hexagons, octagons, cubes, spheres, cylinders, cones, triangular prisms, and rectangular prisms).		
Code	**Standards**	**Annotation**
1.G.1	Distinguish between defining attributes versus non-defining attributes. Use defining attributes to build and draw two-dimensional shapes (**squares, circles, triangles, rectangles, trapezoids, rhombuses, pentagons, hexagons, octagons)**.	Defining attributes: closed or open, number of sides or vertices, etc. Non-defining attributes: color, orientation, size, etc.
1.G.2	Compose a new shape or solid from two-dimensional shapes and/or three-dimensional solids **(squares, circles, triangles, rectangles, trapezoids, rhombuses, pentagons, hexagons, octagons, cubes, spheres, cylinders, cones, triangular prisms, and rectangular prisms)**.	
1.G.3	Partition circles and rectangles into two equal shares. Describe the shares using the word halves, and use the phrase half of. Describe the whole as two of the shares.	Shares may be called parts or pieces.

Mathematics | Grade 2

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Grade 2 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and interpret data.

Geometry

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Domain: Operations and Algebraic Thinking 2.OA

Cluster: Represent and solve problems involving addition and subtraction.

Code	Standards	Annotation
2.OA.1	Use strategies to add and subtract within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.	See Glossary, Table 1. Strategies may include using drawings and equations with a symbol for the unknown number to represent the problem. Some but not all word problems must include standard units of length.

Cluster: Add and subtract within 20.

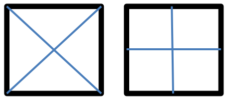
Code	Standards	Annotation
2.OA.2	Use mental strategies to fluently add and subtract within 20.	Students may use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Cluster: Work with equal groups of objects to gain foundations for multiplication.

Code	Standards	Annotation
2.OA.3	Determine whether a given number of objects up to 20 is odd or even. Write an equation to represent an even number using two equal addends or groups of 2.	Strategies may include pairing objects or counting by 2s. Example: 6 = 3 + 3, 6 = 2 + 2 + 2.
2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns. Write an equation to express the total as a sum of equal addends.	Example: 4 + 4 + 4 = 12 or 3 + 3 + 3 + 3 = 12 * * * * * * * * * * * *

Domain: Number and Operations in Base Ten		2.NBT
Cluster: Understand place value.		
Code	**Standards**	**Annotation**
2.NBT.1	Demonstrate understanding that the three digits of a three-digit number represent amounts of hundreds, tens, and ones, including: a. 100 can be thought of as a bundle of ten tens called a "hundred". b. Multiples of 100 represent a number of hundreds, 0 tens, and 0 ones.	Example: 706 represents 7 hundreds, 0 tens, and 6 ones.
2.NBT.2	Count forward and backward from any given number within 1000. Skip-count by 5s, 10s, and 100s.	
2.NBT.3	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	
2.NBT.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, recording the results of comparisons with the symbols >, =, and <.	
Cluster: Use place value understanding and properties of operations to add and subtract.		
Code	**Standards**	**Annotation**
2.NBT.5	Use strategies based on place value, properties of operations, and/or the relationship between addition and subtraction to fluently add and subtract within 100.	
2.NBT.6	Use strategies based on place value and properties of operations to add up to four two-digit numbers.	
2.NBT.7	Demonstrate understanding of place value when adding and subtracting three-digit numbers. Use concrete models or drawings and strategies based on place value, properties of operation, and/or the relationship between addition and subtraction to add and subtract within 1000. Use a written method to explain the strategy.	A written method is an informal recording of a process or observation and may include narrative writing, numbers and symbols, pictures, equations, etc. <u>Note: This does not need to be introduced through standard algorithms. The standard addition and subtraction algorithms are introduced in third grade (3.NBT.2).</u>
2.NBT.8	Mentally add or subtract 10 or 100 to or from a given number between 100 and 900.	

Domain: Measurement and Data		2.MD
Cluster: Measure and estimate lengths in standard units.		
Code	**Standards**	**Annotation**
2.MD.1	Select and use appropriate tools to measure the length of an object.	Tools may include rulers, yardsticks, meter sticks, and measuring tapes.
2.MD.2	Measure the length of an object using two different standard units of measurement. Describe how the two measurements relate to the size of the units chosen.	Different standard units of measurement may include inches and feet, inches and centimeters, feet and yards, yards and meters, etc.
2.MD.3	Estimate lengths using units of inches, feet, centimeters, and meters.	
2.MD.4	Measure to determine how much longer one object is than another, expressing the difference with a standard unit of measurement.	
Cluster: Relate addition and subtraction to equal intervals on a number line.		
Code	**Standards**	**Annotation**
2.MD.5	*This standard has been moved/removed by the committee*	
2.MD.6	Represent whole numbers on a number line diagram with equally spaced points. Represent whole-number sums and differences within 100 on a number line diagram.	Students must demonstrate appropriate spatial representation. Example: 2 would be placed closer to 0 than to 10.
Cluster: Work with time and money.		
Code	**Standards**	**Annotation**
2.MD.7	Tell and write time to the nearest five minutes (including quarter after and quarter to) with a.m. and p.m. using analog and digital clocks.	
2.MD.8	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately.	
Cluster: Represent and interpret data.		
Code	**Standards**	**Annotation**
2.MD.9	Generate data by measuring lengths of objects to the nearest whole unit. Show the measurements by making a line plot, using a horizontal scale marked off in whole-number units.	Students may make repeated measurements of a growing or shrinking object over time or measure several different objects.
2.MD.10	Draw picture graphs and bar graphs with single-unit scales to represent data sets with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.	See Glossary, Table 1.

Domain: Geometry		2.G
Cluster: Reason with shapes and their attributes (squares, circles, triangles, rectangles, trapezoids, rhombuses, pentagons, hexagons, octagons, parallelograms, quadrilaterals, cubes, spheres, cylinders, cones, triangular prisms, and rectangular prisms).		
Code	**Standards**	**Annotation**
2.G.1	Identify trapezoids, rhombuses, pentagons, hexagons, octagons, parallelograms, quadrilaterals, cubes, spheres, cylinders, cones, triangular prisms, rectangular prisms. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.	
2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number.	
2.G.3	Partition circles and rectangles into two, three, or four equal shares. Describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that identical wholes can be equally divided in different ways. Demonstrate understanding that partitioning shapes into more equal shares creates smaller shares.	Example: 

Mathematics | Grade 3

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(4) Students describe, analyze, and compare properties of two dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Grade 3 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten

- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

- Develop understanding of fractions as numbers.

Measurement and Data

- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Represent and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Domain: Operations and Algebraic Thinking		3.OA
Cluster: Represent and solve problems involving multiplication and division.		
Code	**Standards**	**Annotation**
3.OA.1	Interpret and model products of whole numbers.	See Glossary, Table 2. Example: Interpret and model 5 × 7 as the total number of objects in 5 groups of 7 objects each and describe a context in which a total number of objects can be expressed as 5 × 7.
3.OA.2	Interpret and model whole-number quotients of whole numbers, as the number in a group or the number of groups.	
3.OA.3	Using drawings and equations with a symbol for an unknown number, solve multiplication and division word problems within 100 in situations involving equal groups, arrays, and measurement quantities.	See Glossary, Table 2. Emphasis on modeling and constructing meaning is recommended before algorithms are used for computation. Factors, products, divisor, dividends, and quotients will all be less than 100. Example: Students set up 40 chairs in the gymnasium for a concert. They set up 5 rows of chairs. How many chairs were in each row? 5 × c = 40 40 ÷ 5 = c. XXXXXXXX XXXXXXXX XXXXXXXX XXXXXXXX XXXXXXXX
3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers.	Example: Determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ☐ ÷ 3, 6 × 6 = ?
Cluster: Understand properties of multiplication and the relationship between multiplication and division.		
Code	**Standards**	**Annotation**
3.OA.5	Apply properties of operations as strategies to multiply and divide.	Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.) Students need not use formal terms for these properties. Teachers should model accurate vocabulary; however, students will not be assessed on the use of terms at this level.
3.OA.6	Understand division as an unknown-factor problem.	Example: Find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
Cluster: Multiply and divide within 100.		
Code	**Standards**	**Annotation**
3.OA.7	Using mental strategies, fluently multiply and divide within 100.	Use multiple strategies such as count-by/skip counting, doubles, double/doubles, double/double/doubles, derived facts, benchmark numbers, decomposition.

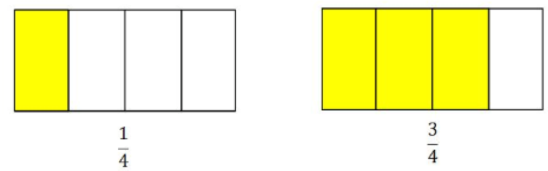
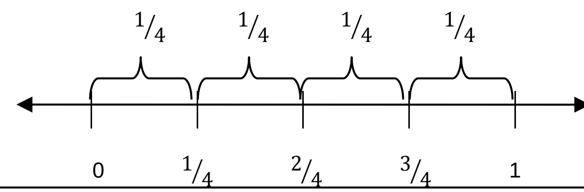
Cluster: Solve problems involving the four operations, and identify and explain patterns in arithmetic. Multiply and divide within 100.		
Code	**Standards**	**Annotation**
3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies.	This standard is limited to problems posed with whole numbers and having whole number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations). Unknown quantity can be referred to as a **variable**. Estimation strategies may include but are not limited to: decomposition, compensation, compatible numbers, estimation, mental math, and rounding.
3.OA.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.	Example: Observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

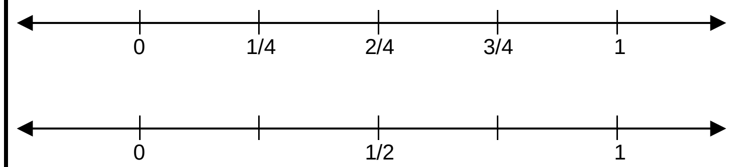
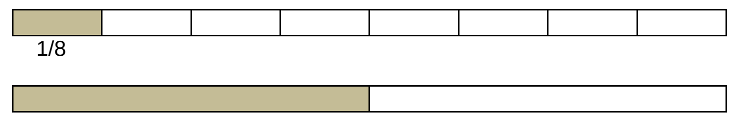
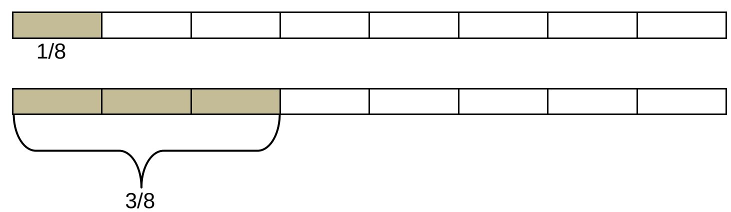
Domain: Number and Operations in Base Ten		3.NBT
Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic. Note: A range of algorithms may be used.		
Code	**Standards**	**Annotation**
3.NBT.1	Use place value understanding to round whole numbers to the nearest 10 or 100.	Possible strategies: using number lines, hundreds charts.
3.NBT.2	Using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction, fluently add and subtract within 1000.	
3.NBT.3	Using strategies based on place value and properties of operations, multiply one-digit whole numbers by multiples of 10 in the range 10-90.	Examples: 9 × 80, 5 × 60.

Domain: Number and Operations - Fractions — 3.NF

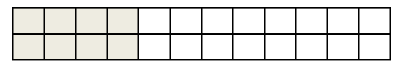
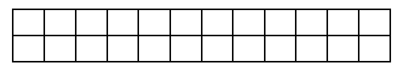
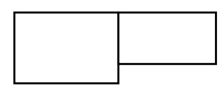
Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Cluster: Develop understanding of fractions as numbers.

Code	Standards	Annotation
3.NF.1	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into *b* equal parts. Understand a fraction a/b as the quantity formed by *a* parts of size 1/b.	Example: 1/4 is the quantity formed by 1 part when a whole is partitioned into 4 equal parts. A fraction 3/4 is the quantity formed by 3 parts of size 1/4. 
3.NF.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram.	
	Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.	Example: A whole is partitioned into 4 equal parts. Recognize that each part is equal to 1/4. 
	Represent a fraction a/b on a number line diagram by marking off *a* lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number *a/b* on the number line.	Students will be able to mark intervals on a number line from 0 to 1 representing the denominators 2, 3, 4, 6, 8. Students will be able to label the number line with corresponding fractions (see number line above).

3.NF.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.	Special cases: each special situation as shown in a, b, c, d. Example: Are $^2/_4$ and $^1/_2$ equivalent fractions? 
	b. Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent.	Example: 1/2 = 2/4, 4/6 = 2/3.
	c. Recognize fractions, a/1 or a/a, that are equivalent to whole numbers. Express whole numbers as fractions, a/1 or a/a.	Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.
	d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions by using a visual fraction model.	Example: When numerators are the same, the fraction with the larger denominator is smaller. $\frac{1}{8} < \frac{1}{2}$  When denominators are the same, the fraction with the larger numerator is greater. $\frac{1}{8} < \frac{3}{8}$ 

Domain: Measurement and Data		3.MD
Cluster: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.		
Code	**Standards**	**Annotation**
3.MD.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve elapsed time word problems on the hour and the half hour, using a variety of strategies.	Intervals of time and elapsed time are synonymous. Problems may be represented on a number line diagram.
3.MD.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units.	Excludes compound units such as cm^3 and finding the geometric volume of a container. Excludes multiplicative comparison problems (problems involving notions of "times as much"; (See Glossary, Table 2).
Cluster: Represent and interpret data		
Code	**Standards**	**Annotation**
3.MD.3	Draw scaled picture graphs and scaled bar graphs to represent data sets with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.	Example: Draw a bar graph in which each square in the bar graph might represent 5 pets.
3.MD.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	
Cluster: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.		
Code	**Standards**	**Annotation**
3.MD.5	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	
3.MD.6	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	

3.MD.7	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning.	Example a = 2, b = 4, c = 8 Area = (4 x 2) + (8 x 2) Area = 8 + 16 Area = 24 sq. units 4 + 8  2 12  2 Area = 12 x 2 Area = 24 sq. units
	d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.	Rectilinear: Connected rectangles in which all of the angles are 90 degrees. Example : A house owner wants to purchase sod for his backyard. The sod is sold in square meters. Determine how many square meters of sod are needed to cover the backyard pictured below.  4 m 6 m 3 m 2 m 4 x 6 = 24 m^2 3 x 2 = 6 m^2 24 + 6 = 30 m^2

Cluster: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Code	Standards	Annotation
3.MD8	Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths. Find an unknown side length. Exhibit rectangles with the same perimeter and different area or with the same area and different perimeters.	*1st experience with perimeter.

Domain: Geometry — 3.G

Cluster: Reason with shapes and their attributes.

Code	Standards	Annotation
3.G.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals. Draw examples of quadrilaterals that do not belong to any of these subcategories.	
3.G.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.	Example: Partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.

Mathematics | Grade 4

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Grade 4 Overview

Operations and Algebraic Thinking

- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- Generate and analyze patterns.

Number and Operations in Base Ten

- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

Measurement and Data

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data.
- Geometric measurement: understand concepts of angle and measure angles.

Geometry

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Domain: Operations and Algebraic Thinking 4.OA

Cluster: Use the four operations with whole numbers to solve problems.

Code	Standards	Annotation
4.OA.1	Interpret a multiplication equation as a comparison~~,~~. Represent verbal statements of multiplicative comparisons as multiplication equations.	Example: Interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Example: Kari has 3 marbles; Greg has 7 times as many. How many marbles does Greg have? Answer: 21 3 x 7 = 21 or 7 x 3 = 21
4.OA.2	Use drawings and equations with a symbol for the unknown number (variable) to represent the problem. Multiply or divide to solve word problems involving multiplicative comparison, distinguishing multiplicative comparison from additive comparison.	See Glossary, Table 2. Example: The giraffe in the zoo is 3 times as tall as the kangaroo. The kangaroo is 6 feet tall. How tall is the giraffe? Solution: Multiplicative method: 6 x 3 = g g = 18 feet Additive method: 6 + 6 + 6 = g g = 18 feet The giraffe is 18 feet tall. The kangaroo is 6 feet tall. How many times taller is the giraffe than the kangaroo? 18/6 = k, k = 3 18 - 6 = 12 12 - 6 = 6 6 – 6 = 0
4.OA.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown (variable) quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	

Cluster: Gain familiarity with factors and multiples.

Code	Standards	Annotation
4.OA.4	Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	

Cluster: Generate and analyze patterns		
Code	**Standards**	**Annotation**
4.OA.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.	Example: Given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers.

Domain: Number and Operations in Base Ten — 4.NBT

Note: Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000

Cluster: Generalize place value understanding for multi-digit whole numbers.

Code	Standards	Annotation
4.NBT.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.	Example: Recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.
4.NBT.2	Read and write multi-digit whole numbers to the one millions place using base-ten numerals, word form, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	
4.NBT.3	Use place value and/or understanding of numbers to round multi-digit whole numbers to any place.	Possible strategies include using number lines, hundreds chart, number sense.

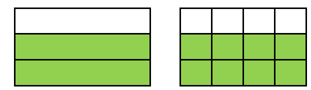
Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.

Code	Standards	Annotation
4.NBT.4	Fluently add and subtract multi-digit whole numbers to the one millions place using strategies flexibly, including the standard algorithm.	Mastery of the addition and subtraction standard algorithms is expected at this stage.
4.NBT.5	Using strategies based on place value and the properties of operations, multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	*The standard multiplication algorithm is a 5th grade standard (5.NBT.5).
4.NBT.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	*The standard division algorithm is a 6th grade standard (6.NS.2).

Domain: Number and Operations – Fractions — 4.NF

Note: Grade 4 expectations in this domain are limited to fractions with denominators 2,3,4,5,6,8,10,12 and 100

Cluster: Extend understanding of fraction equivalence and ordering.

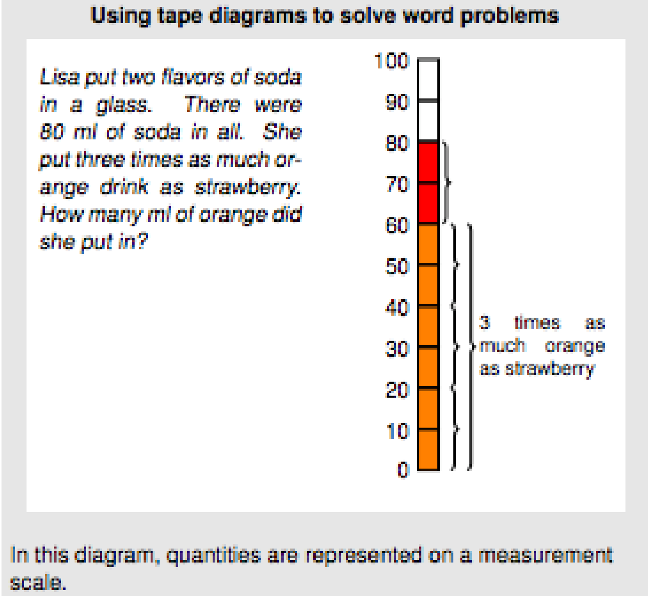
Code	Standards	Annotation
4.NF.1	Using visual fraction models with attention to how the number and size of the parts differ even though the two fractions themselves are the same size, explain why a fraction *a*/*b* is equivalent to a fraction (n × a)/(n × b). Use this principle to recognize and generate equivalent fractions.	*Solutions focus on equivalence, which may include, but does not require simplest form. Example: 2/3 = 8/12 because… Using an area model to show that $\frac{2}{3} = \frac{4*2}{4*3}$  a = 2, b = 3, n = 4
4.NF.2	By creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2, compare two fractions with different numerators and different denominators. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	Example: Compare 3/4 to 5/12 using <, >, =, and justify your conclusion. In 3/4, the numerator 3 is more than 1/2 of the denominator 4, and in 5/12, the numerator 5 is less than 1/2 of the denominator 12; therefore 3/4 is greater than 5/12.

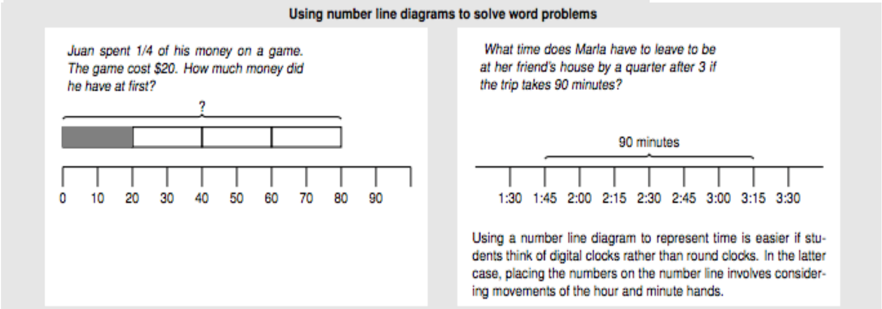
Cluster: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.		
Code	**Standards**	**Annotation**
4.NF.3	Understand a fraction *a*/*b* with *a* > 1 as a sum of unit fractions 1/*b*. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition with an equation. Justify decompositions by using a visual fraction model or other strategies. c. Add and subtract mixed numbers with like denominators. d. Using visual fraction models and equations, solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.	If a = 5, b = 6 5/6 = 1/6 + 1/6 + 1/6 + 1/6 + 1/6 Examples: 3/8 = 1/8 + 1/8 + 1/8 3/8 = 1/8 + 2/8 2 1/8 = 1 + 1 + 1/8 or 8/8 + 8/8 + 1/8 Example: 7 1/5 = 7 + 1/5 = 35/5 +1/5 = 36/5 By replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
4.NF.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a*/*b* as a multiple of 1/*b*. b. Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. c. Using visual fraction models and equations, solve word problems involving multiplication of a fraction by a whole number.	Example: Use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4). Example: Use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b). Example: If each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Cluster: Understand decimal notation for fractions, and compare decimal fractions.		
Code	**Standards**	**Annotation**
4.NF.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100. Use this technique to add two fractions with respective denominators 10 and 100.	Example: Express 3/10 as 30/100 and add 3/10 + 4/100 = 34/100. Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.
4.NF.6	Use decimal notation for fractions with denominators 10 or 100.	Example: Rewrite 62/100 as 0.62; describe a length as 0.62 meters; or locate 0.62 on a number line diagram.
4.NF.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.	

Domain: Measurement and Data		4.MD
Cluster: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.		
Code	**Standards**	**Annotation**
4.MD.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.	Example: Know that 1 ft is 12 times as long as 1 in. Example: Express the length of a 4 ft snake as 48 in. Example: Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36)…

<table>
<tr>
<td>4.M D.2</td>
<td>Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit.

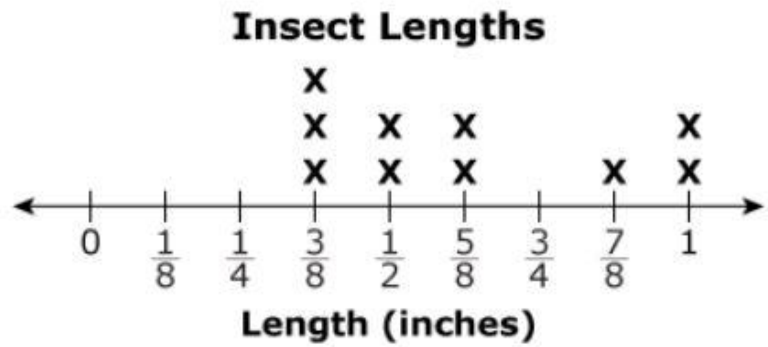
Using diagrams such as number line diagrams that feature a measurement scale, to represent measurement quantities.</td>
<td>Examples: [2]


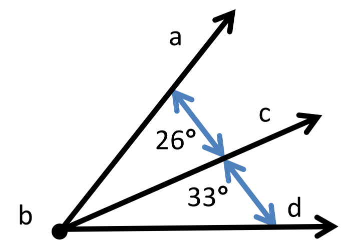

</td>
</tr>
<tr>
<td>4.MD.3</td>
<td>Apply the area and perimeter formulas for rectangles in real-world and mathematical problems.</td>
<td>Example: Find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

<u>Perimeter Formula:</u> $P = l + w + l + w = 2l + 2w = 2(l + w)$

<u>Area Formula</u>: $A = l \times w$</td>
</tr>
</table>

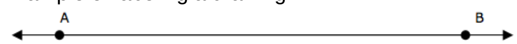
[2] Diagram obtained from: https://commoncoretools.files.wordpress.com/2012/07/ccss_progression_gm_k5_2012_07_21.pdf

Cluster: Represent and interpret data.		
Code	**Standards**	**Annotation**
4.MD.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.	Example: From a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection. 

Cluster: Geometric measurement: understand concepts of angle and measure angles.		
Code	**Standards**	**Annotation**
4.MD.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint. Understand concepts of angle measurement. a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	
4.MD.6	Measure angles in whole-number degrees using a protractor. Using a protractor and ruler, draw angles of a specified measure.	
4.MD.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.	What is the measure of angle ABD? If angle ABD is 59° and angle ABC measures 26°, what is the measure of angle CBD? 

Domain: Geometry 4.G

Cluster: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Code	Standards	Annotation
4.G.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	Focus is on recognition and labeling of drawings, and reading notations, not writing notation in text. Example of notation: $\overleftrightarrow{AB}$ & $\overleftrightarrow{BA}$ Example of labeling a drawing: 
4.G.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of specified size. Recognize right triangles as a category, and identify right triangles.	
4.G.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures. Draw lines of symmetry.	

Mathematics | Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real-world and mathematical problems.

Grade 5 Overview

Operations and Algebraic Thinking

- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data

- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry

- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Domain: Operations and Algebraic Thinking — 5.OA

Cluster: Write and interpret numerical expressions.

Code	Standards	Annotation
5.OA.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	
5.OA.2	Write simple expressions that record calculations with numbers. Interpret numerical expressions without evaluating them.	Example: Express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18{,}932 + 921)$ is three times as large as $18{,}932 + 921$, without having to calculate the indicated sum or product.

Cluster: Analyze patterns and relationships.

Code	Standards	Annotation
5.OA.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns. Graph the ordered pairs on a coordinate plane. Use the graph to verify relationships.	Example: Given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Domain: Number and Operations in Base Ten		5.NBT
Cluster: Understand the place value system.		
Code	**Standards**	**Annotation**
5.NBT.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	
5.NBT.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	
5.NBT.3	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, word form, and expanded form. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Example: 347.392 = 3 × 100 + 4 ×10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).
5.NBT.4	Use place value understanding to round decimals to any place.	
Cluster: Perform operations with multi-digit whole numbers and with decimals to hundredths.		
Code	**Standards**	**Annotation**
5.NBT.5	Fluently multiply multi-digit whole numbers using strategies flexibly, including the standard algorithm.	Mastery of the standard multiplication algorithm is expected at this stage.
5.NBT.6	Using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division, find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	*The standard division algorithm is a 6th-grade standard.(6.NS.2)
5.NBT.7	Using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, add, subtract, multiply, and divide decimals to hundredths. Relate the strategy to a written method and explain the reasoning used.	Written method: an informal recording of a process or observation.

Domain: Number and Operations - Fractions 5.NF

Cluster: Use equivalent fractions as a strategy to add and subtract fractions.

Code	Standards	Annotation
5.NF.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.	*Solutions focus on equivalence, which may include, but does not require simplest form. Example: 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)
5.NF.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, by using visual fraction models and equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.	Example: Recognize an incorrect result 2/5 + 1/2 = 3/7 by observing that 3/7 < 1/2.

Cluster: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Code	Standards	Annotation
5.NF.3	Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers by using visual fraction models and equations to represent the problem.	Example: Interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3 and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?
5.NF.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (a/b) × q as *a* parts of a partition of q into *b* equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles. Represent fraction products as rectangular areas.	Example: Use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)

5.NF.5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case) Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number. c. Relate the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1.	Example: 22 x 36 < 22 x 50, because 36 < 50. 1/7 x 14 < 14, because 1/7 is less than 1. Example: 23 x 13/5 > 23 because 13/5 is greater than 1. 23 x 1/4 < 23 because 1/4 is less than 1. 23 x 2/2 = 23 because 2/2 = 1.
5.NF.6	Solve real-world problems involving multiplication of fractions and mixed numbers using visual fraction models and equations to represent the problem.	
5.NF.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. b. Interpret division of a whole number by a unit fraction, and compute such quotients. c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions using visual fraction models and equations to represent the problem.	Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade. Example: Create a story context for (1/3) ÷ 4 and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. Example: Create a story context for 4 ÷ (1/5) and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) =4. Example: How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

Domain: Measurement and Data		5.MD
Cluster: Convert like measurement units within a given measurement system.		
Code	**Standards**	**Annotation**
5.MD.1	Convert among different-sized standard measurement units within a given measurement system. Use these conversions in solving multi-step, real-world problems.	Include standard and metric systems. Example: Convert 5 cm to 0.05 m.
Cluster: Represent and interpret data.		
Code	**Standards**	**Annotation**
5.MD.2	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.	Example: Given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

Cluster: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.		
Code	**Standards**	**Annotation**
5.MD.3	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	
5.MD.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	
5.MD.5	Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes. Show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. b. Represent threefold whole-number products as volumes to represent the associative property of multiplication. c. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems. d. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problem.	

Domain: Geometry		5.G
Cluster: Graph points on the coordinate place to solve real-world and mathematical problems.		
Code	**Standards**	**Annotation**
5.G.1	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (*x*-coordinate and *x*-axis, *y*-coordinate and *y*-axis).	
5.G.2	Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane. Interpret coordinate values of points in the context of the situation.	
Cluster: Classify two-dimensional figures into categories based on their properties.		
Code	**Standards**	**Annotation**
5.G.3	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.	Example: All rectangles have four right angles and squares are rectangles, so all squares have four right angles.
5.G.4	Classify two-dimensional figures in a hierarchy based on properties.	See Glossary, Table 8.

Mathematics | Grade 6

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Grade 6 Overview

Ratios and Proportional Relationships

- Understand ratio concepts and use ratio reasoning to solve problems.

The Number System

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

Geometry

- Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability

- Develop understanding of statistical variability.
- Summarize and describe distributions.

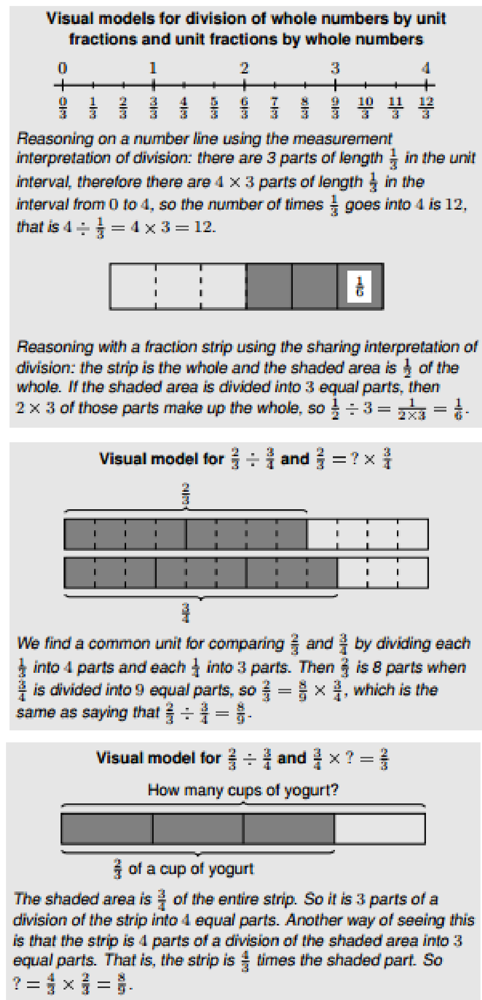
Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Domain: Ratios and Proportional Relationships		6.RP
Cluster: Understand ratio concepts and use ratio reasoning to solve problems.		
Code	**Standards**	**Annotation**
6.RP.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	Example: “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.” This includes part to part and part to whole ratios
6.RP.2	Understand the concept of a unit rate *a*/*b* associated with a ratio *a*:*b* with $b \neq 0$, and use rate language in the context of a ratio relationship.	Example: “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.” The focus should be ratios and rates, but use previous fraction knowledge to support the work.
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, and equations.	**<u>Tape diagram:</u>** A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model (such as metric and inch ruler).
	a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	Example: The recipe calls for 3 cups of flour. How much flour would you need if you doubled the recipe? Tripled the recipe? Make a table to find the missing values. Then plot the pairs of values on a coordinate plane.
	b. Solve unit rate problems including those involving unit pricing and constant speed.	Example: If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? Example: Bananas are 3 lbs for $1.20. What would 1 pound cost? Example: You are traveling 60 mph. You drive 360 miles. How long did you travel?
	c. Find a percent of a quantity as a rate per 100. Solve problems involving finding the whole, given a part and the percent.	Example: 30% of a quantity means 30/100 times the quantity.
	d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	This is the introduction to conversions between measurement units. Example: There are 12 inches in a foot. How many inches are there in three feet? Example: How many cups are in two gallons?

<table>
<tr><th colspan="2">Domain: The Number System</th><th>6.NS</th></tr>
<tr><th colspan="3">Cluster: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</th></tr>
<tr><th>Code</th><th>Standards</th><th>Annotation</th></tr>
<tr><td>6.NS.1</td><td>Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, by using visual fraction models and equations to represent the problem.</td><td>Example: Create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

Example:[3]


</td></tr>
</table>

[3] Common Core Standards Writing Team. (2013, July 4). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 6-8. The Number System; High School, Number.* Tucson, AZ: Institute for Mathematics Education, University of Arizona. For more updates and more information about the Progressions, see http://ime.math.arizona.edu/progressions. For discussion of the Progressions and related topics, see the Tools for the Common Core blog: http://commoncoretools.me.

Cluster: Compute fluently with multi-digit numbers and find common factors and multiples.		
Code	**Standards**	**Annotation**
6.NS.2	Fluently divide multi-digit numbers using strategies flexibly, including the standard algorithm.	**Fluency (Computational):** Having efficient, flexible and accurate methods for computing. **Fluency (Procedural):** Skill in carrying out procedures, flexibly, accurately, efficiently and appropriately.
6.NS.3	Fluently add, subtract, multiply, and divide multi-digit decimals using strategies flexibly, including the standard algorithm for each operation.	
6.NS.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor.	Example: Express 36 + 8 as 4 (9 + 2). This is leading into algebraic topics, including factoring expressions and the distributive property with variables. The focus should not be on simplifying fractions or finding least common denominators.

Cluster: Apply and extend previous understandings of numbers to the system of rational numbers.		
Code	**Standards**	**Annotation**
6.NS.5	Understand that rational numbers are used together to describe quantities having opposite directions or values (may include temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge, etc.). Use rational numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	**Rational number:** A number expressible in the form $\frac{a}{b}$ or $-\frac{a}{b}$ for some fraction $\frac{a}{b}$, when b ≠ 0. The rational numbers include the integers. (See Glossary)
6.NS.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line. Recognize that the opposite of the opposite of a number is the number itself, for example – (–3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane Recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram. Find and position pairs of integers and other rational numbers on a coordinate plane.	**Integer:** (1) A number expressible in the form a or –a for some whole number a (2) the set of whole numbers and their opposites. (See Glossary)

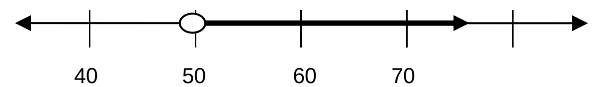
6.NS.7	Understand ordering and absolute value of rational numbers.	**<u>Absolute value</u>:** The distance a number is from zero on a number line. Example: $\lvert -52 \rvert$ = 52 and $\lvert 52 \rvert$ = 52.
	a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.	Example: Interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.
	b. Write, interpret, and explain statements of order for rational numbers in real-world contexts.	Example: Write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.
	c. Understand the absolute value of a rational number as its distance from 0 on the number line. Interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.	Example: For an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.
	d. Distinguish comparisons of absolute value from statements about order.	Example: Recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.
6.NS.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	

Domain: Expressions and Equations — 6.EE

Cluster: Apply and extend previous understandings of arithmetic to algebraic expressions.

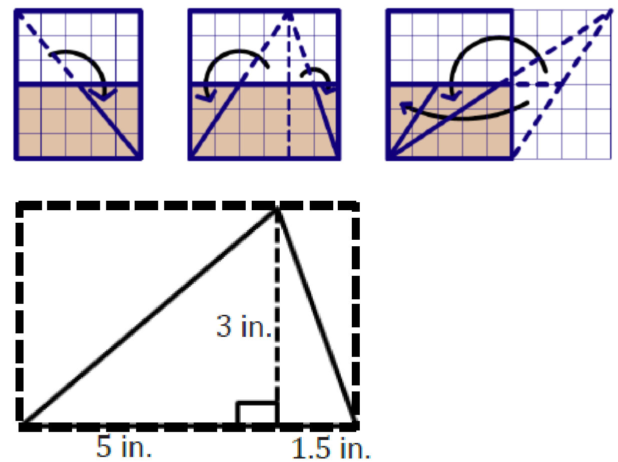
Code	Standards	Annotation
6.EE.1	Write and evaluate numerical expressions involving whole-number exponents.	This standard includes evaluating expressions using the order of operations, including parentheses. **Order of Operations:** 1. Grouping Symbols 2. Exponents 3. Division or Multiplication from left to right 4. Subtraction or Addition from left to right
6.EE.2	Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient, difference, quantity, etc.); view one or more parts of an expression as a single entity. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).	Example: Express the calculation “Subtract y from 5” as 5 – y. **<u>Coefficient</u>:** Any given number multiplied by (in front of) a given variable. Example: In 2x + 3, the 2 is the coefficient. Example: Describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. Part C includes two ideas: (1) simplifying expressions when a value for a variable is given; and (2) using the order of operations with no parentheses to simplify expressions with exponents. The examples below show both of these ideas. Example for idea 1: What is 15 – y when y = 2? Answer: 15 – 2 = 13. Example for idea 2: Evaluate $3x + 2y^2$ when x = 1 and y = 2. Answer: $3(1) + 2(2)^2 = 3(1) + 2(4) = 3 + 8 = 11$. Example: Use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.
6.EE.3	Apply the properties of operations to generate equivalent expressions.	Example: Apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.
6.EE.4	Identify when two expressions are equivalent.	Two expressions are equivalent when the two expressions represent the same number regardless of which value is substituted into them. Example: 5x is equivalent to 2x + 3x. Example: The expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.

Cluster: Reason about and solve one-variable equations and inequalities.		
Code	**Standards**	**Annotation**
6.EE.5	Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	Example: 4x + 3x = 3x + 20. For the equation above, which of the following numbers would make this a true statement? (23/7, 5, 8, 10/2) Answers: x = 5, x = 10/2.
6.EE.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem. Understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	
6.EE.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	Example: Solve the equation x + 7 1/2 = 19. Example: Solve the equation 22.2 = 3x. **Nonnegative rational numbers:** The positive rational numbers and zero. Represent solutions on a number line. (See Standard 6.NS.6)

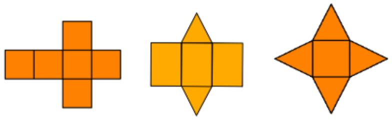
6.EE.8	Write a statement of inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	Note that inequalities are represented by the following symbols: $<, >, \leq, \geq, \neq$. **<u>Constraint</u>:** A limitation; a condition which must be satisfied. **<u>Condition</u>:** An assumption on which rests the validity or effect of something else; a circumstance. Example: A friend would like you to spend more than $50 on her birthday present. Represent this statement as an inequality and represent the solutions of the inequality on a number line. Solution: x > $50.  The constraint of inclusive or exclusive points should be addressed in examples. Additional discussion about a constraint of 0 in a real-world situation could be held. **Compound inequalities should not be addressed at this time.**

Cluster: Represent and analyze quantitative relationships between dependent and independent variables.

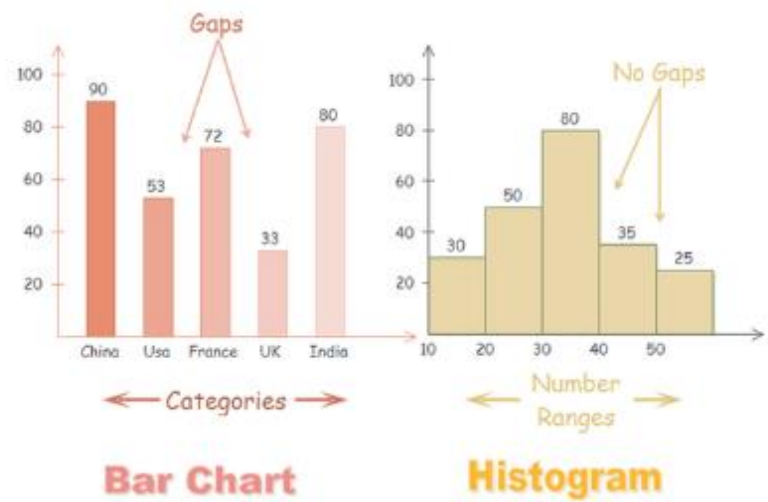
Code	Standards	Annotation
6.EE.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another. Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	Example: Given a formula with two variables where one variable is dependent on the other (e.g., $d = rt$ at a constant rate, or $V = IR$ for a constant resistance), use a table and a graph to show the relationship between the two variables. Example: In a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

Domain: Geometry		6.G
Cluster: Solve real-world and mathematical problems involving area, surface area, and volume.		
Code	**Standards**	**Annotation**
6.G.1	Based on prior knowledge of area of rectangles, decompose or compose triangles to find the area of a triangle. Using knowledge of area of triangles and rectangles, compose and/or decompose triangles, special quadrilaterals, and polygons to find their areas. Apply these techniques in the context of solving real-world mathematical problems.	Example: Find the area of right triangles and other triangles. Students should develop a fluent way of finding the area of a triangle. Example: Find the area of special quadrilaterals and polygons by dividing the shape into triangle and rectangles. Find the area of each part, and add areas together. Example:[4]  Using the shape composition and decomposition skills acquired in earlier grades, students learn to develop area formulas for parallelograms, then triangles. They learn how to address three different cases for triangles: a height that is a side of a right triangle, a height that “lies over the base” and a height that is outside the triangle.

[4] Examples from EngageNY.org and Common Core Standards Writing Team. (2013, July 4). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 6-8. The Number System; High School, Number.* Tucson, AZ: Institute for Mathematics Education, University of Arizona. For more updates and more information about the Progressions, see http://ime.math.arizona.edu/progressions. For discussion of the Progressions and related topics, see the Tools for the Common Core blog: http://commoncoretools.me.

6.G.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	A set of cubes with fractional lengths is needed. Teachers also will need an empty container with fractional edges. The variable B represents area of the base, while b refers to the length of an edge of a polygon.
6.G.3	Draw polygons in the coordinate plane given coordinates for the vertices. Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	Example: The vertices of a rectangle are located at (-2, -1), (2, -1), (2, 3), and (-2, 3). Graph the rectangle. Use the lengths of the sides to find the perimeter. **The focus is not integer operations.**
6.G.4	Represent three-dimensional figures using nets made up of rectangles and triangles (right prisms and pyramids whose bases are triangles and rectangles). Use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	Net: A two-dimensional representation of a three-dimensional shape. 

Domain: Statistics and Probability		6.SP
Cluster: Develop understanding of statistical variability.		
Code	**Standards**	**Annotation**
6.SP.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.	Example: “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.
6.SP.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	Note that measures of center include mean, median, and mode; measures of spread (variation) include range, interquartile range, mean absolute deviation, and standard deviation (high school standard); and overall shape in this context refers to the shape of a graphical representation of data including uniform, skewed, symmetric, and normal (bell-shaped). Additional terminology includes outliers, gaps, clusters, and peaks.
6.SP.3	Recognize that a measure of center for a numerical data set summarizes all of its values using a single number, while a measure of spread (variation) describes how its values vary with a single number.	Example: Given the data set (1, 1, 2, 3, 4): The measures of center are: mean = (1 + 1 + 2 + 3 + 4)/5 = 2.2. median = 2 mode = 1 Note that each measure of center is a single number, although mode may be represented by multiple values or none. The measures of spread are: Range (Max value – Min value): 4 – 1 = 3. Deviations (Mean – data point): 2.2 – 1 = 1.2. 2.2 – 2 = 0.2; 2.2 – 3 = -0.8; 2.2 – 4 = -1.8. Mean absolute deviation (average of the absolute value of each deviation): (1.2 + 1.2 + 0.2 + 0.8 + 1.8)/5 = 1.04. Note that the measures of spread are comparisons between various points within the distribution.

Cluster: Summarize and describe distributions.		
Code	**Standards**	**Annotation**
6.SP.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	**Box plot:** A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[5] (See Glossary) **Histogram:** A bar graph which shows frequency for numerical data within equivalent intervals. Example:[6]  **Dot plot:** A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a line plot. (See Glossary)

[5] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.
[6] Example taken from: www.quora.com/What-is-the-difference-between-a-histogram-and-a-bar-graph

6.SP.5	Summarize numerical data sets in relation to their context, such as by:	
	a. Reporting the number of observations.	
	b. Describing the nature of the attribute being investigated, including how it was measured and its units of measurement.	Describe how the data was collected and what discrepancies may or may not exist.
	c. Calculating quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data was gathered.	**<u>Interquartile range</u>:** A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: first quartile, third quartile. (See Glossary) **<u>Mean absolute deviation</u>:** A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20. (See Glossary)
	d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	This standard does not explicitly include misleading information, including graphs. However, discussion of choice of measures may address this concept. Example: Eight theater critics were asked to score a play on a 12 point scale. The scores were 6, 12, 3, 3, 11, 3, 7, and 3. Find the mean, median, and mode. Which of these is the most appropriate choice of measure to describe this data?

Mathematics | Grade 7

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Grade 7 Overview

Ratios and Proportional Relationships

- Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Expressions and Equations

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry

- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Statistics and Probability

- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Domain: Ratios and Proportional Relationships — 7.RP

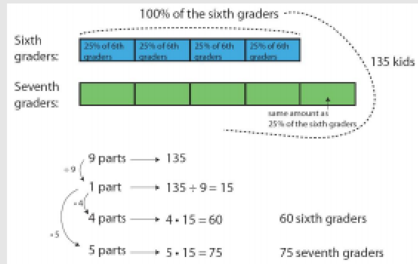
Cluster: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Code	Standards	Annotation
7.RP.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.	**<u>Unit rate</u>:** A rate is simplified so that it has a denominator of 1 unit (e.g., miles per gallon, kilometers per second). Example: If a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2 / 1/4 miles per hour, equivalently 2 miles per hour.
7.RP.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations.	The conceptual understanding is the focus at this point. Cross products and other procedural methods are not necessary. **<u>Proportional relationship</u>:** Varying in the same manner as another quantity, especially increasing if another quantity increases or decreasing if it decreases. In a directly proportional relationship an arbitrary variable (x) is equal to a constant (k) times another variable (y). Formula: x = ky Example: Linda buys pencils for her office supply store every month. A case of pencils has a cost, *c* and she purchases *n* cases every month. She uses this information to find the total cost, *t*, of pencils each month. Write the equation for this proportional relationship. Solution: $t = cn$ Example: If total cost t is proportional to the number n of items purchased at a constant price p, the relationship between total cost and the number of items can be expressed as $t = pn$.
	d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	Example: The equation of a line represents a proportional relationship where the constant of proportionality is a unit rate. For the equation $y = rx$, show how the points (0,0) and (1,r) on this line relate to r, the unit rate. (Slope is introduced in eighth grade.)

7.RP.3	Use proportional relationships to solve multi-step ratio and percent problems.	Examples: Simple interest, tax, markups and markdowns/discounts, gratuities and commissions, fees, percent increase and decrease, percent error. Examples:[7]

Using percentages in comparisons

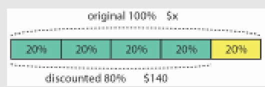
There are 25% more seventh graders than sixth graders in the after-school club. If there are 135 sixth and seventh graders altogether in the after-school club, how many are sixth graders and how many are seventh graders?



"25% more seventh graders than sixth graders means that the number of extra seventh graders is the same as 25% of the sixth graders."

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.

Skateboard problem 1



After a 20% discount, the price is 80% of the original price. So 80% of the original is $140.

percent	dollars
80% →	$140
20% →	$35
100% →	$175

÷ 4; · 5 or add 80%+20%; · 5 or add $140+$35

"To find 20% I divided by 4. Then 80% plus 20% is 100%"

x = original price in dollars

	percent		dollars
discounted / original	$\frac{80}{100}$	=	$\frac{140}{x}$

$$80x = 140 \cdot 100$$

$$x = \frac{140 \cdot 100}{80} = \frac{(2 \cdot 7 \cdot 2 \cdot 5)(2 \cdot 5 \cdot 10)}{2 \cdot 2 \cdot 2 \cdot 10} = 7 \cdot 5 \cdot 5 = 175$$

80% of the original price is $140.

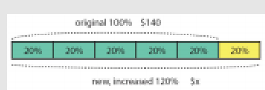
$$\frac{80}{100} \cdot x = 140$$

$$\frac{4}{5} \cdot x = 140$$

$$x = 140 \div \frac{4}{5} = 140 \cdot \frac{5}{4} = \frac{(2 \cdot 7 \cdot 2 \cdot 5) \cdot 5}{4} = 175$$

Before the discount, the price of the skateboard was $175.

Skateboard problem 2



After a 20% increase, the price is 120% of the original price. So the new price is 120% of $140.

percent	dollars
100% →	$140
20% →	$28
100% →	$168

÷ 5; · 6 or add 100%+20%; · 6 or add $140+$28

"To find 20% I divided by 5. Then 100% plus 20% is 120%"

x = increased price in dollars

	percent		dollars
discounted / original	$\frac{120}{100}$	=	$\frac{x}{140}$
	$\frac{12}{10}$	=	$\frac{x}{140}$

$$x = 140 \cdot \frac{12}{10} = 14 \cdot 12 = 168$$

The new, increased price is 120% of $140.

$$x = \frac{120}{100} \cdot 140 = \frac{2 \cdot 6 \cdot 10}{2 \cdot 5 \cdot 10} \cdot 14 \cdot 2 \cdot 5 = 168$$

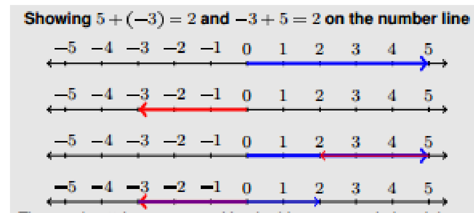
The new price after the increase is $168.

[7] Example obtained from achievethecore.org

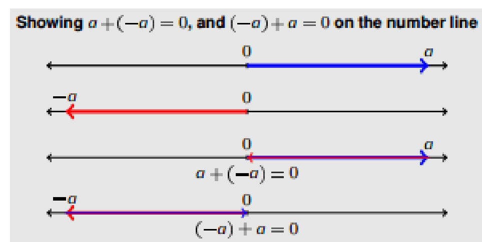
Domain: The Number system — 7.NS

Cluster: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Code	Standards	Annotation
7.NS.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.	
	a. Describe situations in which opposite quantities combine to make 0.	Example: A hydrogen atom has 0 charge because its two constituents are oppositely charged.
	b. Understand *p* + *q* as the number located a distance |*q*| from *p* on a number line, in the direction indicated by the sign of *q*. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.	Example:[8] (see figures below)



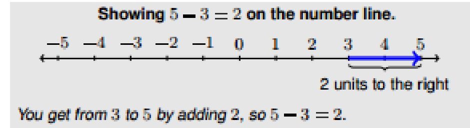
The number 5 is represented by the blue arrow pointing right from 0, and the number −3 is represented by the red arrow pointing left from 0. To add $5+(-3)$ we place the arrow for 5 down first then attach the arrow for −3 to its endpoint. To add $-3+5$ we place the arrow for −3 down first then attach the arrow for 5 to its endpoint.

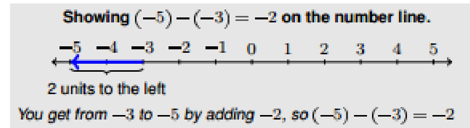


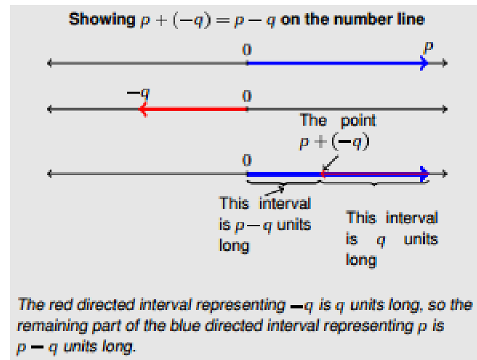
[8] Examples from Common Core Standards Writing Team. (2013, July 4). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 6-8. The Number System; High School, Number.* Tucson, AZ: Institute for Mathematics Education, University of Arizona. For more updates and more information about the Progressions, see http://ime.math.arizona.edu/progressions. For discussion of the Progressions and related topics, see the Tools for the Common Core blog: http://commoncoretools.me.

<table>
<tr><td></td><td>c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$.

Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.</td><td>Example:[9]



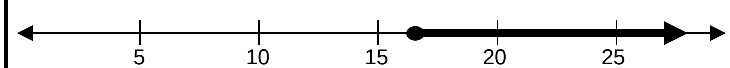



</td></tr>
<tr><td></td><td>d. Apply properties of operations as strategies to add and subtract rational numbers.</td><td>Strategies may include algebra tiles, colored chips, number lines, etc.</td></tr>
</table>

[9] Examples from Common Core Standards Writign Team. (2013, July 4). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 6-8. The Number System; High School, Number.* Tucson, AZ: Institute for Mathematics Education, University of Arizona. For more updates and more information about the Progressions, see http://ime.math.arizona.edu/progressions. For discussion of the Progressions and related topics, see the Tools for the Common Core blog: http://commoncoretools.me.

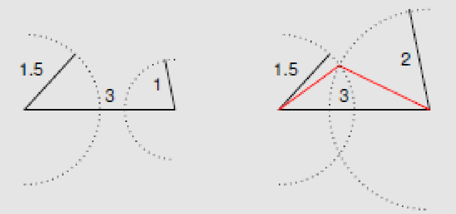
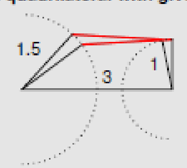
7.NS.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying rational numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$. Interpret products of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division. Know that the decimal form of a rational number terminates or eventually repeats.	Strategies may include algebra tiles, number lines, colored chips, etc. This would be an opportunity to compare the values of rational numbers by writing them in fraction, decimal or percent form.
7.NS.3	Solve real-world and mathematical problems involving the four operations with rational numbers.	Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Domain: Expressions and Equations		7.EE
Cluster: Use properties of operations to generate equivalent expressions.		
Code	**Standards**	**Annotation**
7.EE.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	See Tables 3 and 4 in the Glossary for the properties of operations. Emphasis should be on writing equivalent expressions Example: 7 – 2(3 - 8x) 7 – 6 + 16x 7 – [2(3 - 8x)] 7 + (-2)[3 + (-8)x] 1 + 16x
7.EE.2	Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related.	Example: a + 0.05a = 1.05a means that “increase by 5%” is the same as “multiply by 1.05.”

Cluster: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.		
Code	**Standards**	**Annotation**
7.EE.3	Solve multi-step real-life and mathematical problems posed with rational numbers in any form (positive and negative integers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form. Convert between forms as appropriate. Assess the reasonableness of answers using mental computation and estimation strategies.	Example: If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. Note that tools may include any resource needed, including, but not limited to: equations, operations, inverse operations, technologies, manipulatives, and estimation. (See Mathematical Practice #5). Example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour. Examples: Simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
7.EE.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	
	a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare the algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.	Example: The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? Answer: 2w + 2(6) = 54 or 2(w + 6) = 54.
	b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.	Example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions. Answer : $50 + 3x \geq 100$ $3x \geq 100-50$ $3x \geq 50$ $x \geq 50/3$ $x \geq 16\ 2/3$  In order to earn \$100 in a week, you must make more than 16 2/3 sales. Since fractional sales cannot be made, this means you must make at least 17 sales this week. **This does not include compound inequalities.**

Domain: Geometry 7.G

Cluster: Draw, construct, and describe geometrical figures and describe the relationships between them.

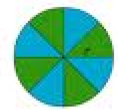
Code	Standards	Annotation
7.G.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	
7.G.2	Draw geometric shapes from given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. Use a variety of methods such as freehand, with ruler and protractor, and with technology.	Example:[10] **Constructing a triangle with given side lengths**  It is not possible to construct a triangle with side lengths 1, 1.5, and 3. No matter how you move the smaller sides around at the ends of the largest side they will never meet, because $1 + 1.5 < 3$. If you increase the 1 to 2, you can create a triangle by finding the intersection of circles as shown. • What does "exactly one" mean? In Grade 7, two triangles with the same side lengths are considered the same if one can be moved on top of the other, so that they match exactly. In Grade 8, the movement will be described in terms of rigid motions. **Constructing a quadrilateral with given side lengths**  The base is fixed and the two sides are of fixed length as they move around circles centered at ends of the base. The top is a rigid rod of fixed length that moves with its endpoints on the circles, creating many quadrilaterals with the same side lengths.
7.G.3	Describe the cross-sections (two-dimensional figures that result from slicing three-dimensional figures, as in plane sections) of right rectangular prisms and right rectangular pyramids.	

[10] Examples from Common Core Standards Writing Team. (2013, July 4). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 6-8. The Number System; High School, Number.* Tucson, AZ: Institute for Mathematics Education, University of Arizona. For more updates and more information about the Progressions, see http://ime.math.arizona.edu/progressions. For discussion of the Progressions and related topics, see the Tools for the Common Core blog: http://commoncoretools.me.

<table>
<tr><th colspan="3">Cluster: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.</th></tr>
<tr><th>Code</th><th>Standards</th><th>Annotation</th></tr>
<tr><td>7.G.4</td><td>Know the formulas for the area and circumference of a circle and use them to solve problems.

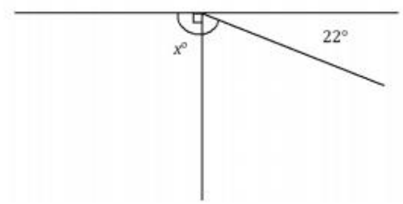
Informally derive the relationship between the circumference and area of a circle.</td><td>To “know the formulas” in this context means to develop and understand formulas, recall them, and apply them in problem solving.

Example:

Draw a circle with divisions. The divisions can then be drawn as a parallelogram.


The circumference of the circle is:
$C = 2\pi r$

The area of the parallelogram (and therefore the circle) is:
$A = bh$

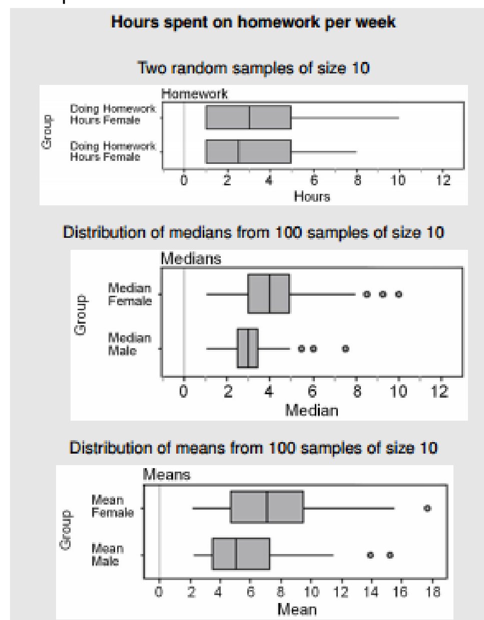
This can be related to the radius of the circle (note: the length of the base of the parallelogram is half the circumference, or πr, and the height is r, $A = \pi r \cdot r$ or A = πr^2.</td></tr>
<tr><td>7.G.5</td><td>Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve equations for an unknown angle in a figure.</td><td>Example:[11]
In a complete sentence, describe the relevant angle relationships in the diagram. Write an equation for the angle relationship shown in the figure, and solve for x. Confirm your answers by measuring the angle with a protractor.


The angles $x°$ and $22°$ are supplementary and sum to $180°$.
$$x + 22 = 180$$
$$x + 22 - 22 = 180 - 22$$
$$x = 158$$
The measure of the angle is $158°$.</td></tr>
</table>

[11] Example obtained from engageNY.org

7.G.6	Solve real-world and mathematical problems involving area of two-dimensional figures composed of polygons and/or circles, including composite figures. Use nets to solve real-world and mathematical problems involving surface area of prisms and cylinders, including composite solids. Solve real-world and mathematical problems involving volumes of right prisms, including composite solids.	Example: Find the surface area of a cylinder, using equations for the area of a circle and a quadrilateral. Prisms are not limited to those with triangular and rectangular bases.

Domain: Statistics and Probability		7.SP
Cluster: Use random sampling to draw inferences about a population.		
Code	**Standards**	**Annotation**
7.SP.1	Understand that statistics can be used to gain information about a population by examining a sample of the population. Understand that generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	
7.SP.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.	Example: Estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Cluster: Draw informal comparative inferences about two populations.		
Code	**Standards**	**Annotation**
7.SP.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.	Example: The mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. The focus is using a graphic to visually display variability between and within two data sets. Example:[12]  *Source: Census at Schools Project, amstat.org/censusatschool/*
7.SP.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.	Example: Decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

[12] Example obtained from Census at School Project, amstat.org/censusatschool/

Cluster: Investigate chance processes and develop, use, and evaluate probability models.		
Code	**Standards**	**Annotation**
7.SP.5	Understand that the probability of a chance event is a number from 0 through 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	
7.SP.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency. Predict the approximate relative frequency given the probability.	Example: When rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. Note: This is the introduction to theoretical probability and experimental/empirical probability.
7.SP.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies. If there is a discrepancy, explain possible sources. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.	Note: This is a comparison of theoretical and experimental/empirical probabilities. Example: If a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. Example: Find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
7.SP.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (such as “rolling double sixes”), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events.	A formal procedure is not necessary at this level. Example: Use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Mathematics | Grade 8

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Grade 8 Overview

The Number System

- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability

- Investigate patterns of association in bivariate data.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

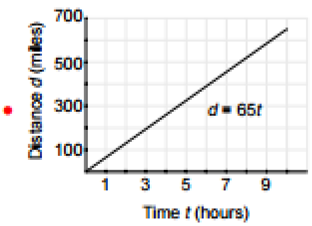
Domain: The Number System		8.NS
Cluster: Know that there are numbers that are not rational, and approximate them by rational numbers.		
Code	**Standards**	**Annotation**
8.NS.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually. Convert a decimal expansion which repeats eventually into a rational number.	
8.NS.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (such as π^2).	Example: By truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Domain: Expressions and Equations		8.EE
Cluster: Work with radicals and integer exponents.		
Code	**Standards**	**Annotation**
8.EE.1	Develop, know and apply the properties of integer exponents to generate equivalent numeric and algebraic expressions.	Conceptual understanding of the rules is necessary. Example: $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. Example: $\frac{2x^{-4}y^5}{6x^2y^{-4}} = \frac{2y^4y^5}{6x^2x^4} = \frac{y^9}{3x^6}$
8.EE.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Classify radicals as rational or irrational.	Example: $x^2 = 25, \sqrt{x^2} = \sqrt{25}$, $x = \pm 5$ Example: $x^3 = 125$, $\sqrt[3]{x^3} = \sqrt[3]{125}$, x = 5
8.EE.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.	Example: Estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.
8.EE.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (such as use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	Scientific notation: a way of representing large or small numbers by using a number from 1 up to (but not including) 10 times an integer power of 10.

<table>
<tr><th colspan="3">Cluster: Understand the connections between proportional relationships, lines, and linear equations.</th></tr>
<tr><th>Code</th><th>Standards</th><th>Annotation</th></tr>
<tr>
<td>8.EE.5</td>
<td>Graph proportional relationships, interpreting the unit rate as the slope of the graph.

Compare two different proportional relationships represented in different ways.</td>
<td>Example:[13] Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

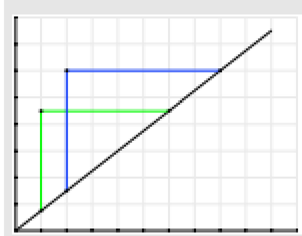
<table>
<tr><th>t (hours)</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr>
<tr><td>$60t$ (miles)</td><td>60</td><td>120</td><td>180</td><td>240</td><td>300</td><td>360</td></tr>
</table>



• In the Grade 8 Functions domain, students see the relationship between the graph of a proportional relationship and its equation $y = mx$ as a special case of the relationship between a line and its equation $y = mx + b$, with $b = 0$.

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.</td>
</tr>
<tr>
<td>8.EE.6</td>
<td>Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane.

Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.</td>
<td>Example:[13]

Why lines have constant slope

The green triangle is similar to the blue triangle because corresponding angles are equal, so the ratio of rise to run is the same in each.</td>
</tr>
</table>

[13] Examples from Common Core Standards Writing Team. (2013, July 4). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 6-8. The Number System; High School, Number.* Tucson, AZ: Institute for Mathematics Education, University of Arizona. For more updates and more information about the Progressions, see http://ime.math.arizona.edu/progressions. For discussion of the Progressions and related topics, see the Tools for the Common Core blog: http://commoncoretools.me.

Cluster: Analyze and solve linear equations and pairs of simultaneous linear equations.		
Code	**Standards**	**Annotation**
8.EE.7	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form *x* = *a*, *a* = *a*, or *a* = *b* results (where *a* and *b* are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	Collecting like terms is combining like terms when they appear on both sides of the equation. Example : 2(x+7) = 3x + 16 2x + 14 = 3x + 16 2x – 3x = 16 – 14 -x = 2 x = -2
8.EE.8	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. c. Solve simple cases by inspection. d. Solve real-world and mathematical problems leading to two linear equations in two variables.	Example: *3x + 2y = 5 and 3x + 2y = 6* have no solution because *3x + 2y* cannot simultaneously be 5 and 6. Methods of solving a system include solving graphically, with substitution, and linear combination (elimination). Example: Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Domain: Functions 8.F

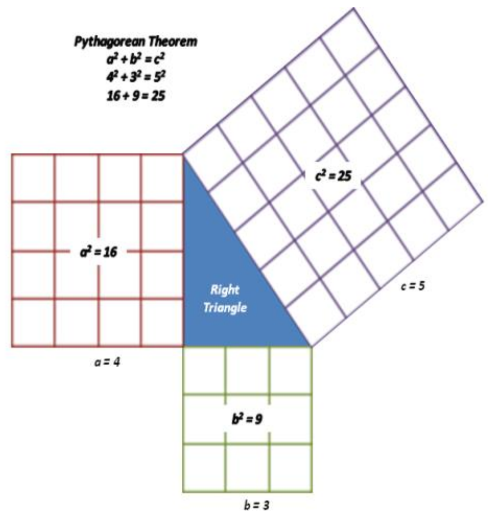
Cluster: Define, evaluate, and compare functions.

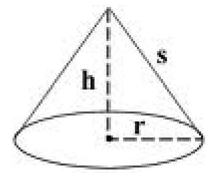
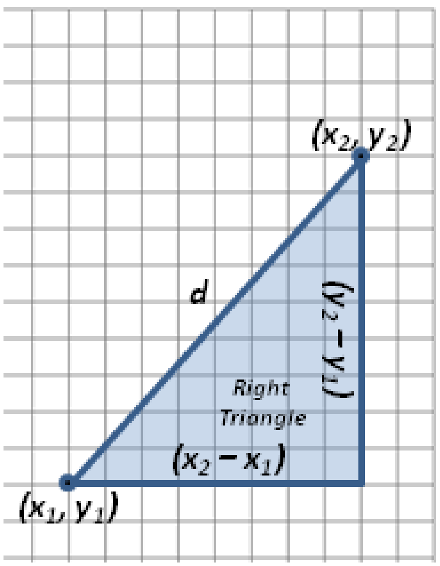
Code	Standards	Annotation
8.F.1	Understand that a function is a rule that assigns to each input exactly one output. Understand that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	Function notation is not required in Grade 8.
8.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, and/or by verbal descriptions).	A variety of methods may be used to demonstrate functions, such as mapping, function table, graph (vertical line test), etc. Example: Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
8.F.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line. Give examples of functions that are not linear.	Example: The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

Cluster: Use functions to model relationships between quantities.

Code	Standards	Annotation
8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	In a linear function, the rate of change is constant and is referred to as slope. Initial value is also referred to as the y-intercept.
8.F.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (may include where the function is increasing or decreasing, linear or nonlinear, etc.). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	A function which is increasing displays a positive slope, while a function which is decreasing displays a negative slope.

Domain: Geometry		**8.G**
Cluster: Understand congruence and similarity using physical models, transparencies, or geometry software.		
Code	**Standards**	**Annotation**
8.G.1	Understand the properties of rotations, reflections, and translations by experimentation: a. Lines are transformed onto lines, and line segments onto line segments of the same length. b. Angles are transformed onto angles of the same measure. c. Parallel lines are transformed onto parallel lines.	
8.G.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. Given two congruent figures, describe a sequence of transformations that exhibits the congruence between them.	
8.G.3	Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.	
8.G.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. Given two similar two-dimensional figures, describe a sequence of transformations that exhibits the similarity between them.	
8.G.5	Use informal arguments to establish facts about the angle sum and exterior angles of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	Angle relationships can be verified experimentally using transformations. Example: Arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Cluster: Understand and apply the Pythagorean Theorem.		
Code	**Standards**	**Annotation**
8.G.6	Explain a proof of the Pythagorean Theorem and its converse.	The Pythagorean Theorem Converse states that if three measures of a triangle form a Pythagorean Triple, then the triangle is a right triangle. One example of a proof of the Pythagorean Theorem: We can form a square off of each side of a right triangle and divide these squares into partitions based on the length of the side. Example: The leg with length *a* will have a length of *a* and width *a.* The area of the square adjacent to *a* is $a \times a = a^2 = 9$. The area of the square adjacent to *b* is $b \times b = b^2 = 16$. The area of the square adjacent to *c* is $c \times c = c^2 = 25$. 9 + 16 =25, therefore $a^2 + b^2 = c^2$. 

8.G.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	Example: A food company is designing ice cream cones. They want the height of the cone to be 4 inches and the radius to be 2.5 inches. Find the length of the sloping side of the cone. $h^2 + r^2 = s^2$  Answer: Since h = 4 and r = 2.5, using the formula: $4^2 + (2.5)^2 = s^2 = 22.25$ s = 4.72 in
8.G.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	Example :  The focus should be on the Pythagorean Theorem and not using the distance formula.

Cluster: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.		
Code	**Standards**	**Annotation**
8.G.9	Know the formulas for the volume of cones, cylinders and spheres. Use the formulas to solve real-world and mathematical problems.	In this context, “know the formulas” means to develop and understand, recall the formula, and apply the formulas in problem solving.

Domain: Statistics and Probability		8.SP
Cluster: Investigate patterns of association in bivariate data.		
Code	**Standards**	**Annotation**
8.SP.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	**Bivariate data:** Pairs of linked numerical observations. (See Glossary) Example: A list of heights and weights for each player on a football team.
8.SP.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	
8.SP.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept(s).	Example: In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
8.SP.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.	**Relative frequency:** The ratio of the number of times that an event happens to the number of trials in which the event can happen or fail to happen.[14] Example: Collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Mathematics Standards for High School –Comments from the Committee

Upon extensive review of the Common Core document, it is the opinion of the North Dakota review committee that there are specific standards that create an advanced expectation for students. The review committee has identified these standards with a ↑ preceding the comment. The committee has made an effort to provide commentary, examples, pre-requisites and real-world applications in order to provide context and relevance to these standards. These specified standards will most likely be the focus of collegial discussion regarding the relevance and meaningful delivery of instruction for the foreseeable future.

The Common Core Mathematics standards at the high school level represent a significant increase in rigor from the expectations in the 2005 version of North Dakota mathematics standards. The conceptual category of Statistics and Probability in the Common Core document is particularly rigorous in comparison to the 2005 state standards and will require further discussion and potential professional development when considering curricular offerings.

[14] Source: Mathematics Dictionary, Glenn James, 1960.

In addition, teachers might need further professional development to address the realignment of topics within existing courses or new courses. To assist all students in achieving these standards, teachers will need to adopt additional research-based instructional strategies, which might require additional professional development. Mastery of the Common Core State Standards at the high school level is dependent upon student proficiency in Grades K-8, in addition to exposure to rigorous mathematics courses at the secondary level.

Mathematics Standards for High School

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics, is indicated by (+), as in this example:

(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

Students may achieve the (+) standards through differentiated instruction in a common mathematics curriculum or in an advanced course.

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students.

Standards without a (+) symbol may also appear in courses intended for all students. The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Mathematics | High School—Number and Quantity

Numbers and Number Systems

During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways. They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities

In real-world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Number and Quantity Overview

The Real Number System

- Extend the properties of exponents to rational exponents.
- Use properties of rational and irrational numbers.

Quantities

- Reason quantitatively and use units to solve problems.

The Complex Number System

- Perform arithmetic operations with complex numbers.
- Represent complex numbers and their operations on the complex plane.
- Use complex numbers in polynomial identities and equations.

Vector and Matrix Quantities

- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Domain: The Real Number System		HS.N-RN
Cluster: Extend the properties of exponents to rational numbers.		
Code	**Standards**	**Annotation**
HS.N-RN.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	Example: We define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.
HS.N-RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.	Example: $\sqrt{x^3} = x^{\frac{3}{2}}$ Example: $(\sqrt{4})^3 = ((4)^{\frac{1}{2}})^3 = 2^3 = 8$
Cluster: Use properties of rational and irrational numbers.		
Code	**Standards**	**Annotation**
HS.N-RN.3	Demonstrate that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.	Example: Evaluate $\sqrt{2} \cdot \sqrt{4}$ and identify which subset of the real number system the solution is in. Solution: $\sqrt{8} = 2\sqrt{2}$, which is irrational.
HS.N-RN.4	Perform basic operations on radicals and simplify radicals to write equivalent expressions.	Basic operations include addition, subtraction, multiplication and division (e.g., rationalizing the denominator).

Note. (+) indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.
** indicates modeling standards.*

Domain: Quantities* (Mathematical Practices 1, 4, and 6)		HS.N-Q
Cluster: Reason quantitatively and use units to solve problems.		
Code	**Standards**	**Annotation**
HS.N-Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems (e.g. unit analysis). Choose and interpret units consistently in formulas. Choose and interpret the scale and the origin in graphs and data displays.	
HS.N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.	Example: When carpeting a room, students might consider whether it is best to use square feet or square yards. When considering a remodeling project, they might choose such units as cost per room, cost per month of the project, or cost per contractor. For further discussion of this standard, please see the Number and Quantity introduction.
HS.N-Q.3	Choose a level of accuracy or precision appropriate to limitations on measurement when reporting quantities.	Example: When using a ruler, students choose to report their measurements based on the precision of the ruler (e.g., to the nearest 1/16 or the nearest 1/32). Example: When using a ruler, students are able to measure accurately. Example: When calculating the cost of a road trip, students are given the cost of gasoline to the thousandths place. When reporting the cost of the trip, students determine what level of precision—to the hundredths place or to the thousandths place—is appropriate and why.

Note. (+) indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.
** indicates modeling standards.*

Domain: The Complex Number System		HS.N-CN
Cluster: Perform arithmetic operations with complex numbers.		
Code	**Standards**	**Annotation**
HS.N-CN.1	Know there is an imaginary number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. Understand the hierarchal relationships among subsets of the complex number system.	Knowledge of complex numbers extends and reinforces student knowledge of the real number system. Example: $\sqrt{8}$ is a complex number because it can be written in the form $\sqrt{8} + 0i$. $\sqrt{8}$ is also a real number since its imaginary coefficient is 0. $\sqrt{8}$ is also an irrational number because it cannot be written as a ratio of two integers.
HS.N-CN.2	Use the definition $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	Knowledge of complex numbers extends and reinforces student knowledge of basic operations and properties of the real number system. Example: $(2 + 3\mathrm{i}) + (4 - 5\mathrm{i}) = 6 - 2\mathrm{i}$ $(2 + 3\mathrm{i}) - (4 - 5\mathrm{i}) = -2 + 8\mathrm{i}$ $(2 + 3\mathrm{i})(4 - 5\mathrm{i}) = 8 - 10\mathrm{i} + 12\mathrm{i} - 15\mathrm{i}^2 = 8 + 2\mathrm{i} + 15 = 23 + 2\mathrm{i}$
(+)HS.N-CN.3	Find the conjugate of a complex number. Use conjugates to find moduli (absolute value) and quotients of complex numbers.	
(+)HS.N-CN.4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers). Explain why the rectangular and polar forms of a given complex number represent the same number.	
(+)HS.N-CN.5	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.	Example: $(1 - \sqrt{3}i)^3 = 8$ because $(1 - \sqrt{3}i)$ has modulus 2 and argument 120°.
(+)HS.N-CN.6	*This standard has been moved/removed by the committee*	

Note. (+) indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.
** indicates modeling standards.*

Cluster: Use complex numbers in polynomial identities and equations.		
Code	**Standards**	**Annotation**
HS.N-CN.7	Solve quadratic equations with real coefficients that have complex solutions.	This topic is also addressed in HS.A-REI.4.
(+)HS.N-CN.8	Extend polynomial identities to the complex numbers.	Example: Rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$. Polynomial identities include but are not limited to: square of a binomial, difference of squares, sum and difference of cubes. (See Table 6 of the Glossary)
(+)HS.N-CN.9	Apply the Fundamental Theorem of Algebra to determine the number of solutions for polynomial functions. Find all solutions to a polynomial equation.	Fundamental Theorem of Algebra: The number of complex solutions to a polynomial equation is equal to the degree of the polynomial.

Note. (+) indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.
** indicates modeling standards.*

Domain: Vector and Matrix Quantities — HS.N-VM

Cluster: Represent and model with vector quantities.

Code	Standards	Annotation
(+)HS.N-VM.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, v).	
(+)HS.N-VM.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	
(+)HS.N-VM.3	Solve problems involving velocity and other quantities that can be represented by vectors.	

Cluster: Perform operations on vectors.

Code	Standards	Annotation
(+)HS.N-VM.4	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand that vector subtraction ***v*** – ***w*** is defined as ***v*** + (–***w***), where –***w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	
(+)HS.N-VM.5	Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction. Perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple *c**v*** using ||*c**v***|| = |*c*|***v***. Compute the direction of *c*v knowing that when |*c*|***v*** ≠ 0, the direction of *c**v*** is either along ***v*** (for *c* > 0) or against ***v*** (for *c* < 0).	

Note. (+) indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.
** indicates modeling standards.*

Cluster: Perform operations on matrices and use matrices in applications.		
Code	**Standards**	**Annotation**
HS.N-VM.6*	Use matrices to represent and manipulate data.	
HS.N-VM.7	Multiply matrices by scalars to produce new matrices.	
HS.N-VM.8	Add, subtract, and multiply matrices of appropriate dimensions.	
HS.N-VM.9	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	
(+)HS.N-VM.10	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	
(+)HS.N-VM.11	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Understand a matrix as a transformation of vectors.	
(+)HS.N-VM.12	Understand a 2 × 2 matrix as a transformation of the plane. Interpret the absolute value of the determinant in terms of area.	

Note. (+) indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.
** indicates modeling standards.*

Mathematics | High School—Algebra

Expression

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid,

$A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Algebra Overview

Seeing Structure in Expressions

- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

Arithmetic with Polynomials and Rational Functions

- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions.

Creating Equations

- Create equations that describe numbers or relationships.

Reasoning with Equations and Inequalities

- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve equations and inequalities in one variable.
- Solve systems of equations.
- Represent and solve equations and inequalities graphically.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Domain: Seeing Structure in Expressions		**HS.A-SSE**
Cluster: Interpret the structure of expressions.		
Code	**Standards**	**Annotation**
HS.A-SSE.1*	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by examining one or more of their parts as a single entity.	Example: Interpret $\frac{1}{2}h(b_1 + b_2)$ as the product of the height of a trapezoid and the average of its base lengths.
HS.A-SSE.2	Use the structure of an expression to identify ways to rewrite it.	Example: See $9a^2 - 4b^2$ as $(3a)^2 - (2b)^2$ and recognize it as a difference of squares that can be factored as $(3a - 2b)(3a + 2b)$. Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$, and further to $(x-y)(x+y)(x^2+y^2)$.
Cluster: Write expressions in equivalent forms to solve problems.		
Code	**Standards**	**Annotation**
HS.A-SSE.3*	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to produce an equivalent expression. c. Use the properties of exponents to transform exponential expressions.	Example: Finding the maximum and minimum of a quadratic function; writing the equation of a circle in standard form to find the center and radius. Example : $8^t = 2^{3t}$. Example: The expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
HS.A-SSE.4*	*This standard has been moved/removed by the committee*	

Note. (+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.*
** indicates modeling standards.*

Domain: Arithmetic with Polynomials and Rational Expressions — HS.A-APR

Cluster: Perform arithmetic operations on polynomials.

Code	Standards	Annotation
HS.A-APR.1	Add, subtract, and multiply polynomials. Understand that polynomials form a system comparable to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.	

Cluster: Understand the relationship between zeros and factors of polynomials.

Code	Standards	Annotation
HS.A-APR.2	Apply the Remainder Theorem.	Remainder Theorem: For a polynomial *p(x)* and a number *a*, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
HS.A-APR.3	Identify zeros of polynomials when suitable factorizations are available. Use the zeros to construct a rough graph of the function defined by the polynomial.	

Cluster: Use polynomial identities to solve problems.

Code	Standards	Annotation
HS.A-APR.4	*This standard has been moved/removed by the committee*	
(+)HS.A-APR.5	Apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n.*	Coefficients in the expansion of $(x + y)^n$ can be determined using Pascal's Triangle or combinations.

Cluster: Rewrite rational expressions.

Code	Standards	Annotation
HS.A-APR.6	Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x)* + *r(x)/b(x)*, where *a(x)*, *b(x)*, *q(x)*, and *r(x)* are polynomials with the degree of r(x) less than the degree of *b(x)*, using inspection, long division, or, for the more complicated examples, a computer algebra system.	Example: Use long division to rewrite: $\frac{3x^3-2x^2+4x-3}{x^2+3x+3}$ In the form: $(3x-11)+\frac{28x+30}{x^2+3x+3}$
HS.A-APR.7	Add, subtract, multiply, and divide rational expressions. Understand that rational expressions form a system comparable to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.	

Note. (+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.*
* *indicates modeling standards.*

Domain: Creating Equations* HS.A-CED

Cluster: Create equations that describe numbers or relationships.

Code	Standards	Annotation
HS.A-CED.1*	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	
HS.A-CED.2*	Create equations in two or more variables to represent relationships between quantities. Graph equations on coordinate axes with appropriate labels and scales.	Example: The cost to rent a car is $50 plus $0.25 per mile driven. Write and graph an equation to represent the situation.
HS.A-CED.3*	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.	Example: Willy Wonka's Chocolate Factory makes *Wonka Bars* and *The Everlasting Gobstopper*, among other amazing treats. Oompa Loompas and Fuzzy Fizzies work on each item. The Oompa Loompas spend 6 minutes making a *Wonka Bar* and 4 minutes mixing the ingredients for an *Everlasting Gobstopper*. There are enough Oompa Loompas for up to 6,000 worker-minutes per day. The Fuzzy Fizzies spend about 1 minute wrapping each Wonka Bar and 2 minutes wrapping each *Everlasting Gobstopper*. There are enough Fuzzy Fizzies for a maximum of 1,200 worker-minutes per day. Write the system of inequalities that represent the situation. Determine whether 500 Wonka Bars and 75 Everlasting Gobstoppers is a viable solution. Solution: Oompa Loompas: (6 min/bar)(x bars) + (4 min/gob) (y gob) $\leq$6,000 min. Fuzzy Fizzies: (1min/bar)(x bars) + (2 min/gob)(y gob) $\leq$1,200 min. Using substitution, y = 150, x = 900 if the maximum number of hours are worked. 500 Wonka Bars and 75 Everlasting Gobstoppers is a viable solution because it satisfies the constraints.
HS.A-CED.4*	Rearrange formulas to isolate a quantity of interest, using the same reasoning as in solving equations.	Example: Rearrange Ohm's law V = IR to isolate resistance R.

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** indicates modeling standards.*

Domain: Reasoning with Equations and Inequalities — HS.A-REI

Cluster: Understand solving equations as a process of reasoning and explain the reasoning.

Code	Standards	Annotation
HS.A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	Use justifiable comments such as "combine like terms," "distributive property," etc. within the explanation.
HS.A-REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	Example: Solve $\sqrt{6-x}=x$ $\left(\sqrt{6-x}\right)^2=x^2$ $6-x=x^2$ $x^2+x-6=0$ $(x-2)(x+3)=0$ $x=2\ or\ x=-3$ Check 2 and -3 in the original equation: $\sqrt{6-2}=\sqrt{4}=2$ $\sqrt{6—3}=\sqrt{9}\neq-3$ Because -3 does not satisfy the original equation, -3 is an extraneous solution. Consequently, 2 is the only solution to the equation.

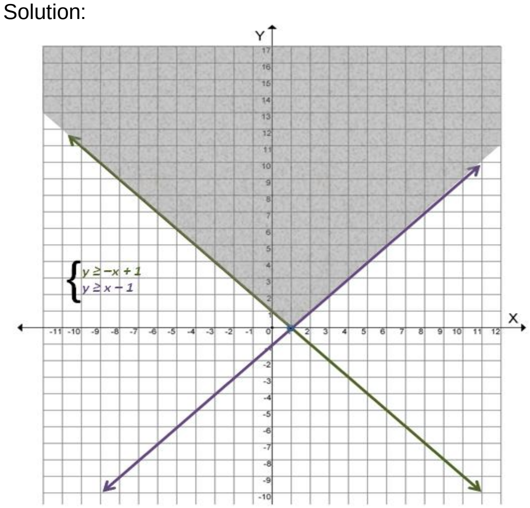
Cluster: Solve equations and inequalities in one variable.

Code	Standards	Annotation
HS.A-REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	Example: Solve for x: 2mx+3mx+mx = 16. Example: Solve for y: 3/4y + 7 > 10.
HS.A-REI.4	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2=q$ that has the same solutions. (+) Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.and *b*.	

Note. (+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.*
** indicates modeling standards.*

Cluster: Solve systems of equations.		
Code	**Standards**	**Annotation**
HS.A-REI.5	*This standard has been moved/removed by the committee*	
HS.A-REI.6	Solve systems of linear equations exactly and approximately focusing on pairs of linear equations in two variables.	Methods for solving may include substitution, linear combination, graphing or matrices.
HS.A-REI.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	Example: Find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$.
(+)HS.A-REI.8	Represent a system of linear equations as a single matrix equation.	Example: Represent the system as a matrix equation: $\begin{cases} 2x + 3y = 0 \\ x + 4y = 8 \end{cases}$ Solution: $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$
(+)HS.A-REI.9	Find the inverse of a matrix, if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).	Example: Solve $\begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & -1 \\ 5 & 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ using technology.

Note. (+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.*
** indicates modeling standards.*

Cluster: Represent and solve equations and inequalities graphically.		
Code	**Standards**	**Annotation**
HS.A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.	
HS.A-REI.11	Using graphs, technology, tables, or successive approximations, show that the solution(s) to the equation f(x) = g(x) are the x-value(s) that result in the y-values of f(x) and g(x) being the same.	
HS.A-REI.12	Graph the solutions to a linear inequality in two variables as a half-plane. Graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	Example: Solve by graphing: $\begin{cases} y \geq -x+1 \\ y \geq x-1 \end{cases}$ Solution: 

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** indicates modeling standards.*

Mathematics | High School—Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Functions Overview

Interpreting Functions

- Understand the concept of a function and use function notation.
- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations.

Building Functions

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

Linear, Quadratic, and Exponential Models

- Construct and compare linear, quadratic, and exponential models and solve problems.
- Interpret expressions for functions in terms of the situation they model.

Trigonometric Functions

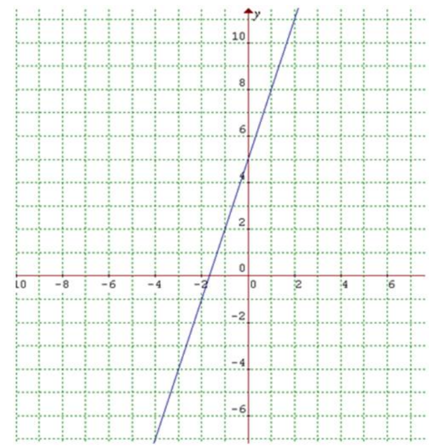
- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Domain: Interpreting Functions		HS.F-IF
Cluster: Understand the concept of a function and use function notation.		
Code	**Standards**	**Annotation**
HS.F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.	
HS.F-IF.2*	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
HS.F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	Example: The Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n > 1. Example: Write a recursive formula in function notation for the sequence generated by adding 5 to each successive term beginning with 2. Solution: $\begin{cases} f(1) = 2 \\ f(n) = f(n-1) + 5, n > 1 \end{cases}$

Note. (+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.*
** indicates modeling standards.*

Domain: Seeing Structure in Expressions		HS.F-IF
Cluster: Interpret functions that arise in applications in terms of the context		
Code	**Standards**	**Annotation**
HS.F-IF.4*	Use tables, graphs, verbal descriptions and equations to interpret and sketch the key features of a function modeling the relationship between two quantities.	Key features may include intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
HS.F-IF.5*	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	Example: If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
HS.F-IF.6*	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	Example: Estimate the rate of change given the graph below:  Solution: The average rate of change of a function $y = f(x)$ over an interval $[a, b]$ is $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$. Therefore, the estimated average rate of change for the function graphed above is: $\frac{(5-0)}{(0-(-1.75)} \approx 2.86$

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** indicates modeling standards.*

Cluster: Analyze functions using different representations.		
Code	**Standards**	**Annotation**
HS.F-IF.7*	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima where appropriate. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior. f. Graph f(x) = sin x and f(x) = cos x as representations of periodic phenomena. g. (+)Graph trigonometric functions, showing period, midline, phase shift and amplitude.	Example: Solve the annual compound interest formula $A = P(1 + r)^t$ for t and draw a graph of time vs. amount for a given rate and principle amount, showing intercepts and end behavior. Compare this graph to the graph of amount vs. time.
HS.F-IF.8*	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions.	Example: Identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
HS.F-IF.9*	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	Example: Given a graph of one quadratic function and an algebraic representation for another function, say which has the larger maximum.

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** indicates modeling standards.*

Domain: Building Functions* HS.F-BF

Cluster: Build a function that models a relationship between two quantities

Code	Standards	Annotation
HS.F-BF.1*	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. c. (+)Compose functions.	Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Example: If T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
HS.F-BF.2*	Write arithmetic and geometric sequences both recursively and with an explicit formula and convert between the two forms. Use sequences to model situations.	

Cluster: Build new functions from existing functions.

Code	Standards	Annotation
HS.F-BF.3*	Identify the effect on the graph of replacing f(x) by f(x) + k, f(x + k), k f(x), and f(kx), for specific values of k (both positive and negative); find the value of k given the graphs. Recognize even and odd functions from their graphs.	Technology may be used to experiment with the effects of transformations on a graph.
HS.F-BF.4*	Find inverse functions. a. Write an equation for the inverse given a function has an inverse. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or a table, given that the function has an inverse. d. Produce an invertible function from a non-invertible function by restricting the domain.	Example: $f(x) = 2x^3$ or $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$.
HS.F-BF.5*	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	

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** indicates modeling standards.*

Domain: Linear, Quadratic, and Exponential Models*		HS.F-LE
Cluster: Construct and compare linear, quadratic, and exponential models and solve problems.		
Code	**Standards**	**Annotation**
HS.F-LE.1*	Identify situations that can be modeled with linear and exponential functions. Justify the most appropriate model for a situation based on the rate of change over equal intervals. Include situations in which a quantity grows or decays.	
HS.F-LE.2*	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description, or two input-output pairs given their relationship.	
HS.F-LE.3*	Compare the end behavior of linear, quadratic and exponential functions using graphs and/or tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing as a linear or quadratic function.	
HS.F-LE.4*	Use logarithms to express the solution to $ab^{ct} = d$ where a, c, and d are real numbers and b is a positive real number. Evaluate the logarithm using technology when appropriate.	Example: $3e^{2t} = 317$ $e^{2t} = \frac{317}{3}$ $\ln e^{3t} = \ln\left(\frac{317}{3}\right)$ $3t = \ln\left(\frac{317}{3}\right)$ $t = \frac{1}{3}\ln\left(\frac{317}{3}\right)$ Using a calculator and rounding t to the nearest hundredth: $t \approx 1.55$.
Cluster: Interpret expressions for functions in terms of the situation they model.		
Code	**Standards**	**Annotation**
HS.F-LE.5*	Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.	**<u>parameter</u>:** A constant or a variable in a mathematical expression, which distinguishes various specific cases. For example, in the equation y = mx + b, m and b are parameters which specify the particular straight line represented by the equation. (From “Mathematics Dictionary, edited by Glenn James and Robert James, 1960, Princeton, New Jersey). Example: A cell phone plan includes $40 a month, plus 2 cents per minute of usage. Write a function that shows the monthly cost of using your cell phone, and interpret the parameters. Answer: C = 0.02m + 40 The monthly cost is 40 dollars plus 2 cents times the number of minutes used. The minimum cost per month is $40 (at m = 0).

Note. (+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.*
** indicates modeling standards.*

Domain: Trigonometric Functions — HS.F-TF

Cluster: Extend the domain of trigonometric functions using the unit circle.

Code	Standards	Annotation
HS.F-TF.1	Understand that the radian measure of an angle is the ratio of the length of the arc to the length of the radius of a circle.	
HS.F-TF.2	Extend right triangle trigonometry to the four quadrants. (+)Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	Example: Find *sin* 210°
HS.F-TF.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6. (+)Use the unit circle to express the values of sine, cosine, and tangent for π - *x*, π + *x*, and 2π - *x* in terms of their values for *x*, where *x* is any real number.	
(+)HS.F-TF.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	

Cluster: Model periodic phenomena with trigonometric functions.

Code	Standards	Annotation
(+)HS.F-TF.5*	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	Example*:* The depth of the ocean at a swim buoy reaches a maximum of 6 feet at 3 A.M. and a minimum of 2 feet at 9:00 A.M. Write a trigonometric function that models the water depth *y* (in feet) as a function of the time *t* (in hours). Assume that *t = 0* represents 12:00 A.M.
(+)HS.F-TF.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	
(+)HS.F-TF.7*	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	

Cluster: Prove and apply trigonometric identities.

Code	Standards	Annotation
HS.F-TF.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.	Students might "prove" by providing a formal proof, demonstrating, or justifying. See the Glossary for a definition of mathematical proof. Example: Given θ is a Quadrant II angle and sin θ = 4/5, find cos θ using the Pythagorean Identity.
(+)HS.F-TF.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	

Note. (+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.*
** indicates modeling standards.*

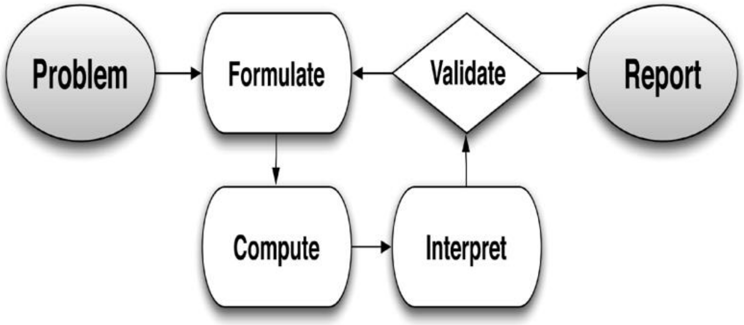
Mathematics | High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for seven players at a club with four tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.



In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).*

Mathematics | High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations.

The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Geometry Overview

Congruence

- Experiment with transformations in the plane.
- Understand congruence in terms of rigid motions.
- Prove and apply geometric theorems.
- Make geometric constructions.

Similarity, Right Triangles, and Trigonometry

- Understand similarity.
- Prove theorems involving similarity.
- Define trigonometric ratios and solve problems involving right triangles.
- Apply trigonometry to general triangles.

Circles

- Understand and apply theorems about circles.
- Find arc lengths and areas of sectors of circles.

Expressing Geometric Properties with Equations

- Understand and use conic sections.
- Use coordinates to verify simple geometric theorems algebraically.

Geometric Measurement and Dimension

- Explain surface area and volume formulas and use them to solve problems.
- Visualize relationships between two-dimensional and three-dimensional objects.

Modeling with Geometry

- Apply geometric concepts in modeling situations

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Mathematical Practices

1. Apply geometric concepts in modeling situations.
2. Mathematical Practices.
3. Make sense of problems and persevere in solving them.
4. Reason abstractly and quantitatively.
5. Construct viable arguments and critique the reasoning of others.
6. Model with mathematics.
7. Use appropriate tools strategically.
8. Attend to precision.
9. Look for and make use of structure.
10. Look for and express regularity in repeated reasoning.

Domain: Congruence		**HS.G-CO**
Cluster: Experiment with transformations in the plane.		
Code	**Standards**	**Annotation**
HS.G-CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, and plane.	
HS.G-CO.2	Represent transformations in the plane. Describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	
HS.G-CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	
HS.G-CO.4	Develop or verify experimentally the characteristics of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	Example: Using patty paper or geometry software, develop/verify that the reflection line is the perpendicular bisector of the segment that connects the pre-image to its image.
HS.G-CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	
Cluster: Understand congruence in terms of rigid motions.		
Code	**Standards**	**Annotation**
HS.G-CO.6	Use geometric descriptions of rigid motions to predict the effect of a given rigid motion on a given figure. Use the definition of congruence in terms of rigid motions to decide if two figures are congruent.	**Congruent:** Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations). (See Glossary) **Rigid motion:** A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures. (See Glossary)
HS.G-CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	
HS.G-CO.8	Prove two triangles are congruent using the congruence theorems such as ASA, SAS, and SSS.	

Note. (+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.*
** indicates modeling standards.*

Cluster: Prove and apply geometric theorems.		
Code	**Standards**	**Annotation**
HS.G-CO.9	Prove and apply theorems about lines and angles.	“Proof“ may take on a variety of forms (flow, paragraph, 2-column, informal). Theorems include: Vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.
HS.G-CO.10	Prove and apply theorems about triangles.	“Proof “ may take on a variety of forms (flow, paragraph, 2-column, informal). Theorems include: Measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
HS.G-CO.11	Prove and apply theorems about parallelograms.	“Proof “ may take on a variety of forms (flow, paragraph, 2-column, informal). Theorems include: Opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Cluster: Make geometric constructions.		
Code	**Standards**	**Annotation**
HS.G-CO.12	Make basic geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).	Basic constructions include: Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
(+)HS.G-CO.13	Apply basic constructions to create polygons such as equilateral triangles, squares, and regular hexagons inscribed in circles.	

Note. (+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.*
** indicates modeling standards.*

Domain: Similarity, Right Triangles, and Trigonometry		HS.G-SRT
Cluster: Understand similarity.		
Code	**Standards**	**Annotation**
HS.G-SRT.1	Verify experimentally the properties of dilations given by a center and a scale factor.	
HS.G-SRT.2	Given two figures, use similarity transformations to decide if they are similar. Apply the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	
HS.G-SRT.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	"Establish" may mean justify or prove the AA Similarity Theorem.
Cluster: Prove theorems involving similarity.		
Code	**Standards**	**Annotation**
HS.G-SRT.4	Prove theorems about triangles.	Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely.
HS.G-SRT.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
Cluster: Define trigonometric ratios and solve problems involving right triangles.		
Code	**Standards**	**Annotation**
HS.G-SRT.6	Verify experimentally that the side ratios in similar right triangles are dependent upon the measure of an acute angle in the triangle, due to the preservation of angle measure in similarity. Use this discovery to develop definitions of the trigonometric ratios for acute angles.	
HS.G-SRT.7	Explain and use the relationship between the sine and cosine of complementary angles.	
HS.G-SRT.8*	Use special right triangles (30-60-90 and 45-45-90), trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	

Note. (+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.*
** indicates modeling standards.*

Cluster: Apply trigonometry to general triangles.		
Code	**Standards**	**Annotation**
(+)HS.G-SRT.9	Derive the formula A = 1/2 *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	
(+)HS.G-SRT.10*	Prove the Laws of Sines and Cosines and use them to solve problems.	
(+)HS.G-SRT.11*	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (such as surveying problems, resultant forces).	

Note. (+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.*
** indicates modeling standards.*

Domain: Circles — HS.G-C

Cluster: Understand and apply theorems about circles.

Code	Standards	Annotation
HS.G-C.1	Understand and apply theorems about relationships with line segments and circles including radii, diameter, secants, tangents, and chords.	
HS.G-C.2	Understand and apply theorems about relationships with angles formed by radii, diameter, secants, tangents, and chords. Understand and apply properties of angles for a quadrilateral inscribed in a circle.	
HS.G-C.3	Construct the inscribed and circumscribed circles of a triangle.	Constructions include locating the incenter and circumcenter.
(+)HS.G-C.4	Construct a tangent line from a point outside a given circle to the circle.	

Cluster: Find arc lengths and areas of sectors of circles.

Code	Standards	Annotation
HS.G-C.5	Explain and use the formulas for arc length and area of sectors of circles.	

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** indicates modeling standards.*

Domain: Expressing Geometric Properties with Equations — HS.G-GPE

Cluster: Understand and use conic sections.

Code	Standards	Annotation
HS.G-GPE.1	Derive the equation of a circle of given center and radius. Derive the equation of a parabola given a focus and directrix. (+) Derive the equations of ellipses and hyperbolas given foci, using the fact that the sum or difference of distances from the foci is constant.	
HS.G-GPE.2	Convert between the standard and general form equations of conic sections.	Conic sections include the circle, ellipse, parabola and hyperbola.
HS.G-GPE.3	Identify key features of conic sections given their equations. Apply properties of conic sections in real-world situations.*	Key features include: Circle – center, radius Parabola – vertex, focus, directrix Ellipse – center, foci, vertices, length of major and minor axis Hyperbola – center, foci, asymptotes

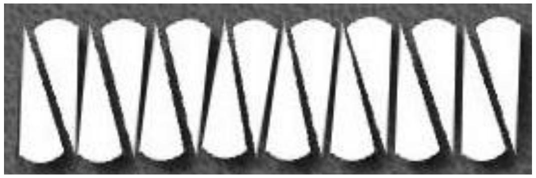
Cluster: Use coordinates to verify simple geometric theorems algebraically.

Code	Standards	Annotation
HS.G-GPE.4	Use coordinates to verify simple geometric theorems algebraically. Use coordinates to verify algebraically that a given set of points produces a particular type of triangle or quadrilateral.	Example: Given a kite with coordinates (2,0), (-2,0), (0,3) and (0,-4), verify that the diagonals are perpendicular. This standard allows for a coordinate proof. Example: Verify algebraically whether a figure defined by four given points in the coordinate plane is a rectangle.
HS.G-GPE.5	Develop and verify the slope criteria for parallel and perpendicular lines. Apply the slope criteria for parallel and perpendicular lines to solve geometric problems using algebra.	Example: Find the equation of a line parallel or perpendicular to a given line that passes through a given point.
HS.G-GPE.6	Use coordinates to find the midpoint or end point of a line segment. (+)Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	Example: Find the coordinate pair that is 2/3 the distance from the point (2,3) to (-4,7).
HS.G-GPE.7*	Use coordinates to compute perimeters of polygons and areas of triangles, parallelograms, trapezoids and kites.	

Note. (+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.*
** indicates modeling standards.*

Domain: Geometric Measurement and Dimension

Cluster: Explain surface area and volume formulas and use them to solve problems.

Code	Standards	Annotation
HS.G-GMD.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, or informal limit arguments.	**Cavalieri's Principle:** 2D: Suppose two regions in a plane are included between two parallel lines in that plane. If every line parallel to these two lines intersects both regions in line segments of equal length, then the two regions have equal areas. 3D: Suppose two regions in three-space (solids) are included between two parallel planes. If every plane parallel to these two planes intersects both regions in cross-sections of equal area, then the two regions have equal volumes. Example: The area of a circle can be deduced by rearranging sectors of two semi-circles to form a rough rectangle. Area : $= r \cdot \frac{1}{2} \cdot \text{Circumference}$ $= r \cdot \frac{1}{2} \cdot 2\pi r$ $= \pi r^2$ 
HS.G-GMD.2	Calculate the surface area for prisms, cylinders, pyramids, cones, and spheres to solve problems.	
HS.G-GMD.3*	Know and apply volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems.	

Cluster: Visualize relationships between two-dimensional and three-dimensional objects.

Code	Standards	Annotation
HS.G-GMD.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	

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** indicates modeling standards.*

Domain: Modeling with Geometry*		**HS.G-MG**
Cluster: Apply geometric concepts in modeling situations.		
Code	**Standards**	**Annotation**
HS.G-MG.1*	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	
HS.G-MG.2*	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	
HS.G-MG.3*	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	Example: Students design a soft drink package that minimizes surface area and cost.

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** indicates modeling standards.*

Mathematics | High School—Statistics and Probability*

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling.

Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Statistics and Probability Overview

Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable.
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- Interpret linear models.

Making Inferences and Justifying Conclusions

- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments and observational studies.

Conditional Probability and the Rules of Probability

- Understand independence and conditional probability and use them to interpret data.
- Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Using Probability to Make Decisions

- Calculate expected values and use them to solve problems.
- Use probability to evaluate outcomes of decisions.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

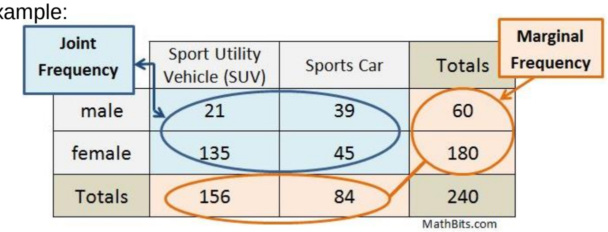
Domain: Interpreting Categorical and Quantitative Data* HS.S-ID

Cluster: Summarize, represent, and interpret data on a single count or measurement variable.

Code	Standards	Annotation
HS.S-ID.1*	Represent data with plots on the real number line (dot plots, histograms, and box plots).	
HS.S-ID.2*	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	Students may use technology to find the standard deviation.
HS.S-ID.3*	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	
HS.S-ID.4*	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, or tables to estimate areas under the normal curve.	Example: An example of a data set that does not fit to a normal distribution is age at retirement. Most people retire in their mid 60s or older, with increasingly fewer retiring at increasingly earlier ages. This results in a negatively skewed distribution.

Cluster: Summarize, represent, and interpret data on two categorical and quantitative variables.

Code	Standards	Annotation
HS.S-ID.5*	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	Example: (see table below) The joint relative frequency of being male and owning an SUV is 21/240. The marginal relative frequency of owning an SUV is 156/240. The conditional relative frequency of owning a SUV given you are a male is 21/60.
HS.S-ID.6*	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data (with or without technology). Use functions fitted to data to solve problems in the context of the data. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. **Residual:** The observed value minus the predicted value. It is the difference of the results obtained by observation, and by computation from a formula.



	Sport Utility Vehicle (SUV)	Sports Car	Totals
male	21	39	60
female	135	45	180
Totals	156	84	240

Note: (+) indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.
** indicates modeling standards.*

Cluster: Interpret linear models		
Code	**Standards**	**Annotation**
HS.S-ID.7*	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Interpolate and extrapolate the linear model to predict values.	
HS.S-ID.8*	Compute (using technology) and interpret the correlation coefficient of a linear fit.	
HS.S-ID.9*	Distinguish between correlation and causation.	**Correlation:** A mutual relationship between two or more things. **Causation:** The producer of an effect, result, or consequence. Example: It is noted there is a high correlation between people who eat ice cream daily and their annual job salary. Does eating ice cream predict salary or vice-versa?

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** indicates modeling standards.*

Domain: Making Inferences and Justifying Conclusions*		HS.S-IC
Cluster: Understand and evaluate random processes underlying statistical experiments.		
Code	**Standards**	**Annotation**
HS.S-IC.1*	Understand the process of making inferences about population parameters based on a random sample from that population.	
HS.S-IC.2*	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.	Example: A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?
Cluster: Make inferences and justify conclusions from sample surveys, experiments, and observational studies.		
Code	**Standards**	**Annotation**
HS.S-IC.3*	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	Example: Design a simple study and explain the impact of sampling methods, bias and the phrasing of questions asked during data collection.
HS.S-IC.4*	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	
HS.S-IC.5*	*This standard has been moved/removed by the committee*	
HS.S-IC.6*	Evaluate reports based on data. a. Evaluate articles, reports or websites based on data published in the media by identifying the source of the data, the design of the study, and the way the data are analyzed and displayed. b. Identify and explain misleading use of data; recognize when claims based on data confuse correlation and causation. c. Recognize and describe how graphs and data can be distorted to support different points of view.	

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** indicates modeling standards.*

Domain: Conditional Probability and the Rules of Probability*		HS.S-CP
Cluster: Understand independence and conditional probability and use them to interpret data.		
Code	**Standards**	**Annotation**
HS.S-CP.1*	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).	Example: Given a classroom of 30 students, list the subset for students in the room who are blonde and have blue eyes.
HS.S-CP.2*	Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	Understand that event A is independent from event B if the probability of event A does not change in response to the occurrence of event B.
HS.S-CP.3*	Understand the conditional probability of *A* given *B* as $P(A \text{ and } B)/P(\text{B})$, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	Understand that the conditional probability of an event A given B is the probability that event A will occur given the knowledge that event B has already occurred.
HS.S-CP.4*	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.	Example: Collect data from a random sample of students in your school on their favorite subject among mathematics, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in 10th grade. Do the same for other subjects and compare the results.
HS.S-CP.5*	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.	Example: Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

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** indicates modeling standards.*

Cluster: Use the rules of probability to compute probabilities of compound events in a uniform probability model.		
Code	**Standards**	**Annotation**
HS.S-CP.6*	Find the conditional probability of *A* given *B* and interpret the answer in terms of the model.	Example: A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test? Solution: P(A and B) = Probability that a student passed both tests = 0.25. P(A) = Probability that a student passed the first test = 0.42. Find P(B) given that P(A) is true = . P(A and B)/P(A) = . 0.25/0.42 =.0.6 =.60%. Therefore 60% of those students who passed the first test also passed the second test.
HS.S-CP.7*	Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.	
HS.S-CP.8*	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.	
HS.S-CP.9*	Use permutations and combinations to determine the number of outcomes in terms of the model. (+)Use permutations and combinations to compute probabilities of compound events and solve problems.	Example: Given a football team of 60 athletes, what is the probability that the star quarterback and star linebacker are not chosen for drug testing? $\frac{58^{C_2}}{60^{C_2}} \approx 0.934 \approx 93.4\%$

Note: (+) *indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.*
** indicates modeling standards.*

Domain: Using Probability to Make Decisions*		HS.S-MD
Cluster: Calculate expected values and use them to solve problems.		
Code	**Standards**	**Annotation**
(+)HS.S-MD.1*	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space. Graph the corresponding probability distribution using the same graphical displays as for data distributions.	
(+)HS.S-MD.2*	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	
(+)HS.S-MD.3*	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.	Example: Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected value.
(+)HS.S-MD.4*	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.	Example: Find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?

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** indicates modeling standards.*

Cluster: Use probability to evaluate outcomes of decisions.		
Code	**Standards**	**Annotation**
(+)HS.S-MD.5*	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. b. Evaluate and compare strategies on the basis of expected values.	 Example: Find the expected winnings from a state lottery ticket or a game at a fast-food restaurant. Example: Compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.
(+)HS.S-MD.6*	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	
(+)HS.S-MD.7*	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	

Note: (+) indicates additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.
** indicates modeling standards.*

Note on courses and transitions

The high school portion of the Standards for Mathematical Content specifies the mathematics all students should study for college and career readiness. These standards do not mandate the sequence of high school courses. However, the organization of high school courses is a critical component to implementation of the standards. To that end, sample high school pathways for mathematics – in both a traditional course sequence (Algebra I, Geometry, and Algebra II) as well as an integrated course sequence (Mathematics 1, Mathematics 2, Mathematics 3) – can be found in Mathematics Appendix A. It is expected that additional model pathways based on these standards will become available as well.

The standards themselves do not dictate curriculum, pedagogy, or delivery of content. In particular, states may handle the transition to high school in different ways. For example, many students in the United States today take Algebra I in the 8th grade, and in some states this is a requirement. The K-7 standards contain the prerequisites to prepare students for Algebra I by 8th grade, and the standards are designed to permit states to continue existing policies concerning Algebra I in 8th grade.

A second major transition is the transition from high school to post-secondary education for college and careers. The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as Grades 6-8. It is important to note as well that cut scores or other information generated by assessment systems for college and career readiness should be developed in collaboration with representatives from higher education and workforce development programs, and should be validated by subsequent performance of students in college and the workforce.

Glossary

Absolute value. The distance a number is from zero on a number line. Example: $|-52|$ = 52 and $|52|$ = 52.

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 - 5 = 9 is a subtraction within 20, and 55 - 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Array. An arrangement of objects, pictures, or numbers in columns or rows.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Attribute. A characteristic or property of an object.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[15]

Cardinality. The number of elements in a given mathematical set.

Cavalieri's Principle. 2D: Suppose two regions in a plane are included between two parallel lines in that plane. If every line parallel to these two lines intersects both regions in line segments of equal length, then the two regions have equal areas.
3D: Suppose two regions in three-space (solids) are included between two parallel planes. If every plane parallel to these two planes intersects both regions in cross-sections of equal area, then the two regions have equal volumes.

Coefficient. Any given number multiplied by (in front of) a given variable. Example: In 2x + 3, the 2 is the coefficient.

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Compose. To put together parts or elements.

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[15] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Condition. An assumption on which rests the validity or effect of something else; a circumstance.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Constraint. A limitation; a condition which must be satisfied.

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. Example: If a stack of books is known to have eight books and three more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Decompose. To separate into parts or basic elements.

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Elapsed time. A time interval.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. Example: 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[16] See also: median, third quartile, interquartile range.

Fluency (Computational). Having efficient, flexible and accurate methods for computing.

Fluency (Procedural). Skill in carrying out procedures, flexibly, accurately, efficiently and appropriately.

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[16] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E.,"Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. (1) A number expressible in the form a or –a for some whole number a. (2) the set of whole numbers and their opposites.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 - 6 = 9. See also: first quartile, third quartile.

Know from memory. The ability to compute mathematics facts with automaticity (3-5 seconds) and confidence.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[17]

Mathematical Proof. A carefully reasoned argument for verifying a conjecture that would meet the standards of the broader mathematics community. (Principles and Standards for School Mathematics, NCTM, 2000).

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[18] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

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[17] Adapted from Wisconsin Department of Public Instruction, *op. cit*.
[18] To be more precise, this defines the *arithmetic mean*.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Nonnegative rational numbers. The positive rational numbers and zero.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

One-to-one correspondence. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

Parameter. A constant or a variable in a mathematical expression, which distinguishes various specific cases. Example: In the equation y = mx + b, m and b are parameters which specify the particular straight line represented by the equation. (From Mathematics Dictionary, edited by Glenn James and Robert James, 1960, Princeton, New Jersey).

Partition. Divide up into pieces.

Percent rate of change. A rate of change expressed as a percent. Example: If a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number from 0 through 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See also: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or – a/b for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. Connected rectangles in which all of the angles are 90 degrees.

Relative frequency. The ratio of the number of times that an event happens to the number of trials in which the event can happen or fail to happen. (Source: Mathematics Dictionary, Glenn James and Robert James, 1960, Princeton, New Jersey)

Repeating decimal. The decimal form of a rational number. See also: terminating decimal.

Residual. The observed value minus the predicted value. It is the difference of the results obtained by observation, and by computation from a formula.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. Example: The heights and weights of a group of people could be displayed on a scatter plot.[19]

Scientific notation. A way of representing large or small numbers by using a number from 1 up to (but not including) 10 times an integer power of 10.

Shares. Groups or sets.

Similarity transformation. A rigid motion followed by a dilation.

Standard algorithm. A step-by-step procedure specifying how to solve a problem.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. See also: median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. See also: probability model.

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[19] Adapted from Wisconsin Department of Public Instruction, *op. cit*.

Unit rate. A rate is simplified so that it has a denominator of 1 unit (e.g., miles per gallon, kilometers per second).

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Variable. Unknown value within an equation.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, …

Table 1. Common addition and subtraction situations.

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**[20]
Put Together/ Take Apart[21]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 - 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0, 5 = 1 + 4, 5 = 4 + 1, 5 = 2+ 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[22]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 - 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 - 3 = ?, ? + 3 = 5

[20] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[21] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[22] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2. Common multiplication and division situations.

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[23] area[24]	There are 3 rows of apples with 6 apples in each row. How many apples are there? Area example. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[23] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[24] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3. The properties of operations. Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $–a$ so that $a + (–a) = (–a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a – c = b – c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. Here a, b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $–a < –b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Table 6. Polynomial Identities include but are not limited to:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a+b)(c+d) = ac + ad + bc + bd$$
$a^2 - b^2 = (a+b)$(a-b) (Difference of squares)

$a^2 \pm b^3 = (a \pm b)(a^2 \pm ab + b^2)$ (Sum and Difference of Cubes)

$$x^2 + (a+b)x + ab = (x+a)(x+b)$$

Table 7: Standard Algorithms for division.

The standard algorithm for division is long division. For example:

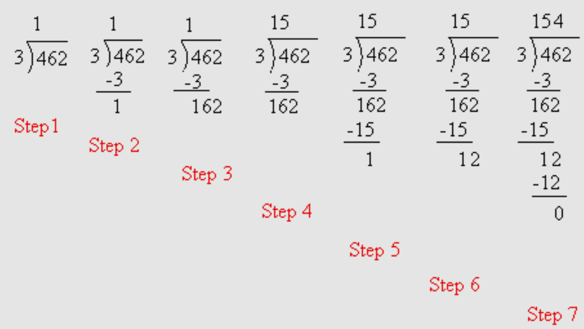
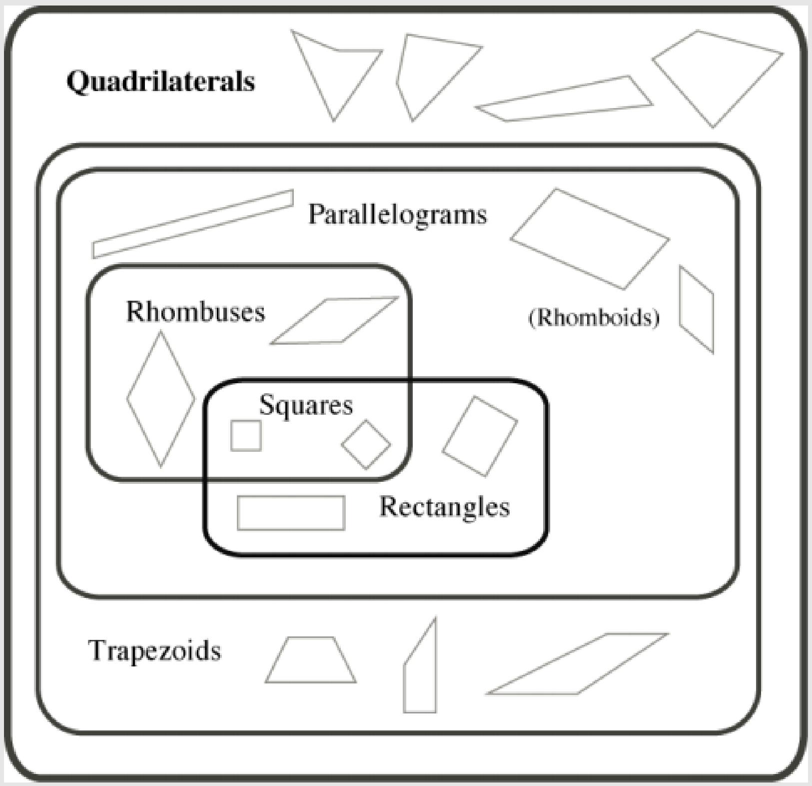


Table 8. Venn diagram showing classification of quadrilaterals.

Venn diagram showing classification of quadrilaterals



Note that rhomboids are parallelograms that are not rhombuses or rectangles. This example uses the inclusive definition of trapezoid (see p. [pageref “T(E)”)]).

Figure	Defining Characteristic
Quadrilateral	A polygon with 4 sides
Trapezoid	A quadrilateral with **at least 1** pair of parallel opposite sides

Parallelogram	A quadrilateral with 2 pairs of parallel sides
Rectangle	A quadrilateral with 4 right angles
Rhombus	A quadrilateral with 4 congruent sides
Square	A quadrilateral with 4 congruent sides and 4 right angles