NORTH DAKOTA
DEPARTMENT OF
PUBLIC INSTRUCTION

North Dakota
Mathematics
Content Standards
K-12

July 2023

North Dakota Department of Public Instruction
Kirsten Baesler, State Superintendent
600 East Boulevard Avenue, Dept. 201
Bismarck, North Dakota 58505-0440

www.nd.gov/dpi

Document Revision Log

Date Revised	Description of Revision	Page(s)
5/22/24	Added Clarification: **3.NO.NF.4** Within this standard, learners are going beyond 1.	21
5/22/24	Corrected Standard Description: **4.DPS.D.2** Generate data and create line plots to display a data set of fractions of a unit (½, ¼, ⅛). Solve problems involving addition and subtraction of fractions by using information presented in line plots.	27, 102, 103
5/22/24	Corrected Standard Description: **5.NO.NBT.1** Understand that in a multi-digit number, a digit in one place represents ten times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	28, 76, 81
5/22/24	Corrected Standard Description: **5.DPS.D.1** Generate data and create line plots to display a data set of fractions of a unit (½, ¼, ⅛). Use grade-level operations for fractions to solve problems involving information presented in line plots.	31, 102, 103
5/22/24	Moved Clarification as it was in the incorrect place: From 6.AR.RP.3 **-** To **6.AR.RP.5**.	33
6/27/24	Added clarification “Ordering numbers may be included for number comparisons” to standards 1.NO.NBT.2, 2.NO.NBT.2, 3.NO.NBT.2, 3.NO.NF.5, 4.NO.NBT.2, 5.NO.NBT.2.	14, 17, 20, 21, 24, 29
6/27/24	Added clarification “Properties of two-dimensional shapes would include identifying the number of sides and angles found in polygons and triangles” to standard 4.GM.G.2.	26
6/27/24	Added clarification “Properties of two-dimensional shapes and the names of specific shapes would include identifying the number of sides, angles, and lines of symmetry found in polygons and triangles (equilateral, isosceles, scalene)” to standard 5.GM.G.1.	31

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SUPERINTENDENT'S FOREWORD

As the North Dakota State Superintendent, it is my honor to introduce the new North Dakota K-12 mathematics content standards. These standards represent a collective effort by North Dakota teachers and content experts to provide our learners with the necessary mathematical knowledge and skills to tackle the challenges of our ever-changing world.

Today, problem-solving and creative thinking are highly valued skills. Our students must be equipped with algebraic and geometric reasoning abilities, allowing them to navigate novel situations and find innovative solutions. However, these skills can only be fully developed on a strong foundation of numeracy. Therefore, it is crucial that our learners possess robust numeracy skills, which serve as the bedrock for developing their reasoning abilities.

The new mathematics content standards have been carefully crafted to embed the foundational skills necessary for students to develop their geometric and algebraic reasoning. This progression of skills will guide students throughout their K-12 academic journey, building the necessary background knowledge to become skilled and innovative problem solvers.

While the North Dakota content standards serve as a statewide reference point for teaching mathematics, we encourage local school districts to utilize them as a guide in developing their own customized curricula. We recognize the importance of tailoring education to the unique needs and contexts of each community while ensuring a solid alignment with the state standards.

The process of developing these standards involved collaboration between the North Dakota Department of Public Instruction, North Dakota State University, and a team of dedicated North Dakota educators. Starting in July 2022 and continuing through June 2023, this collective effort produced drafts that were made available for public comment. We received invaluable feedback from teachers, administrators, parents, and the community, which greatly contributed to the refinement of the standards.

Drawing upon previous North Dakota standards, standards from other states, and extensive research on mathematics content and skill development, the writing committee identified the foundational knowledge and skills that learners need to solve a variety of mathematical problems. Notably, in this 2023 version, the high school standards are divided into two grade spans rather than broadly categorized. This change allows for more specific identification of assessed standards in grades 9-10 and provides districts with the opportunity to align standards within their courses.

I would like to express my gratitude to the review committee, comprised of interested stakeholders from the general public, who provided another layer of scrutiny and feedback. Their dedication and insights greatly enhanced the quality and relevance of the standards. I am truly thankful to those who devoted their time and talent to reviewing the draft standards and providing recommendations to the writing committee.

The primary architects of these standards are our North Dakota educators. Their expertise and commitment to our students are unmatched. They have exemplified the best in North Dakota education by openly, transparently, and collaboratively working on this document. Each member of the writing committee deserves our heartfelt appreciation for their extensive research, rigorous analysis, and thoughtful deliberations. It is through their hard work that these standards are now ready to be implemented in classrooms across our great state.

I am confident that the adoption of these new mathematics content standards will empower our students, equip them with the necessary mathematical skills, and foster their critical thinking abilities. Together, we can prepare the next generation of problem solvers and innovators, ensuring a bright future for North Dakota and beyond.

Sincerely,

Kirsten Baesler
North Dakota State Superintendent

INTRODUCTION

Educational standards are statements designed to describe a clear path for students to gain the proficiency required to learn increasingly complex material. The standards provide educational guidelines but do not prescribe teaching practices, curricula, or assessment methods. The North Dakota Mathematics Content Standards provide a rigorous and developmentally appropriate framework for instruction to increase student achievement and provide students with a quality, equitable education. These standards help develop critical and innovative thinking and problem-solving skills students will apply when meeting future postsecondary and workforce demands.

The development of these new mathematics standards was a multi-phase process. State Superintendent of Public Instruction Kirsten Baesler established a statewide committee through an application process that included educators and higher education faculty. Over five two-day sessions, the committee reviewed the existing standards, drafted new standards; and revised their work based upon input from two rounds of public comments and two reviews by a content standards review committee representing business interests, parents, and the public. The committee began its work in July 2022 and completed the development of new standards in May 2023.

The 2023 Mathematics Content Standards identify math attributes that develop mathematical thought processes woven within the content. The math attributes describe processes within mathematical concepts. The math attributes summarize the mathematical practices found in the 2017 North Dakota Mathematics Content Standards and are aligned with the 2022 North Dakota Learning Continuum, which identified essential knowledge, skills, and dispositions learners need to demonstrate throughout their lives.

The content standards identify essential skills and concepts across four categories which focus on developing a conceptual understanding of math concepts as a learner progresses from learning foundational arithmetic skills to applying those skills in algebra and geometry. The new standards require procedural skills and fluency in using mathematical skills and concepts in various authentic problem situations. The North Dakota Mathematics Content Standards identify skills in which proficiency is needed.

MATH ATTRIBUTES

The math attributes contained in these standards summarize the mathematical practices found in the 2017 North Dakota Mathematics Content Standards and align with the 2022 North Dakota Learning Continuum. These attributes will help learners solve authentic problems while connecting concepts, providing supporting evidence, explaining the reasoning and efficiency of strategies used, and proving the accuracy of solutions. The three attributes identified will be used by learners throughout their education and future careers. The attributes are arranged by grade span.

A chart showing the progression of the Math Attributes is shown below:

Math Attributes (MA) *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement through participation in various relevant learning experiences.*		
Problem-Solving (P)	**Connections (C)**	**Reasoning and Proof (R)**
Analyze, execute, evaluate, and adapt approaches and solutions when solving novel situations.	*Create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.*	*Reason logically, citing relevant evidence to explain and critique what they see, think, and conclude through exploration, generalization, and validation.*
K-2.MA.P Learners can identify and use strategies to problem-solve situations and determine an appropriate solution.	**K-2.MA.C** Learners can make connections and demonstrate relationships using words, pictures, or symbols.	**K-2.MA.R** Learners can use prior knowledge and experiences to explain their thinking.
3-5.MA.P Learners can develop and carry out a logical plan to problem-solve situations, reflect on the reasonableness of solutions, and explore alternate strategies with guidance.	**3-5.MA.C** Learners can make connections and summarize related ideas using supporting evidence.	**3-5.MA.R** Learners can reason logically based on experience and knowledge, citing evidence to support their reasoning and conclusions.
6-8.MA.P Learners can analyze information and formulate a flexible, systematic plan to problem-solve authentic situations and reflect on the reasonableness of the solution, making revisions when necessary.	**6-8.MA.C** Learners can create connections within and across concepts and provide examples of how they relate to other learning and ideas using supporting evidence	**6-8.MA.R** Learners can reason logically, citing evidence to evaluate and explain what they see, think, and conclude through exploration and justification.
9-12.MA.P Learners can analyze, execute, critique, and adapt approaches and solutions when problem-solving in novel situations.	**9-12.MA.C** Learners can create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.	**9-12.MA.R** Learners can reason logically, citing evidence to critique and explain what they see, think, and conclude through exploration, generalization, and validation.
Lifelong MA.P Learners can integrate their cumulative knowledge and life experiences to discern and prioritize information in authentic situations, consider and apply alternative methods of resolution, and evaluate the relevance, efficacy, and accuracy of solutions.	**Lifelong MA.C** Learners can apply connections and develop generalizations within and across concepts to execute effective decision-making or generate new ideas.	**Lifelong MA.R** Learners can reason logically to discern the validity of information and synthesize it to formulate, investigate, and critique claims and evidence.

(2022 North Dakota Learning Continuum)

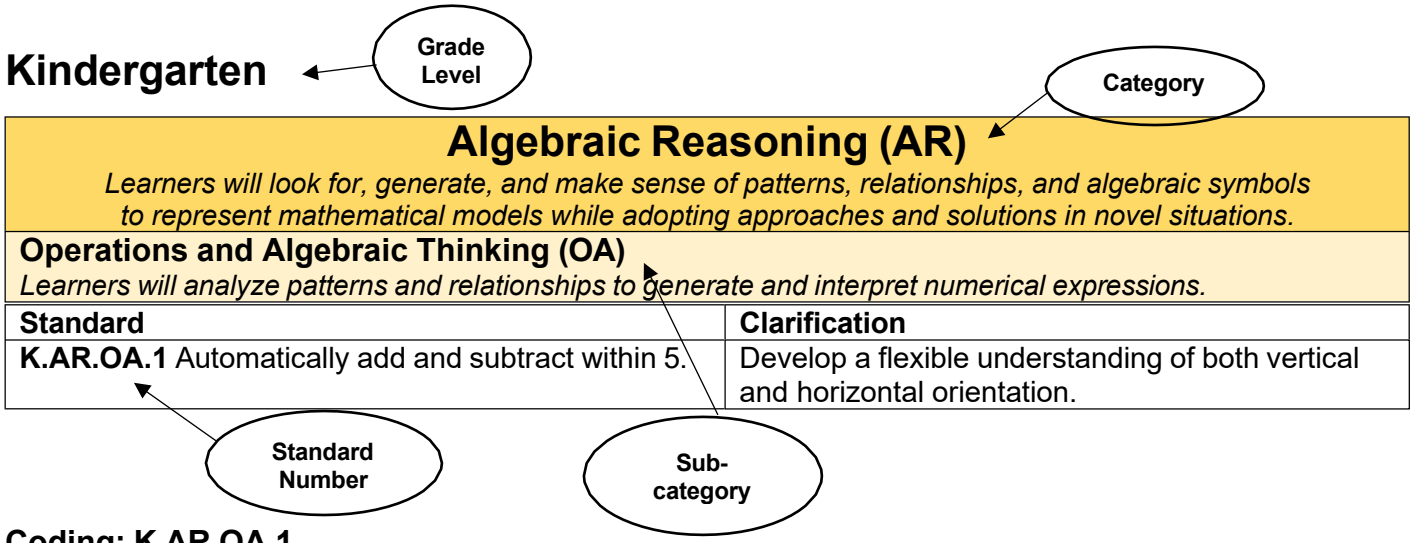
HOW TO READ THIS DOCUMENT

The content standards serve as a guide for districts to use as they develop curricula and select instructional materials. These standards do not define how teachers teach.

The document is organized by category, sub-category, and standard and includes four categories defined below:

Category	Definition
Number and Operations	Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across disciplines.
Algebraic Reasoning	Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.
Geometry and Measurement	Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.
Data, Probability, and Statistics	Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic concepts of probability.

Each category progresses from kindergarten through grade 12, with the high school level divided into two grade-span groups. Each category is split into sub-categories which are made up of the standards. The elementary level focuses on building arithmetic skills and concepts, the middle level moves toward applying those skills in pre-algebraic concepts, and the high school level refines and hones the skills needed to develop the algebraic and geometric strategies to solve problems in the post-high school world.



Coding: K.AR.OA.1

K – Grade Level or Grade Span

AR – Category

OA – Sub-category

1 – Standard Number

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NORTH DAKOTA
MATHEMATICS
K-12 STANDARDS

KINDERGARTEN

Math Attributes (MA) *Learners will practice and demonstrate broad, transferable, and enduring the skills necessary for advancement through participation in various relevant learning experiences.*		
Problem-Solving (P)	**Connections (C)**	**Reasoning and Proof (R)**
Analyze, execute, evaluate, and adapt approaches and solutions when problem-solving in novel situations.	*Create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.*	*Reason logically, citing relevant evidence to explain and critique what they see, think, and conclude through exploration, generalization, and validation.*
K-2.MA.P Learners can identify and use strategies to problem-solve situations and determine an appropriate solution.	**K-2.MA.C** Learners can make connections and demonstrate relationships using words, pictures, or symbols.	**K-2.MA.R** Learners can use prior knowledge and experiences to explain their thinking.

(2022 North Dakota Learning Continuum)

Number and Operations (NO)
Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

Counting and Cardinality (CC)
Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

Standards	Clarification
K.NO.CC.1 Count verbally in sequential order by ones and tens to 100, making accurate decuple transitions (e.g., 89 to 90). Count verbally forward from any given number within 100.	
K.NO.CC.2 Count backward from 20 by ones and from a given number within 10.	
K.NO.CC.3 Identify and write any given numeral within 20.	In a progression, students may identify and write numerals sequentially prior to being able to identify and write any numeral within 20.
K.NO.CC.4 Recognize and verbally label arrangements, without counting, for briefly shown collections up to 10 (e.g., "I saw 5." How do you know?" "I saw 3 and 2, that is 5.").	Recognize without counting. Use scattered arrangements for combinations up to 7. Structured arrangements such as ten frames (utilizing 5+ and pair-wise patterns) can be utilized for combinations up to 10.
K.NO.CC.5 Count and tell how many objects up to 20 are in an arranged pattern or up to 10 objects in a scattered configuration. Represent a quantity of up to 20 with a numeral.	

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

Standards	Clarification
K.NO.NBT.1 Compose and decompose numbers from 11 to 19 using a group of ten ones and some more ones using a model, drawing, or equation.	
K.NO.NBT.2 Compare two numbers between 1 and 20 using words greater than, less than, or equal to.	In a progression, students will use groups of objects for comparison prior to the end-of-year standard of comparing numerals within 20. Ordering numbers may be included for number clarifications.

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

NOTE: Standards begin in first grade.

Algebraic Reasoning (AR)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

Standards	Clarification
K.AR.OA.1 Automatically add and subtract within 5.	Develop a flexible understanding of both vertical and horizontal orientation. See Appendix B for recommended automatically.
K.AR.OA.2 For any number from 1 to 9, find the number that makes 10 when added to the given number, sharing the answer with a model, drawing, or equation.	
K.AR.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way using verbal explanations, objects, or drawings.	
K.AR.OA.4 Solve authentic word problems with addition by putting together or adding to within 10.	Develop a flexible understanding of both vertical and horizontal orientation.
K.AR.OA.5 Solve authentic word problems with subtraction by taking apart or taking from within 10.	Develop a flexible understanding of both vertical and horizontal orientation.
K.AR.OA.6 Recognize, duplicate, complete, and extend repeating patterns in a variety of contexts (e.g., shape, color, size, objects, sounds, movements).	Use AB-ab, abc, aabb type patterns.

Geometry and Measurement (GM)
Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.

Standards	Clarification
K.GM.G.1 Name shapes and identify them as two-dimensional (squares, circles, triangles, rectangles) regardless of their orientations or overall sizes.	
K.GM.G.2 Name shapes and identify them as three-dimensional (cubes and spheres) regardless of their orientations or overall sizes.	
K.GM.G.3 Compare and classify two-dimensional shapes to describe their similarities, differences, and attributes (squares, circles, triangles, rectangles).	
K.GM.G.4 Compose a geometric shape by combining two or more simple shapes.	

Measurement (M) *Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.*	
Standards	**Clarification**
K.GM.M.1 Compare and order two objects with a common measurable attribute.	In a progression, students will describe and understand common measurable attributes (e.g., length and weight) for ordering and comparisons.
K.GM.M.2 Tell time related to daily life (today, yesterday, tomorrow, morning, afternoon, night).	This concept is foundational learning for time. Days of the week and concepts of a.m. and p.m. are included.
Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*	
Data (D) *Learners will represent and interpret data.*	
Standard	**Clarification**
K.DPS.D.1 Sort and classify objects (up to 10) based on attributes and explain the reasoning used.	

FIRST GRADE

Math Attributes (MA) *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement through participation in various relevant learning experiences.*		
Problem-Solving (P)	**Connections (C)**	**Reasoning and Proof (R)**
Analyze, execute, evaluate, and adapt approaches and solutions when problem-solving in novel situations.	*Create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.*	*Reason logically, citing relevant evidence to explain and critique what they see, think, and conclude through exploration, generalization, and validation.*
K-2.MA.P Learners can identify and use strategies to problem-solve situations and determine an appropriate solution.	**K-2.MA.C** Learners can make connections and demonstrate relationships using words, pictures, or symbols.	**K-2.MA.R** Learners can use prior knowledge and experiences to explain their thinking.

(2022 North Dakota Learning Continuum)

Number and Operations (NO) *Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.*	
Counting and Cardinality (CC) *Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.*	
Standards	**Clarification**
1.NO.CC.1 Count forward by ones and tens from any given point within 120.	Students practice their understanding of numbers in the range of 120 by reading, writing, and verbally counting.
1.NO.CC.2 Count backward by ones and tens from a given number within 120.	Students practice their understanding of numbers in the range of 120 by reading, writing, and verbally counting.
1.NO.CC.3 Represent several objects with a written numeral up to 120.	In a progression, students may write their numeral patterns sequentially prior to being able to represent any numeral or quantity in the range of 120.
1.NO.CC.4 Recognize and verbally label arrangements, without counting, for briefly shown collections up to 20 (e.g., “I saw 16.” How do you know?” “I saw 10 and 6, that is 16.”).	Recognize without counting. Structured arrangements such as twenty frames (utilizing 10+ and pair wise patterns) can be utilized for combinations up to 20.
1.NO.CC.5 Skip count forward and backward by 5s and 10s from multiples and recognize the patterns of up to 10 skip counts.	**Assessment Boundary**: Start from any multiple and move forward or backward by 5 or 10 (e.g., 15, 10, 5, etc.). Range of 5-50 and 10-100.
Base Ten (NBT) *Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.*	
Standards	**Clarification**
1.NO.NBT.1 Demonstrate that the two digits of a two-digit number represent a composition of some tens and some ones.	Students may use concrete models, drawings, or written numerals to show a place value understanding of tens and ones.
1.NO.NBT.2 Compare two two-digit numbers using symbols >, <, and =. Justify comparisons based on the value of tens and ones.	Ordering numbers may be included for number comparisons.

1.NO.NBT.3 Add within 100 using a two-digit number and a one-digit number. Use concrete models, drawings, and strategies that reflect an understanding of place value.	Develop a flexible understanding of both vertical and horizontal orientation.
1.NO.NBT.4 Subtract multiples of 10 within 100 using concrete models, drawings, and strategies that reflect an understanding of place value.	Develop a flexible understanding of both vertical and horizontal orientation.
1.NO.NBT.5 Mentally add or subtract 10 to or from a given two-digit number and explain the reasoning used.	

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

Standard	Clarification
1.NO.NF.1 Partition circles and rectangles into two and four equal shares using the language halves and fourths.	

Algebraic Reasoning (AR)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

Standards	Clarification
1.AR.OA.1 Automatically add and subtract within 10.	Develop a flexible understanding of both vertical and horizontal orientation. See Appendix B for recommended automaticity.
1.AR.OA.2 For any number from 1 to 19, find the number that makes 20 when added to the given number, sharing the answer with a model, drawing, or equation.	Students use composition and decomposition strategies for combinations of 20 (e.g., “I have 17 and I need 3 to make 20” or “Tell me two numbers that go together to make 20.”).
1.AR.OA.3 Decompose numbers less than or equal to 20 into pairs in more than one way.	
1.AR.OA.4 Solve authentic word problems with addition, including three numbers and unknowns, within 20.	Develop a flexible understanding of both vertical and horizontal orientation.
1.AR.OA.5 Solve authentic word problems with subtraction, including unknowns, within 20.	Develop a flexible understanding of both vertical and horizontal orientation.
1.AR.OA.6 Distinguish and use the +, -, and = symbols accurately in an equation.	In a progression, students learn the meaning of an equal sign, including if equations are true and false, solving on both sides if needed.
1.AR.OA.7 Identify, create, complete, and extend patterns that are repeating, increasing, and decreasing in a variety of contexts.	Example: shape, color, size, objects, and/or numerical patterns.

Geometry and Measurement (GM)
Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.

Standards	Clarification
1.GM.G.1 Name shapes and identify them as two-dimensional (trapezoids, rhombuses, pentagons, hexagons, octagons).	**Assessment Boundary**: Includes shapes from K.GM.G.1.

1.GM.G.2 Name and identify solids as three-dimensional (cylinders, cones, triangular prisms, and rectangular prisms).	**Assessment Boundary**: Includes shapes from K.GM.G.2.
1.GM.G.3 Determine geometric attributes of two-dimensional and three-dimensional shapes (squares, circles, triangles, rectangles, trapezoids, rhombuses, pentagons, hexagons, octagons, cubes, spheres, cylinders, cones, triangular prisms, and rectangular prisms).	**Assessment Boundary**: Includes shapes from K.GM.G.3.
1.GM.G.4 Compose a geometric shape or solid by combining multiple two-dimensional shapes and/or three-dimensional solids (squares, circles, triangles, rectangles, trapezoids, rhombuses, pentagons, hexagons, octagons, cubes, spheres, cylinders, cones, triangular prisms, and rectangular prisms).	
Measurement (M) *Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.*	
Standards	**Clarification**
1.GM.M.1 Measure the length of an object as a whole number of same-size, non-standard units from end to end.	Non-standard units may include paperclips, cubes, popsicle sticks, etc.
1.GM.M.2 Compare the lengths of three objects using a common measurable attribute.	
1.GM.M.3 Tell and write time to the hour and half-hour (including o-clock and half past) using analog and digital clocks.	
1.GM.M.4 Identify and tell the value of a dollar bill, quarter, dime, nickel, and penny.	
1.GM.M.5 Count collections of coins (pennies, nickels, and dimes) relating to counting patterns by 1s, 5s, and 10s up to one dollar.	This standard includes a mixture of coins (pennies, nickels, and dimes) up to one dollar. Students may start by counting one coin up to one dollar but are expected to apply their counting patterns of 1s, 5s, and 10s.
Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic concepts of probability.*	
Data (D) *Learners will represent and interpret data.*	
Standards	**Clarification**
1.DPS.D.1 Collect, organize and represent data with up to three categories using picture and bar graphs.	
1.DPS.D.2 Analyze data by answering descriptive questions.	Ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

SECOND GRADE

Math Attributes (MA) *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement through participation in various relevant learning experiences.*		
Problem-Solving (P)	**Connections (C)**	**Reasoning and Proof (R)**
Analyze, execute, evaluate, and adapt approaches and solutions when problem-solving in novel situations.	*Create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.*	*Reason logically, citing relevant evidence to explain and critique what they see, think, and conclude through exploration, generalization, and validation.*
K-2.MA.P Learners can identify and use strategies to problem-solve situations and determine an appropriate solution.	**K-2.MA.C** Learners can make connections and demonstrate relationships using words, pictures, or symbols.	**K-2.MA.R** Learners can use prior knowledge and experiences to explain their thinking.

(2022 North Dakota Learning Continuum)

Number and Operations (NO) *Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.*	
Counting and Cardinality (CC) *Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.*	
Standards	**Clarification**
2.NO.CC.1 Count forward from any given number within 1000.	
2.NO.CC.2 Count backward from any given number within 1000.	
2.NO.CC.3 Read and write numbers up to 1000 using standard, word, and expanded forms.	Spelling is not assessed.
2.NO.CC.4 Skip count forward and backward by 2s and 100s and recognize the patterns of skip counts.	**Assessment Boundary**: Start from any multiple and move forward or backward by 2s or 100 (e.g., 20, 18, 16, etc.). Range 2-20 and 100-1,000.
Base Ten (NBT) *Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.*	
Standards	**Clarification**
2.NO.NBT.1 Understand that the three digits of a three-digit number represent a composition of some hundreds, some tens, and some ones.	
2.NO.NBT.2 Compare two three-digit numbers using symbols >, <, and =. Justify comparisons based on the value of hundreds, tens, and ones.	Ordering numbers may be included for number comparisons.
2.NO.NBT.3 Add within 100 using place value strategies and/or the relationship between addition and subtraction.	Develop a flexible understanding of both vertical and horizontal orientation. The representation of whole-number sums within 100 on a number line diagram may be included.
2.NO.NBT.4 Subtract within 100 using place value strategies and/or the relationship between addition and subtraction.	Develop a flexible understanding of both vertical and horizontal orientation. The representation of whole-number differences within 100 on a number line diagram may be included.
2.NO.NBT.5 Mentally add or subtract 10 or 100 to or from a given number between 100 and 900.	

Fractions (NF) *Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.*	
Standards	**Clarification**
2.NO.NF.1 Partition circles and rectangles into two, three, or four equal shares. Describe the shares using the language of halves, thirds, fourths, half of, a third of, and a fourth of.	
2.NO.NF.2 Recognize that identical wholes can be equally divided in different ways.	
2.NO.NF.3 Recognize that partitioning shapes into more equal shares creates smaller shares.	
Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*	
Operations and Algebraic Thinking (OA) *Learners will analyze patterns and relationships to generate and interpret numerical expressions.*	
Standards	**Clarification**
2.AR.OA.1 Automatically add and subtract within 20.	See Appendix B for recommended automaticity.
2.AR.OA.2 Apply the properties of operations to solve addition and subtraction equations within 100 and justify thinking.	Properties of Operations – See Appendix A, Table 1.
2.AR.OA.3 Solve one- and two-step authentic word problems with addition within 100, including the use of unknowns.	Develop a flexible understanding of both vertical and horizontal orientation.
2.AR.OA.4 Solve one- and two-step authentic word problems with subtraction within 100, including the use of unknowns.	Develop a flexible understanding of both vertical and horizontal orientation.
2.AR.OA.5 Use repeated addition to find the total number of objects arranged in a rectangular array.	**Assessment Boundary**: Proficiency is limited up to a 5 x 5 rectangular array.
2.AR.OA.6 Identify a group of objects from 0 to 20 as even or odd by showing even numbers as a sum of two equal parts.	
Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*	
Geometry (G) *Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.*	
Standards	**Clarification**
2.GM.G.1 Identify two-dimensional shapes (parallelograms and quadrilaterals).	**Assessment Boundary**: Include shapes from K.GM.G.1 and 1.GM.G.1 while adding parallelograms and quadrilaterals.
2.GM.G.2 Identify two-dimensional shapes found within three-dimensional shapes.	
2.GM.G.3 Compose geometric shapes having specified geometric attributes, such as a given number of edges, angles, faces, vertices, and/or sides.	Composition includes drawing, building, or creating.

Measurement (M) *Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.*	
Standards	**Clarification**
2.GM.M.1 Measure the length of an object using two different standard units of measurement. Describe how the two measurements relate to the size of the units chosen.	**Assessment Boundary**: Different standard units of measurement may include inches, feet, centimeters, and meters.
2.GM.M.2 Estimate and measure to determine how much longer one object is than another, expressing the difference with a standard unit of measurement.	
2.GM.M.3 Tell and write time to the nearest five minutes (including quarter after and quarter to) with a.m. and p.m. using analog and digital clocks.	
2.GM.M.4 Count collections of money (quarters, dimes, nickels, and pennies) relating to counting patterns by 1s, 5s, and 10s up to one dollar.	
Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic concepts of probability.*	
Data (D) *Learners will represent and interpret data.*	
Standards	**Clarification**
2.DPS.D.1 Formulate questions and collect, organize, and represent data with up to four categories using single-unit scaled picture and bar graphs.	
2.DPS.D.2 Generate data and create line plots marked in whole-number units.	
2.DPS.D.3 Analyze data and interpret the results to solve one-step comparison problems using information from the graphs.	

THIRD GRADE

Math Attributes (MA) *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement through participation in various relevant learning experiences.*		
Problem-Solving (P)	**Connections (C)**	**Reasoning and Proof (R)**
Analyze, execute, evaluate, and adapt approaches and solutions when problem-solving in novel situations.	*Create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.*	*Reason logically, citing relevant evidence to explain and critique what they see, think, and conclude through exploration, generalization, and validation.*
3-5.MA.P Learners can develop and carry out a logical plan to problem-solve situations, reflect on the reasonableness of solutions, and explore alternate strategies with guidance.	**3-5.MA.C** Learners can make connections and summarize related ideas using supporting evidence.	**3-5.MA.R** Learners can reason logically based on experience and knowledge, citing evidence to support their reasoning and conclusions.

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Number and Operations (NO) *Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.*	
Counting and Cardinality (CC) *Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.*	
Standard	**Clarification**
3.NO.CC.1 Read and write numbers up to 10,000 using objects or visual representations, including standard, word, and expanded forms.	Spelling is not assessed.
Base Ten (NBT) *Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.*	
Standards	**Clarification**
3.NO.NBT.1 Compare two four-digit numbers using symbols >, <, and =. Justify comparisons based on the value of thousands, hundreds, tens, and ones.	
3.NO.NBT.2 Apply place value understanding to round whole numbers to the nearest 10 or 100.	Ordering numbers may be included for number comparisons.
3.NO.NBT.3 Add and subtract within 1000 using place value strategies, algorithms, and/or the relationship between addition and subtraction.	Apply a flexible understanding of both vertical and horizontal orientation.
3.NO.NBT.4 Multiply one-digit whole numbers by multiples of 10 within 100.	Apply a flexible understanding of both vertical and horizontal orientation.
Fractions (NF) *Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.*	
Standards	**Clarification**
3.NO.NF.1 Partition two-dimensional figures into equal areas and express the area of each part as a unit fraction of the whole. Describe using the language of sixths, eighths, a sixth of, and an eighth of.	Two-dimensional figures are partitioned into halves, fourths, and thirds in prior grades (see 1.NO.NF.1 and 2.NO.NF.1).

3.NO.NF.2 Represent and understand a fraction as a number on a number line.	
3.NO.NF.3 Represent equivalent fractions using visual representations and number lines.	
3.NO.NF.4 Recognize whole numbers as fractions and express fractions that are equivalent to whole numbers.	Within this standard, learners will go beyond 1.
3.NO.NF.5 Compare fractions of the same whole having the same numerators or denominators, using symbols >, <, and = by reasoning about their size (fractions should be limited to denominators of 2, 3, 4, 6, and 8 and should not exceed the whole).	Ordering numbers may be included for number comparisons.
Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*	
Operations and Algebraic Thinking (OA) *Learners will analyze patterns and relationships to generate and interpret numerical expressions.*	
Standards	**Clarification**
3.AR.OA.1 Using mental strategies, multiply and divide basic facts within 100. Automatically multiply and divide up to 5 x 5 and 10s facts.	Develop a flexible understanding of both vertical and horizontal orientation. Students will continue to learn multiplication and division within the range of basic facts to 100, but automaticity is expected within the range of 5 x 5 and 10s facts. Continued automaticity of facts continues in 4.AR.OA.1. See Appendix B for recommended automaticity.
3.AR.OA.2 Apply the properties of operations to solve multiplication and division equations and justify thinking.	Apply a flexible understanding of both vertical and horizontal orientation. Properties of Operations – See Appendix A, Table 1. **Assessment Boundary**: Learners utilize commutative, associative, and distributive properties without formal language.
3.AR.OA.3 Solve two-step authentic word problems using addition and subtraction within 1000, including equations with a letter as an unknown.	
3.AR.OA.4 Use strategies and visual models to solve authentic word problems with multiplication within 100, including unknowns, using grouping models and equations.	
3.AR.OA.5 Use strategies and visual models to solve authentic word problems with division within 100, including unknowns, using grouping models and equations.	
3.AR.OA.6 Identify arithmetic patterns and explain them using the properties of operations.	Properties of Operations – See Appendix A, Table 1. Example: Observe that 4 times a number is always even and explain why 4 times a number can be decomposed into two equal addends. **Assessment Boundary**: Learners utilize commutative, associative, and distributive properties without formal language.

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*	
Geometry (G) *Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.*	
Standards	**Clarification**
3.GM.G.1 In two-dimensional shapes, identify lines, angles (right, acute, obtuse), and perpendicular and parallel lines.	Two-dimensional shapes include quadrilaterals and right triangles.
3.GM.G.2 Sort quadrilaterals into categories based on attributes.	Quadrilaterals may share attributes, and attributes can define a larger category. (See Appendix D)
3.GM.G.3 Identify lines of symmetry in quadrilaterals.	
Measurement (M) *Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.*	
Standards	**Clarification**
3.GM.M.1 Measure lengths using rulers marked with halves and fourths of an inch.	
3.GM.M.2 Measure and estimate liquid volumes and masses of objects using standard units. Solve one-step authentic word problems involving masses or volume given in the same units.	Standard units: grams (g), kilograms (kg), and liters (l).
3.GM.M.3 Tell and write time to the nearest minute and measure time intervals in minutes.	
3.GM.M.4 Solve elapsed time authentic word problems on the hour and the half-hour, using a variety of strategies.	
3.GM.M.5 Solve authentic word problems involving dollar bills, quarters, dimes, nickels, and pennies using the $ and ¢ symbols appropriately.	**Assessment Boundary**: Word problems do not include the use of decimals.
3.GM.M.6 Solve problems involving the perimeters of rectangles given the side lengths or when given the perimeter and unknown side length(s).	Use rectangles with the same perimeter and different areas or with the same area and different perimeters.
3.GM.M.7 Recognize area as an attribute of plane figures and understand concepts of area measurement.	A square with a side length of 1 unit, called “a unit square,” is said to have “one square unit” of area and can be used to measure area. A plan figure, which can be covered without gaps or overlaps by n unit squares, is said to have an area of n square units.
3.GM.M.8 Find the area of a rectangle with whole-number side lengths by modeling with unit squares; show that area can be additive and is the same as would be found by multiplying the side lengths.	

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic concepts of probability.*	
Data (D) *Learners will represent and interpret data.*	
Standards	**Clarification**
3.DPS.D.1 Formulate questions to collect, organize, and represent data with more than four categories using scaled picture and bar graphs.	This includes collecting observations, surveys, or experiments to collect data to best-fit hypotheses or questions.
3.DPS.D.2 Generate data and create line plots marked in whole numbers, halves, and fourths of a unit.	
3.DPS.D.3 Analyze data and make simple statements to solve one- and two-step problems using information from the graphs.	

FOURTH GRADE

Math Attributes (MA) *Learners will practice and demonstrate broad, transferable, and enduring necessary skills for advancement through participation in various relevant learning experiences.*		
Problem-Solving (P)	**Connections (C)**	**Reasoning and Proof (R)**
Analyze, execute, evaluate, and adapt approaches and solutions when problem-solving in novel situations.	*Create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.*	*Reason logically, citing relevant evidence to explain and critique what they see, think, and conclude through exploration, generalization, and validation.*
3-5.MA.P Learners can develop and carry out a logical plan to problem-solve situations, reflect on the reasonableness of solutions, and explore alternate strategies with guidance.	**3-5.MA.C** Learners can make connections and summarize related ideas using supporting evidence.	**3-5.MA.R** Learners can reason logically based on experience and knowledge, citing evidence to support their reasoning and conclusions.

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Number and Operations (NO) *Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.*	
Counting and Cardinality (CC) *Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.*	
Standard	**Clarification**
4.NO.CC.1 Read numbers to the millions place, including word, standard, and expanded form. Write numbers to the millions place, including standard and expanded form.	Students are not expected to write word form to the millions. Spelling is not assessed.
Base Ten (NBT) *Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.*	
Standards	**Clarification**
4.NO.NBT.1 Understand that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.	
4.NO.NBT.2 Compare two numbers to the millions place and decimals to the hundredths place, using symbols >, <, and =. Justify comparisons based on the value of the digits.	Students compare two numbers to the millions place and decimals to the hundredth place. In a progression, students may practice reading and writing numbers and decimals prior to comparing. Ordering numbers may be included for number comparisons.
4.NO.NBT.3 Apply place value understanding to round multi-digit whole numbers to any place.	
4.NO.NBT.4 Add and subtract multi-digit whole numbers to the one million place using strategies, including the algorithm.	Apply a flexible understanding of both vertical and horizontal orientation.
4.NO.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number and multiply two two-digit numbers. Show and justify the calculation using equations, rectangular arrays, and models.	Apply a flexible understanding of both vertical and horizontal orientation.

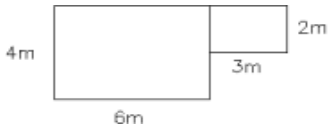
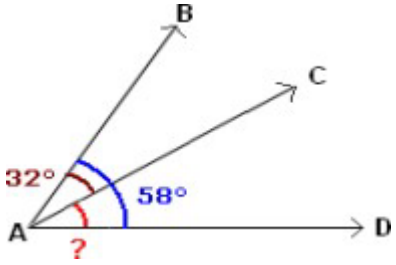
4.NO.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors using place value strategies. Show and justify the calculation using equations, rectangular arrays, and models.	

Fractions (NF) *Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.*	
Standards	**Clarification**
4.NO.NF.1 Express equivalent fractions with a denominator of 10 and a denominator of 100 to generate a decimal notation.	
4.NO.NF.2 Explain and demonstrate how a mixed number is equivalent to a fraction greater than one and how a fraction greater than one is equal to a mixed number using visual fraction models and reasoning strategies (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).	Example: $1\frac{1}{3}=\frac{4}{3}$ and $\frac{4}{3}=1\frac{1}{3}$
4.NO.NF.3 Generate equivalent fractions using numerical representations, visual representations, and number lines (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).	
4.NO.NF.4 Demonstrate how equivalent fractions are generated by multiplying a fraction equivalent to 1 or the properties of multiplication (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).	
4.NO.NF.5 Compare and order fractions having, unlike numerators or denominators. Record comparisons using the symbols >, <, and =. Justify using a visual fraction model (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).	
4.NO.NF.6 Solve authentic word problems by adding and subtracting fractions and mixed numbers with like denominators (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).	In a progression, students would learn how to add and subtract fractions and mixed numbers to apply the understanding to word problems.
4.NO.NF.7 Solve problems by multiplying fractions and whole numbers using visual fraction models (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).	**Assessment Boundary**: Model with visuals how fractions are multiplied, rather than using a standard algorithm for multiplication with fractions.

Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*	
Operations and Algebraic Thinking (OA) *Learners will analyze patterns and relationships to generate and interpret numerical expressions.*	
Standards	**Clarification**
4.AR.OA.1 Automatically multiply and divide through 10 x 10.	Apply a flexible understanding of both vertical and horizontal orientation. Automaticity in the range of 5 x 5 and 10s facts is in standard 3.AR.OA.1. Continued automaticity of facts continues in standard 5.AR.OA.1. See Appendix B for recommended automaticity.
4.AR.OA.2 Identify and apply the properties of operations for addition, subtraction, multiplication, and division and justify thinking.	Apply a flexible understanding of both vertical and horizontal orientation. Properties of Operations – See Appendix A, Table 1. **Assessment Boundary**: Learners utilize commutative, associative, identity, and distributive properties.
4.AR.OA.3 Solve multi-step authentic word problems using the four operations, including problems with interpreted remainders. Represent problems using equations, including a symbol as an unknown.	**Assessment Boundary**: Use drawings and equations with symbols for the unknown number (variable) to represent a problem.
4.AR.OA.4 Find factor pairs and multiples within the range of 1-36 while classifying numbers as prime or composite.	
4.AR.OA.5 Interpret multiplication equations as a comparison. Represent multiplicative comparisons as multiplication equations.	Example: Interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Example: Kari has 3 marbles; Greg has 7 times as many. How many marbles does Greg have? 3 x 7 = 21 or 7 x 3 = 21.
4.AR.OA.6 Generate a number or shape pattern that follows a given rule while identifying apparent features of the pattern that were not explicit in the rule itself.	(e.g., Given a rule “add 3” and the starting number of 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers.)
Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*	
Geometry (G) *Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.*	
Standards	**Clarification**
4.GM.G.1 Identify, label, and draw points, lines, line segments, rays, and angles (right, acute, obtuse).	

4.GM.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of specified size.	See Appendix D for guidance. Properties of two-dimensional shapes would include identifying the sides and angles found in polygons and triangles. **Assessment Boundary**: Shapes are classified by their attributes and not their formal name.
4.GM.G.3 Draw lines of symmetry in two-dimensional figures.	

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

Standards	Clarification
4.GM.M.1 Know the relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min., sec. Record measurement equivalents in a two-column table.	
4.GM.M.2 Generate simple conversions from a larger unit to a smaller unit to solve authentic problems within a single system of measurement, both customary and metric systems.	
4.GM.M.3 Identify and use the appropriate tools, operations, and units of measurement, both customary and metric, to solve problems involving time, length, weight, mass, and capacity.	
4.GM.M.4 Solve authentic word problems involving dollar bills, quarters, dimes, nickels, and pennies using $ and ¢ symbols and decimal notation appropriately.	
4.GM.M.5 Apply the area and perimeter formulas for rectangles, including connected rectangular figures, in problems.	Example: A house owner wants to buy sod for his backyard. The sod is sold in square meters. Determine how many square meters of sod are needed to cover the backyard pictured below.  $4 \times 6 = 24\ m^2$ $3 \times 2 = 6\ m^2$ $24 + 6 = 30\ m^2$
4.GM.M.6 Measure angles in whole-number degrees using a protractor. Using a protractor and ruler, draw angles of a specified measure.	
4.GM.M.7 Recognize angle measures as additive and solve addition and subtraction problems to find unknown angles on a diagram.	Example: If angle BAD is 58° and angle BAC measures 32°, what is the measure of angle CAD? 

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic concepts of probability.*	
Data (D) *Learners will represent and interpret data.*	
Standards	**Clarification**
4.DPS.D.1 Formulate questions to collect, organize, and represent data to reason with math and across disciplines.	Choose the visual representation that best displays the data collected (e.g., pictograph, bar graph, and tallies).
4.DPS.D.2 Generate data and create line plots to display a data set of fractions of a unit (½, ¼, ⅛). Solve problems involving addition and subtraction of fractions by using information presented in line plots.	
4.DPS.D.3 Utilize graphs and diagrams to represent and solve authentic word problems using the four operations involving whole numbers, benchmark fractions, and decimals.	**Assessment Boundary**: This includes distances, intervals of time, liquid volumes, masses of objects, and money.

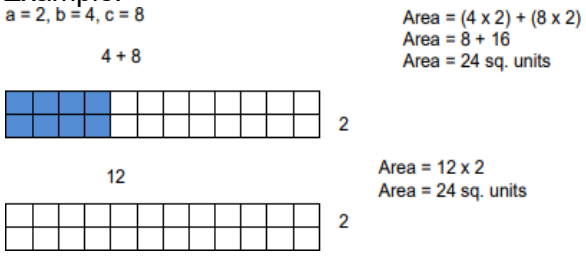
FIFTH GRADE

Math Attributes (MA) *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement through participation in various relevant learning experiences.*		
Problem-Solving (P)	**Connections (C)**	**Reasoning and Proof (R)**
Analyze, execute, evaluate, and adapt approaches and solutions when problem-solving in novel situations.	*Create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.*	*Reason logically, citing relevant evidence to explain and critique what they see, think, and conclude through exploration, generalization, and validation.*
3-5.MA.P Learners can develop and carry out a logical plan to problem-solve situations, reflect on the reasonableness of solutions, and explore alternate strategies with guidance.	**3-5.MA.C** Learners can make connections and summarize related ideas using supporting evidence.	**3-5.MA.R** Learners can reason logically based on experience and knowledge, citing evidence to support their reasoning and conclusions.

(2022 North Dakota Learning Continuum)

Number and Operations (NO) *Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.*	
Counting and Cardinality (CC) *Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.*	
Standard	**Clarification**
5.NO.CC.1 Read and write decimals to the thousandths, including standard, word, and expanded forms.	Spelling is not assessed.
Base Ten (NBT) *Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals*	
Standards	**Clarification**
5.NO.NBT.1 Understand that in a multi-digit number, a digit in one place represents ten times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	
5.NO.NBT.2 Compare two decimals to the thousandth place using symbols >, <, and =. Justify comparisons based on the value of the digits.	Ordering numbers may be included for number comparisons.
5.NO.NBT.3 Apply place value understanding to round decimals to any place.	
5.NO.NBT.4 Multiply multi-digit whole numbers using strategies flexibly, including the algorithm.	Apply a flexible understanding of both vertical and horizontal orientation. Mastery of the multiplication algorithm is expected.
5.NO.NBT.5 Use concrete models, drawings, place value strategies, properties of operations and/or relationships to add, subtract, and multiply decimals to hundredths.	Properties of Operations – See Appendix A, Table 1. Division of decimals is found within the sixth-grade standards.

5.NO.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and two-digit divisors using place value strategies. Show and justify the calculation by using equations, rectangular arrays, and/or area models.	Division procedures, including the algorithm, are included in sixth grade.
5.NO.NBT.7 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	
Fractions (NF) *Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.*	
Standards	**Clarification**
5.NO.NF.1 Generate equivalent forms of commonly used fractions and decimals (e.g., halves, fourths, fifths, tenths).	This standard includes writing fractions in the lowest terms.
5.NO.NF.2 Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number and explain why multiplying a given number by a fraction less than one results in a product smaller than the given number.	
5.NO.NF.3 Solve authentic word problems by adding and subtracting fractions and mixed numbers with unlike denominators using visual fraction models and equations.	In a progression, students may practice adding and subtracting fractions and mixed numbers with unlike denominators prior to using the understanding in word problems.
5.NO.NF.4 Solve authentic word problems by multiplying fractions and mixed numbers using visual fraction models and equations.	**Assessment Boundary**: Model with visuals how fractions are multiplied, rather than using the standard algorithm for multiplication with fractions.
Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*	
Operations and Algebraic Thinking (OA) *Learners will analyze patterns and relationships to generate and interpret numerical expressions.*	
Standards	**Clarification**
5.AR.OA.1 Automatically multiply and divide through 12 x 12.	Apply a flexible understanding of both vertical and horizontal orientation. Automaticity of facts is also in standards 3.AR.OA.1 and 4.AR.OA.1. See Appendix B for recommended automaticity.
5.AR.OA.2 Analyze problems using the order of operations to solve and evaluate expressions while justifying thinking.	Apply a flexible understanding of both vertical and horizontal orientation.

Operations and Algebraic Thinking (OA) *Learners will analyze patterns and relationships to generate and interpret numerical expressions.*	
5.AR.OA.3 Write simple expressions that record calculations with numbers. Interpret numerical expressions without evaluating them.	Example: Express the calculation "add 8 and 7, then multiply by 2 as 2x(8 + 7). Recognize that 3x(18,932 + 921) is three times as large as 18,932 + 921 without having to calculate the indicated sum or product.
5.AR.OA.4 Find factor pairs and multiples within the range of 1-100 while classifying numbers as prime or composite.	
5.AR.OA.5 Generate two numerical patterns using two given rules and form ordered pairs consisting of corresponding terms from the two patterns. (Graphing on a coordinate plane).	Example: Given the rule "add 3" and the starting number of 0 and given the rule "add 6" and the starting number of 0, generate terms in the resulting sequences and, in this case, observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.
Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*	
Geometry (G) *Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.*	
Standards	**Clarification**
5.GM.G.1 Classify two-dimensional figures in a hierarchy based on properties.	See Appendix D. Properties of two-dimensional shapes and the names of the specific shapes would include identifying the number of sides, angles, and lines of symmetry found in polygons and triangles (equilateral, isosceles, scalene).
5.GM.G.2 Identify the x-coordinate and y-coordinate to graph and name points in the first quadrant of the coordinate plane.	In a progression, students may begin by learning about the origin in direction with the axis and how the coordinates correspond.
5.GM.G.3 Form ordered pairs and graph points in the first quadrant on the coordinate plane to solve authentic word problems.	
Measurement (M) *Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.*	
Standards	**Clarification**
5.GM.M.1 Generate conversions among different-sized standard measurement units within a given measurement system, both customary and metric. Use these conversions to solve multi-step, authentic word problems.	
5.GM.M.2 Find the area and perimeter of a rectangle, including connected rectangular figures, with fractional side lengths.	Example: 

5.GM.M.3 Recognize volume as an attribute of rectangular prisms and measure volume by counting unit cubes.	In a progression, students may begin by recognizing that volume is additive when measuring volume by counting unit cubes.

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic concepts of probability.*	
Data (D) *Learners will represent and interpret data.*	
Standards	**Clarification**
5.DPS.D.1 Generate data and create line plots to display a data set of fractions of a unit (½, ¼, ⅛). Use grade-level operations for fractions to solve problems involving information presented in line plots.	
5.DPS.D.2 Utilize graphs and diagrams to represent, analyze, and solve authentic word problems using information presented in one or more tables or line plots including whole numbers, fractions, and decimals.	The DPS.D category in K-4 describes graphs and tables that students are expected to learn. 5.DPS.D.2 encompasses all graphs, and the problem now dictates which visual representation students should use.

SIXTH GRADE

Math Attributes (MA) *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement through participation in various relevant learning experiences.*		
Problem-Solving (P)	**Connections (C)**	**Reasoning and Proof (R)**
Analyze, execute, evaluate, and adapt approaches and solutions when problem-solving in novel situations.	*Create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.*	*Reason logically, citing relevant evidence to explain and critique what they see, think, and conclude through exploration, generalization, and validation.*
6-8.MA.P Learners can analyze information and formulate a flexible, systematic plan to problem-solve authentic situations and reflect on the reasonableness of the solution, making revisions when necessary.	**6-8.MA.C** Learners can create connections within and across concepts and provide examples of how they relate to other learning and ideas using supporting evidence.	**6-8.MA.R** Learners can reason logically, citing evidence to evaluate and explain what they see, think, and conclude through exploration and justification.

(2022 North Dakota Learning Continuum)

Number and Operations (NO) *Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.*	
Number Systems (NS) *Learners will expand their knowledge of the number system to create connections and solve problems within and across concepts.*	
Standards	**Clarification**
6.NO.NS.1 Explain and show the relationship between non-zero rational numbers and their opposites using horizontal and vertical number lines, including authentic problems. Use rational numbers to represent quantities in authentic contexts and explain the meaning of 0 in certain situations.	This is the concept of absolute value, but formal notation is not required at this level.
6.NO.NS.2 Write, interpret, and explain statements of order for rational numbers on a number line and in authentic contexts.	A statement of order could be a list of numbers, a statement of inequality, or a description.
Operations (O) *Learners will expand their computational fluency to create connections and solve problems within and across concepts.*	
Standards	**Clarification**
6.NO.O.1 Divide multi-digit whole numbers up to four-digit dividends and two-digit divisors using strategies or procedures.	Learners should be able to reason using number relationships and logic to choose an efficient strategy to solve each problem. Procedures may include the standard algorithm.
6.NO.O.2 Add and subtract fractions and decimals up to the hundredth place, including authentic problems.	Fractions include mixed numbers and improper fractions.

6.NO.O.3 Apply multiplication and division of fractions and decimals to solve and interpret problems using visual models, including authentic problems.	Fractions include mixed numbers and improper fractions. **Assessment Boundary**: Decimal division is limited to problems where either the dividend or divisor is a whole number, and the other is a decimal up to the hundredth place.
6.NO.O.4 Determine the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.	This leads to algebraic topics, including factoring expressions and the distributive property with variables. The focus should not be on simplifying fractions or finding the least common denominators. See Appendix B for recommended automaticity.

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

Ratios and Proportional Relationships (RP)

Learners will use ratios, rates, and proportions to model relationships and solve problems.

Standards	Clarification
6.AR.RP.1 Describe the concept of a ratio relationship between two quantities using ratio language and visual models.	Visual models may include tables of equivalent ratios, tape diagrams, double number line diagrams, etc. This includes part-to-part and part-to-whole ratios.
6.AR.RP.2 Describe and calculate a unit rate when given a ratio relationship between two quantities using rate language and visual models.	Visual models may include tables of equivalent ratios, tape diagrams, double number line diagrams, etc. The focus should be ratios and rates but use previous fraction knowledge to support the work.
6.AR.RP.3 Make and use tables of equivalent ratios, tape diagrams, double number line diagrams, and equations to solve problems involving ratios, rates, and unit rates, including authentic problems.	
6.AR.RP.4 Calculate a percent of a quantity as a rate per 100. Solve problems using ratio reasoning involving finding the whole when given a part and the percent.	
6.AR.RP.5 Convert measurement units within and between measurement systems using ratio reasoning given conversion factors.	This is the introduction to conversions between measurement units.

Expressions and Equations (EE)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

Standards	Clarification
6.AR.EE.1 Read, write, and evaluate numerical expressions, including expressions with whole number exponents and grouping symbols.	This standard includes evaluating expressions using the order of operations, including parentheses.
6.AR.EE.2 Read and evaluate algebraic expressions, including expressions with whole number exponents and grouping symbols. Write algebraic expressions to represent simple and authentic situations.	

6.AR.EE.3 Identify when two expressions are equivalent. Apply the properties of operations to generate equivalent expressions.	Properties of Operations – See Appendix A, Table 1. Both numeric and algebraic expressions are included. Two expressions are equivalent when they represent the same number regardless of which value is substituted into them.
6.AR.EE.4 Describe the concept of a solution of an equation and an inequality. Determine whether a given number is a solution to an equation or an inequality.	
6.AR.EE.5 Write and solve equations of the form $x + p = q$ and $px = q$ for cases in which p and q are non-negative whole numbers or decimals, including authentic problems.	
6.AR.EE.6 Write a statement of inequality of the form $x > c$ or the form $x < c$ to represent a constraint or condition. Recognize that inequalities of the form $x > c$ or the form $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	Inequalities are represented by the following $<$, $>$, $\leq$, $\geq$, $\neq$. **Assessment Boundary**: This does not include compound inequalities at this level.
Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*	
Area and Volume (AV) *Learners will use visualization and spatial reasoning to solve problems involving the area, surface area, and volume of geometric figures.*	
Standards	**Clarification**
6.GM.AV.1 Derive the relationship of the areas of triangles using the area of rectangles. Calculate the areas of triangles and quadrilaterals by composing and/or decomposing them into rectangles and triangles, including authentic problems.	Learners should develop a fluent way of finding the area of a triangle. Using the shape composition and decomposition skills acquired in earlier grades, Learners learn to develop area formulas for parallelograms and then triangles. They learn how to address three different cases for triangles: • a height that is a side of a right triangle, • a height that "lies over the base," • and a height that is outside the triangle.
6.GM.AV.2 Describe the concept of volume of a right rectangular prism. Apply given formulas to calculate the volume of right rectangular prisms, including fractional edge lengths, including authentic problems.	In fifth grade, there is a similar standard with whole numbers only. This understanding is extended to fractional sizes.

Geometric Figures (GF) *Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics of figures, perform transformations, and construct logical arguments.*	
Standards	**Clarification**
6.GM.GF.1 Identify and position ordered pairs of rational numbers in all four quadrants of a coordinate plane.	
6.GM.GF.2 Draw polygons in the coordinate plane given coordinates for the vertices. Determine the length of a side joining points with the same first or second coordinate, including authentic problems.	The focus is <u>not</u> on integer operations. The sides of polygons should <u>not</u> be diagonal.
6.GM.GF.3 Represent three-dimensional figures using nets made up of rectangles and triangles (right prisms and pyramids whose bases are triangles and rectangles). Calculate the surface area of prisms with rectangular and triangular bases using nets, including authentic problems.	**Assessment Boundary**: This standard does not include knowing and applying surface area formulas; the focus is on individual areas of the net.
Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic concepts of probability.*	
Data Analysis (D) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences, conclusions, and making predictions.*	
Standards	**Clarification**
6.DPS.D.1 Write a statistical question that can be answered using measures of center or variability of a data set.	**Assessment Boundary**: Measures of center: mean and median. Measures of variability: range and mean absolute deviation.
6.DPS.D.2 Calculate measures of center (median and mean) and variability (range and mean absolute deviation) to answer a statistical question. Identify mode(s) if they exist.	
6.DPS.D.3 Identify outliers by observation and describe their effect on measures of center and variability. Justify which measures would be appropriate to answer a statistical question.	
6.DPS.D.4 Display numerical data in plots on a number line, including dot plots and histograms. Describe any overall patterns in data, such as gaps, clusters, and skews.	Overall shape in this context refers to the shape of a graphical representation of data including uniform, skewed, symmetric, and normal (bell-shaped).
Probability (P) *Learners will understand and apply basic concepts of probability.*	
NOTE: Standards begin in seventh grade.	

SEVENTH GRADE

Math Attributes (MA) *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement through participation in various relevant learning experiences.*		
Problem-Solving (P)	**Connections (C)**	**Reasoning and Proof (R)**
Analyze, execute, evaluate, and adapt approaches and solutions when problem-solving in novel situations.	*Create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.*	*Reason logically, citing relevant evidence to explain and critique what they see, think, and conclude through exploration, generalization, and validation.*
6-8.MA.P Learners can analyze information and formulate a flexible, systematic plan to problem-solve authentic situations and reflect on the reasonableness of the solution, making revisions when necessary.	**6-8.MA.C** Learners can create connections within and across concepts and provide examples of how they relate to other learning and ideas using supporting evidence.	**6-8.MA.R** Learners can reason logically, citing evidence to evaluate and explain what they see, think, and conclude through exploration and justification.

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Number and Operations (NO) *Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.*	
Number Systems (NS) *Learners will expand their knowledge of the number system to create connections and solve problems within and across concepts.*	
Standards	**Clarification**
7.NO.NS.1 Describe the absolute value of a number as its distance from zero on a number line.	Learners should be introduced to the notation of absolute value at this level.
7.NO.NS.2 Recognize common fractions and decimal equivalencies up to a denominator of 10. Convert a rational number to a decimal using technology.	See Appendix B for recommended automaticity.
Operations (O) *Learners will expand their computational fluency to create connections and solve problems within and across concepts.*	
Standards	**Clarification**
7.NO.O.1 Add, subtract, multiply, and divide integers using visual models and properties of operations in multi-step problems, including authentic problems.	Properties of Operations – See Appendix A, Table 1. Visual models may include algebra tiles, colored chips, number lines, etc.
7.NO.O.2 Add, subtract, multiply, and divide non-negative fractions in multi-step problems, including authentic problems.	

7.NO.O.3 Add, subtract, multiply, and divide non-negative decimals to the hundredth place in multi-step problems using strategies or procedures, including authentic problems.	Learners should be able to reason using number relationships and logic to choose an efficient strategy to solve each problem. Procedures can include the standard algorithm. **Assessment Boundary**: Division is limited to problems where either the dividend or divisor is a whole number, and the other is a decimal up to the hundredth place.
Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*	
Ratios and Proportional Relationships (RP) *Learners will use ratios, rates, and proportions to model relationships and solve problems.*	
Standards	**Clarification**
7.AR.RP.1 Calculate unit rates associated with ratios of rational numbers, including ratios of lengths, areas, and other quantities measured in like or different units.	Unit rates may be represented as fractions, decimals, and/or percents.
7.AR.RP.2 Analyze the relationship between the dependent and independent variables of a proportional relationship using graphs and tables. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, k) where k is the unit rate.	
7.AR.RP.3 Identify the constant of proportionality in tables, graphs, equations, diagrams, and descriptions of proportional relationships. Represent proportional relationships by an equation of the form y = kx, where k is the constant of proportionality, and describe the meaning of each variable (y, k, x) in the context of the situation.	
7.AR.RP.4 Use proportional relationships to solve multi-step problems involving ratios, percents, and scale drawings of geometric figures, including authentic problems.	The focus should be on the conceptual understanding of a proportional relationship, not the procedural methods of solving these problems. Conceptual methods can include using ratio tables, tape diagrams, double number lines, etc.
Expressions and Equations (EE) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.*	
Standards	**Clarification**
7.AR.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions involving variables, integers, and/or nonnegative fractions and decimals with an emphasis on writing equivalent expressions.	Properties of Operations – See Appendix A, Table 1.

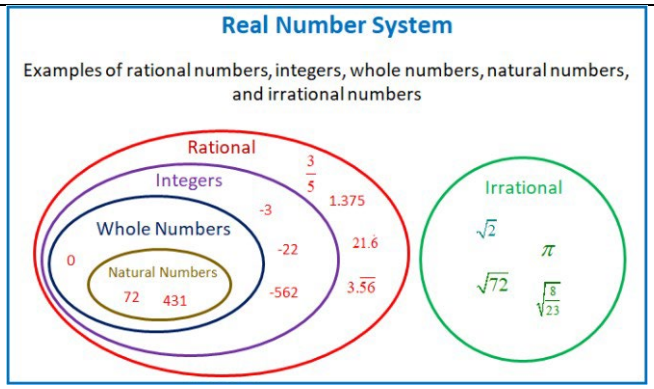
7.AR.EE.2 Write and solve equations of the form px + q = r and p(x + q) = r , including authentic problems.	Properties of Equality – See Appendix A, Table 2. **Assessment Boundary**: q and r are integers, and p is an integer or a positive fraction/decimal.
7.AR.EE.3 Write and solve one- or two-step inequalities where coefficients and solutions are integers and/or non-negative fractions and decimals, including authentic problems. Graph the solution set of the inequality and interpret it in the context of the problem.	Properties of Inequality – See Appendix A, Table 3. **Assessment Boundary**: At this level, compound inequalities are not included.
Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*	
Area and Volume (AV) *Learners will use visualization and spatial reasoning to solve authentic and mathematical problems involving area, surface area, and volume of geometric figures.*	
Standards	**Clarification**
7.GM.AV.1 Describe the relationship between the circumference and diameter of a circle (pi). Apply given formulas to calculate the area and circumference of a circle, including authentic problems.	The focus of the first part is to develop an understanding of the concept of pi.
7.GM.AV.2 Calculate areas of polygons by composing and/or decomposing them into rectangles and triangles, including authentic problems. Solve problems involving the surface area of prisms and right pyramids using nets, including authentic problems.	**Assessment Boundary**: The standard does not include knowing and applying surface area formulas; the focus is on the individual areas of the net.
7.GM.AV.3 Solve problems involving the volume of prisms and composite solids, including authentic problems.	Any problem can be used, provided the base can be decomposed into triangles and/or rectangles.
Geometric Figures (GF) *Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics of figures, perform transformations, and construct logical arguments.*	
Standards	**Clarification**
7.GM.GF.1 Draw triangles from given conditions using appropriate tools. Defend whether a unique triangle, multiple triangles, or no triangle can be constructed when given three measures of angles or sides.	Appropriate tools could include protractors, rulers, compasses, and/or technology. Ensure learners understand the triangle classifications and vocabulary.
7.GM.GF.2 Describe the following angle-pair relationships: supplementary angles, complementary angles, vertical angles, and adjacent angles. Solve for an unknown angle in a figure by applying facts about these angles.	**Assessment Boundary**: Solving for unknown angles does not include algebraic expressions involving operations. The focus is finding unknown angle measures, not solving equations.

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic concepts of probability.*	
Data Analysis (D) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, and making predictions.*	
Standards	**Clarification**
7.DPS.D.1 Identify the strengths and weaknesses of a population sample, including bias in the process of the data collection.	
7.DPS.D.2 Analyze and draw inferences about a population using single and multiple random samples by using given measures of center and variability for the numerical data set.	
Probability (P) *Learners will understand and apply basic concepts of probability.*	
Standards	**Clarification**
7.DPS.P.1 Develop a probability model to find probabilities of theoretical events and contrast probabilities from an experimental model.	This is the first time learners have been exposed to the concept of probability. The basic concepts of probability and likelihood will need to be developed before fully addressing this standard.
7.DPS.P.2 Develop a probability model to find theoretical probabilities of independent compound events.	Examples of probability models can include organized lists, tree diagrams, area models, and simulations. **Assessment Boundary**: Learners are not expected to use formulas (formal procedures).

EIGHTH GRADE

Math Attributes (MA) *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement through participation in various relevant learning experiences.*		
Problem-Solving (P)	**Connections (C)**	**Reasoning and Proof (R)**
Analyze, execute, evaluate, and adapt approaches and solutions when problem-solving in novel situations.	*Create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.*	*Reason logically, citing relevant evidence to explain and critique what they see, think, and conclude through exploration, generalization, and validation.*
6-8.MA.P Learners can analyze information and formulate a flexible, systematic plan to problem-solve authentic situations and reflect on the reasonableness of the solution, making revisions when necessary.	**6-8.MA.C** Learners can create connections within and across concepts and provide examples of how they relate to other learning and ideas using supporting evidence.	**6-8.MA.R** Learners can reason logically, citing evidence to evaluate and explain what they see, think, and conclude through exploration and justification.

(2022 North Dakota Learning Continuum)

Number and Operations (NO) *Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.*	
Number Systems (NS) *Learners will expand their knowledge of the number system to create connections and solve problems within and across concepts.*	
Standards	**Clarification**
8.NO.NS.1 Compare and classify real numbers within the real number system.	
8.NO.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them on a number line diagram, and estimate the value of irrational expressions involving one operation.	Expressions can include examples such as 2π or $\sqrt{2} + 11$.
8.NO.NS.3 Use scientific notation to represent very large or very small quantities. Interpret scientific notation generated by technology. Compare and order numbers in both scientific and standard notation.	

Operations (O) *Learners will expand their computational fluency to create connections and solve problems within and across concepts.*	
Standards	**Clarification**
8.NO.O.1 Evaluate mentally the square roots of perfect squares up to 225 and cube roots of perfect cubes up to 1000.	This standard supports standard 8.NO.NS.2. This is the first learners are introduced to the idea of radicals. Connections should be made to the area of a square and the volume of a cube. See Appendix B for recommended automaticity.
8.NO.O.2 Add, subtract, multiply, and divide rational numbers using strategies or procedures.	Learners should be able to reason using number relationships and logic to choose an efficient strategy to solve each problem. Procedures can include the standard algorithm.
Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*	
Expressions and Equations (EE) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.*	
Standards	**Clarification**
8.AR.EE.1 Explain the relationship between repeated multiplication and the properties of integer exponents. Apply a single exponent property to generate equivalent numeric and algebraic expressions that include numerical coefficients.	
8.AR.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a non-negative rational number.	Attention should be drawn to the conceptual understanding of the number of solutions to these equations.
8.AR.EE.3 Explain the characteristics of a linear relationship, including identifying the slope and y-intercept in tables, graphs, equations, and descriptions.	
8.AR.EE.4 Represent linear relationships using tables, graphs, equations, and descriptions when given a relationship in one of these forms.	**Assessment Boundary**: Equations must be of the form $y = mx + b$.
8.AR.EE.5 Solve linear equations with rational number coefficients and variables on both sides, including equations that require using the distributive property and/or combining and collecting like terms. Interpret the number of solutions. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions.	

8.AR.EE.6 Read, write, and evaluate numerical and algebraic expressions including expressions involving absolute value. Solve and graph equations of the form |x|=r where r is a nonnegative rational number.	Attention should be drawn to the conceptual understanding about the number of solutions to these equations and why r cannot be a negative value. Graphs should be done on a number line with attention being drawn to the symmetry of the solutions.
8.AR.EE.7 Solve and graph inequalities in one variable with rational number coefficients and variables on both sides, including inequalities that require using the distributive property and/or combining like terms.	**Assessment Boundary**: This level does not include compound inequalities.
8.AR.EE.8 Graph linear inequalities in two variables on a coordinate plane. Interpret the possible solutions in the context of authentic problems.	**Assessment Boundary**: This level does not include compound inequalities. Inequalities must be given in the slope-intercept form.

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

Standards	**Clarification**
8.AR.F.1 Defend whether a relation is a function from various representations using appropriate function language.	**Assessment Boundary**: Function language does not include function notation at this level.
8.AR.F.2 Compare and contrast properties of two linear functions, each represented in a different way (algebraically, graphically, numerically in tables, and/or by descriptions).	
8.AR.F.3 Compare and contrast linear and non-linear functions represented in different ways (algebraically, graphically, numerically in tables, and/or by descriptions).	Non-linear is a general term that refers to any function that does not change at a constant rate. This standard is not requiring any specific type of non-linear function, such as quadratic or exponential.
8.AR.F.4 Model a linear function between two quantities by creating a table, graph, and equation. Interpret the rate of change and initial value of a linear function in terms of the situation it models.	Modeling is applying mathematics learners know to solve problems arising in everyday life, society, and the workplace. See Appendix C for the modeling process.
8.AR.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph including where the function is constant, increasing, or decreasing; linear or nonlinear; and discrete or continuous. Create a graph that exhibits the qualitative features of a function described.	

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*	
Area and Volume (AV) *Learners will use visualization and spatial reasoning to solve problems involving area, surface area, and volume of geometric figures.*	
Standard	**Clarification**
8.GM.AV.1 Apply given formulas to solve problems involving the volume of cones, cylinders, and spheres, including authentic problems.	
Geometric Figures (GF) *Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics of figures, perform transformations, and construct logical arguments.*	
Standards	**Clarification**
8.GM.GF.1 Perform single transformations to a figure on the coordinate plane and determine whether the figures are congruent or similar.	**Assessment Boundary**: Reflections on the coordinate plane are limited to over the x- or y-axis. Rotations are limited to multiples of 90° rotations about the origin. Centers for dilation on the coordinate plane are limited to the origin. Formal (coordinate) notations are not expected at this level.
8.GM.GF.2 Describe the characteristics of transformations on the coordinate plane using transformation language.	**Assessment Boundary**: For translations, use distance and direction. For reflection, use an axis as a line of reflection. For rotations about the origin, use direction (clockwise and counterclockwise) and degree (90, 180, 270, 360).
8.GM.GF.3 Name the type of transformation(s) needed to map a pre-image to its image.	**Assessment Boundary**: Sequences should be limited to two transformations.
8.GM.GF.4 Describe the following angle-pair relationships: interior and exterior angles of triangles and angles formed when a transversal cuts parallel lines or intersecting lines. Solve for an unknown angle in a figure by applying facts about these angles.	
8.GM.GF.5 Describe the relationship between the leg lengths and the hypotenuse length of a right triangle. Determine whether a triangle is a right triangle using this relationship.	
8.GM.GF.6 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in two and three dimensions on and off a coordinate plane, including authentic problems.	This does not include the distance formula.

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic concepts of probability.*	
Data Analysis (D) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, and making predictions.*	
Standards	**Clarification**
8.DPS.D.1 Interpret scatter plots for bivariate measurement data to investigate patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	
8.DPS.D.2 Draw an informal trend line on a given scatter plot with a linear association and justify its fit by describing the closeness of the data points to the line.	
8.DPS.D.3 Solve authentic problems in the context of bivariate measurement data by interpreting the slope and intercept(s) and making predictions using a linear model.	
8.DPS.D.4 Construct and interpret a two-way table summarizing bivariate categorical data collected from the same subjects.	Interpretations can include calculating joint and marginal relative frequencies.
Probability (P) *Learners will understand and apply basic concepts of probability.*	
NOTE: There are no probability standards at this level. Probability concepts are further developed in ninth and tenth grade.	

NINTH AND TENTH GRADES

Math Attributes (MA) *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement through participation in various relevant learning experiences.*		
Problem-Solving (P)	**Connections (C)**	**Reasoning and Proof (R)**
Analyze, execute, evaluate, and adapt approaches and solutions when problem-solving in novel situations.	*Create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.*	*Reason logically, citing relevant evidence to explain and critique what they see, think, and conclude through exploration, generalization and validation.*
9-12.MA.P Learners can analyze, execute, critique, and adapt approaches and solutions when problem-solving in novel situations.	**9-12.MA.C** Learners can create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.	**9-12.MA.R** Learners can reason logically, citing evidence to critique and explain what they see, think, and conclude through exploration, generalization, and validation.

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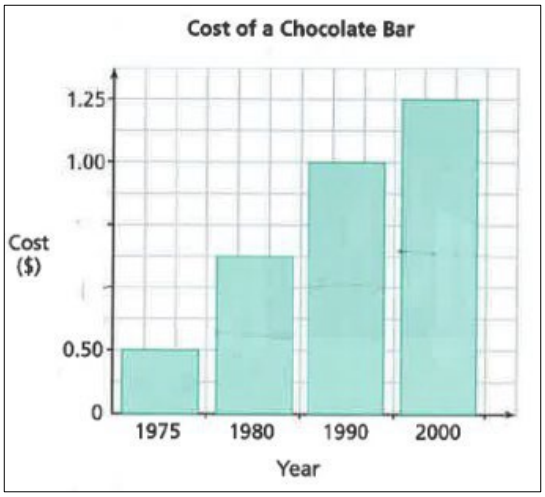
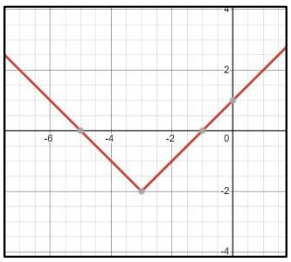
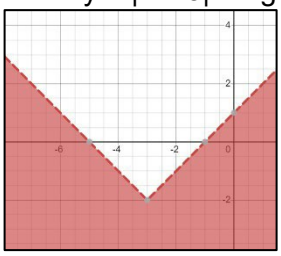
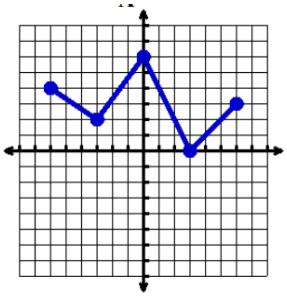
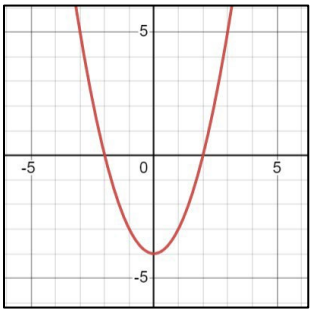
Number and Operations (NO) *Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.*	
Standards	**Clarifications**
9-10.NO.1 Explain how the definition of rational exponents follows from extending the properties of integer exponents; rewrite simple expressions involving radicals and rational exponents using the properties of exponents.	Example: $\sqrt{x^3}=x^{\frac{3}{2}}$ Example: $\sqrt{4^3}=\left(4^{\frac{1}{2}}\right)^3=2^3=8$
9-10.NO.2 Perform basic operations on simple radical expressions to write a simplified equivalent expression.	Basic operations include addition, subtraction, multiplication, and division (e.g., rationalizing the denominator (no conjugation)). Example: Simplify: $2\sqrt{9}-3\sqrt{9}+\sqrt{4}\sqrt{7}$ Example: Rationalize: $\frac{1}{\sqrt{2}}$

9-10.NO.3 Choose and interpret the scale and the units in graphs and data displays.	Example: Are there any false impressions in this graph? If so, how could you change the scale to alleviate false impressions?  Example: Gary sold candy bars to raise money for the German Club. He raised a total of $1000 for selling 400 candy bars. Graph the relationship between candy bars sold and the total raised.
9-10.NO.4* Define appropriate quantities and units for the purpose of descriptive modeling.	Example: When carpeting a room, learners may consider whether it is best to use square feet or square yards. When considering a remodeling project, they may choose such units as cost per room, cost per month of the project, or cost per contractor. Example: It takes Jeb 4.5 hours to run 50 kilometers. What is Jeb's rate in minutes per mile? Solution: $\frac{4.5\text{hr}}{50\text{km}} \cdot \frac{1.609\text{ km}}{1\text{ mi}} \cdot \frac{60\text{min}}{1\text{ hr min/ m}} \approx 8.689\text{ min/mi}$
9-10.NO.5 Choose a level of accuracy or precision appropriate to limitations on measurement when reporting quantities.	This standard applies across all high school grade levels. Example: When using a ruler, learners report their measurements based on the ruler's precision (e.g., to the nearest $^1/_{16}$ or the nearest $^1/_{32}$). They are able to measure accurately. Example: If you play soccer and you always hit the left goalpost instead of scoring, then you are not accurate; you are precise. Example: When using a ruler, learners are able to measure accurately.

	Example: When calculating the cost of a road trip, learners are given the cost of gasoline to the thousandth place. When reporting the trip cost, learners determine what level of precision (to the hundredths place or thousandths place) is appropriate and why.
Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*	
Standards	**Clarification**
9-10.AR.1 Use the structure of an expression (i.e., quadratic and exponential) to identify ways to rewrite it.	This standard includes rewriting expressions by factoring, combining like terms, using factoring techniques, applying distributive property, applying operations with polynomials, and recognizing patterns and structures in expressions. Example: See $9a^2 - 4b^2$ as $(3a)^2 - (2b)^2$ and recognize it as a difference of squares that can factor as (3a-2b)(3a+2b). Example: See $x^4 - y^4$ as $\left(x^2\right)^2 - \left(y^2\right)^2$, thus recognizing it as a difference of squares that can be factored as $\left(x^2 - y^2\right)\left(x^2 + y^2\right)$ and further to $\left(x - y\right)\left(x + y\right)\left(x^2 + y^2\right)$.
9-10.AR.2 Rearrange formulas to isolate a quantity or variable(s) of interest using the same reasoning as in solving equations.	This standard applies across all high school grade levels. Example: Rearrange V=IR to solve for the resistance R in Ohm's Law.
9-10.AR.3* Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions.	Limit inequalities to linear and quadratic.

9-10.AR.4* Create linear and exponential equations in two or more variables to represent relationships between quantities. Graph equations on coordinate axes with appropriate labels and scales.	Limit to situations requiring evaluation of exponential functions at integer inputs. Example: The cost to rent a car is $50 plus $0.25 per mile driven. Write and graph an equation to represent the situation.
9-10.AR.5 Justify each step in solving a linear equation that may or may not have a solution.	Use justifiable comments such as "combine like terms," and "distributive property," within the explanation. Example: $2(3x - 5) + 3x - 2(6 + x) = 5x - 3 + 6x + 17$ Solution: $2(3x-5)+3x-2(6-x)=11x+14$ Combine like terms $6x-10+3x-12+2x=11x+14$ Distributive Prop. $11x-22=11x+14$ Combine like terms $-22=0x+14$ Additive Inverse $-22 \neq 14$
9-10.AR.6 Solve linear equations and inequalities (to include compound inequalities) in one variable.	Examples: Solve: $-4(x - 3) + 8 < -10 + 2x$ Solve: $-\frac{3}{4}y + 7 > 10$ Solve: $6 + 7d < 6d - 5$ or $3d - 7 < 5 + 6d$ Solve: $\frac{4}{3}(4x - 3) = \frac{1}{7}(5x - 5)$
9-10.AR.7* Solve a system of linear equations graphically and algebraically. Create and solve a system of linear equations in context.	Example: Solve: $\begin{cases} -21 = -3y - 12x \\ 2 = -x - 4y \end{cases}$ Example: Sarah had $12,100 to invest. She decided to invest her money in bonds and mutual funds. She invested a portion of the money in bonds paying 8% interest per year and the remainder in a mutual fund paying 9% per year. After one year, the total income she had earned from the investments was $1,043. How much had she invested in each rate?

9-10.AR.8 Graph the solution set to a two-variable system of linear inequalities. Create and graph the solution set to a two-variable system of linear inequalities in context.	Example: Solve by graphing: $\begin{cases} y \geq -x+1 \\ y \geq x-1 \end{cases}$ Example: The girls' swim team is hosting a fundraiser. They would like to raise at least $500. They are selling candles for $5 and flower arrangements for $6. The girls estimate that, at most, they will sell 200 items. • Write a system of inequalities to represent this situation. • Graph each inequality on a grid.
9-10.AR.9 Solve absolute value equations and inequalities in one or two variables.	Example: Solve: $3\|x + 2\| - 6 = 6$ Solution: $x = 2$ or $x = -6$ Example: Solve $y = \|x + 3\| - 2$ graphically.  Example: Solve $y < \|x + 3\| - 2$ graphically. 
9-10.AR.10 Solve quadratic equations in one variable by inspection (e.g., for $x^2 = 49$) taking square roots, the quadratic formula, and factoring, as appropriate to the initial form of the equation.	
9-10.AR.11 Add, subtract, and multiply polynomials.	Focus on polynomial expressions that simplify to forms that are linear or quadratic.

Functions (F) *Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.*	
Standards	**Clarification**
9-10.AR.F.1 Determine whether a relationship is a function given a table, graph, or words, identifying x as an element of the domain and f(x) as an element in the range. Determine the domain and range of a function in context.	Example: State the domain and range of the function graphed at the right. Solution: Domain: $-6 \leq x \leq 6$ or $[-6,6]$ Range: $0 \leq y \leq 6$ or $[0,6]$ 
9-10.AR.F.2* Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in context.	Example: Suppose f(d) = 0.5d + 50 where f(d) represents the cost of renting a car driven d miles. Evaluate f(200) and interpret the result. Example: Given P(s)=4s, where P(s) represents the perimeter of a square whose side length is s, P is a function of s.
9-10.AR.F.3* Sketch the key features (to include intercepts, maximums, minimums, and lines of symmetry, where applicable) of linear, exponential, and quadratic functions modeling the relationship between two quantities using tables, graphs, written descriptions, and equations.	Example: Given $f(x) = x^2 - 4$. Graph the function and identify the intercepts, maximums, minimums, and any symmetry. Solution:  Intercepts: (-2,0), (2,0), (0,-4) Relative Minimum: (0,-4) Symmetric to the y-axis
9-10.AR.F.4* Relate the domain of a linear, quadratic, or exponential function to its graph and, where applicable, to the quantitative relationship it describes.	Example: A vehicle depreciates roughly 20% per year. Suppose the situation is modeled by the equation $f(x) = 39{,}900\,(0.8)^x$. Graph the function and justify the domain.

<table>
<tr>
<td>9-10.AR.F.5* Calculate and interpret the rate of change of linear, quadratic, or exponential functions (presented algebraically or as a table) over specified intervals.

Estimate the rate of change from a graph.</td>
<td>Focus on linear, quadratic, and exponential functions whose domains are a subset of the integers.

Example:
Jamie went on a bike trip and stopped regularly at half-hour intervals. At each break, he recorded his total distance since leaving home.

<table>
<tr><th>Stops</th><th>Time (h)</th><th>Distance (km)</th></tr>
<tr><td>1st</td><td>0.5</td><td>7</td></tr>
<tr><td>2nd</td><td>1</td><td>15</td></tr>
<tr><td>3rd</td><td>1.5</td><td>21</td></tr>
<tr><td>4th</td><td>2</td><td>24</td></tr>
<tr><td>5th</td><td>2.5</td><td>28</td></tr>
<tr><td>6th</td><td>3</td><td>36</td></tr>
</table>

What was Jamie’s average speed, in km/h, during the first half-hour? During the last half-hour? Justify why the speeds are different.</td>
</tr>
<tr>
<td>9-10.AR.F.6* Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function.
a. Use appropriate forms of linear, quadratic, and exponential functions to show zeros, extreme values, and symmetry (where applicable) and interpret them in context.
b. Use the properties of an exponential function to classify it as growth or decay.</td>
<td>Example:
Identify the percent rate of change in functions such as $y=(1.02)^t$, $y=(0.97)^t$, $y=(1.01)^{12t}$, $y=(1.2)^{\frac{t}{10}}$ and classify them as representing exponential growth or decay.

Example:
Given a quadratic function, explain the meaning of the zeros of the function. That is if
$f(x) = x^2 - 7x + 12 = (x - 4)(x - 3)$, then
$f(4) = 0$ and $f(3) = 0$.

Example:
A toy rocket is launched at 128 ft/sec from a height of 5 feet. What is the maximum height of the rocket, and when does the rocket reach that height?

Example: $8^t = 2^{3t}$

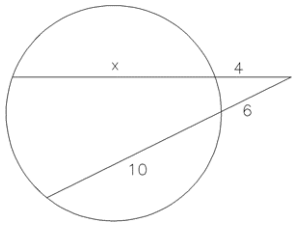
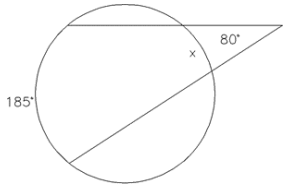
Example: The expression 1.15^t can be rewritten as $\left(1.15^{\frac{1}{12}}\right)^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</td>
</tr>
<tr>
<td>9-10.AR.F.7* Compare key features of two linear, exponential, or quadratic functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</td>
<td>Example:
Given a graph of one quadratic function and an algebraic representation for another function, say which has the larger maximum.

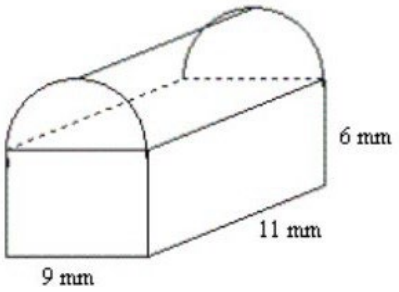
Example:
Compare the intercepts of two functions, one represented graphically, and the other is represented symbolically.</td>
</tr>
</table>

9-10.AR.F.8* Identify situations that can be modeled with linear, quadratic, and exponential functions. Justify the most appropriate model for a situation based on the rate of change over equal intervals. Include situations in which a quantity grows or decays.	Example: A person earning $10 per hour experiences a constant rate of change in salary given the number of hours worked. Example: The number of bacteria on a dish doubles every hour and will have equal factors over equal intervals.
9-10.AR.F.9* Identify the effect of transformations on the graph of a linear, absolute value, or quadratic function by replacing f(x) with af(x), f(x - h), and f(x) + k, for specific values of a, h, and k (both positive and negative). Find the value of a, h, and k given the graph of the function.	
9-10.AR.F.10* Find the inverse of a linear function and describe the relationship between the domain, range, and graph of the function and its inverse in context.	Example: An internet provider charges an initial equipment fee of $500 and an additional $100 per month for using their satellite internet service. The following function represents this linear relationship: C(x) = 100x + 500 where x is the number of months and C(x) is the combined cost. Find $C^{-1}(x)$ and describe this inverse relationship.
9-10.AR.F.11* Interpret the parameters in a linear, quadratic, or exponential function in context.	Parameters for a linear: values of m and b Parameters of a quadratic: values of a and c Parameters for an exponential: values of a and b Example: In the equation $y = mx + b$, m and b are parameters that specify the particular line represented by the equation. Example: A cell phone plan costs $40 a month plus 2 cents per minute of usage. Write a function that shows the monthly cost of using your cell phone and interpret the parameters. Answer: C = 0.02m + 40 The monthly cost is 40 dollars plus two cents times the number of minutes used. The minimum cost per month is $40 (at m = 0). Example: Interpret ½h $(b_1 + b_2)$ as the product of the height of a trapezoid and the average of its base lengths.
9-10.AR.F.12 Identify, using graphs or tables, the solution(s) to linear and exponential functions f(x) = g(x) as x-value(s) that result in equivalent y-values.	Example: Use a graphing calculator to find and justify the approximate solution(s) to the system below. $\begin{cases} f(x) = 2^x \\ g(x) = 4x - 4 \end{cases}$ Solution(s): x = 2 therefore f(2) = 4, g(2) = 4

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*	
Standards	**Clarification**
9-10.GM.1 Know precise definitions and notations of angle, circle, perpendicular line, parallel line, and line segment based on the undefined notions of point, line, and plane.	Example: An angle is composed of two rays that share a common initial point.
9-10.GM.2 Represent transformations in the plane. Describe transformations as functions taking points in the plane as inputs and giving other points as outputs. Compare transformations that preserve distance and angle to those that do not (i.e., rigid versus non-rigid motion).	
9-10.GM.3 Describe the rotations and reflections of a triangle, rectangle, parallelogram, trapezoid, or regular polygon that map each figure onto itself or another figure.	
9-10.GM.4 Develop or verify the characteristics of rotations, reflections, and translations in angles, circles, perpendicular lines, parallel lines, and line segments.	Example: Using patty paper or geometry software, develop/verify that the reflection line is the perpendicular bisector of the segment that connects the pre-image to its image.
9-10.GM.5 Draw the image of a figure that has undergone a series of transformations [rotation(s), reflection(s), or translation(s)] of a geometric figure using a variety of methods (e.g., graph paper, tracing paper, or geometry software).	Learners must be able to perform and draw a series of transformations as well as describe said transformations to successfully produce the resulting image.
9-10.GM.6 Predict the effect of a specified rigid motion on a given figure using geometric descriptions of rigid motions. Determine whether two figures are congruent using the definition of congruence in terms of rigid motions.	Learners must be able to predict and recognize rigid motions and use them to justify congruence.
9-10.GM.7 Use the definition of congruence, based on rigid motions, to show two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent.	
9-10.GM.8 Prove two triangles are congruent using the congruence theorems.	"Proof" may take on various forms (flow, paragraph, 2-column, informal).
9-10.GM.9 Prove and apply theorems about lines and angles.	"Proof" may take on a variety of forms (flow, paragraph, 2-column, informal). Theorems include but are not limited to vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

9-10.GM.10 Prove and apply theorems about triangles.	"Proof" may take on a variety of forms (flow, paragraph, 2-column, informal). Theorems include but are not limited to measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
9-10.GM.11 Prove and apply theorems about parallelograms.	"Proof" may take on a variety of forms (flow, paragraph, 2-column, informal). Theorems include but are not limited to opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and rectangles are parallelograms with congruent diagonals.
9-10.GM.12 Make basic geometric constructions (e.g., segment, angle, bisectors, parallel and perpendicular lines) with a variety of tools and methods.	Tools may include a compass and straightedge, string, reflective devices, paper folding, or dynamic geometric software.
(+) 9-10.GM.13 Apply basic constructions to create polygons such as equilateral triangles, squares, and regular hexagons inscribed in circles.	Learners can use technology or compass and straightedge to accomplish the construction.
9-10.GM.14 Verify experimentally and justify the properties of dilations given by a center and a scale factor.	
9-10.GM.15 Use transformations to decide if two given figures are similar. Apply the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	
9-10.GM.16 Prove similarity theorems about triangles.	"Proof" may take on a variety of forms (flow, paragraph, 2-column, informal).
9-10.GM.17 Apply knowledge of congruence and similarity criteria for triangles to solve problems and to prove relationships in various geometric figures.	
9-10.GM.18 Recognize how the properties of similar right triangles allow the trigonometric ratios to be defined and determine the sine, cosine, and tangent of an acute angle in a right triangle.	Example: Verify experimentally that the side ratios in similar right triangles depend upon the measure of an acute angle in the triangle due to the preservation of angle measure in similarity. Use this discovery to develop definitions of the trigonometric ratios for acute angles.
(+) 9-10.GM.19 Explain and use the relationship between the sine and cosine of complementary angles.	
9-10.GM.20* Solve applied problems involving right triangles using trigonometric ratios, the Pythagorean Theorem, and special right triangles (30°-60°-90° and 45°-45°-90°).	

(+) 9-10.GM.21* Solve unknown sides and angles of non-right triangles using the Laws of Sines and Cosines.	
9-10.GM.22 Apply theorems about relationships between line segments and circles or angles and circles formed by radii, diameter, secants, tangents, and chords to find unknown lengths or angles.	Example: solve for x: Example: Solve for x:   Solution: x = 20 Solution: $x = 25^{\circ}$
(+) 9-10.GM.23 Construct the incenter and circumcenter of a triangle. Relate the incenter and circumcenter to the inscribed and circumscribed circles.	Learners may use technology to perform the constructions.
(+) 9-10.GM.24 Construct a tangent line from a point outside a given circle to the circle.	
9-10.GM.25 Explain and use the formulas for arc length and area of sectors of circles.	
9-10.GM.26 Recognize that the radian measure of an angle is the ratio of the length of the arc to the length of the radius of a circle.	
9-10.GM.27 Develop and verify the slope criteria for parallel and perpendicular lines. Apply the slope criteria for parallel and perpendicular lines to solve problems.	Example: Find the equation of a line parallel or perpendicular to a given line that passes through a given point.
9-10.GM.28 Verify simple geometric theorems algebraically using coordinates. Verify algebraically, using coordinates, that a given set of points produces a particular type of triangle or quadrilateral.	This standard allows for coordinate proof. Example: Given a rhombus with vertices at (2, 0), (-2, 0), (0, 3) and (0, -3), verify that the diagonals are perpendicular. Example: Verify algebraically whether a figure defined by four given points in the coordinate plane is a rectangle.
9-10.GM.29 Determine the midpoint or endpoint of a line segment using coordinates. (+) Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	(+) Example: Find the coordinate pair that is 2/3 the distance from the point (2, 3) to (-4, 7).
9-10.GM.30* Compute perimeters of polygons and areas of triangles, parallelograms, trapezoids, and kites using coordinates.	

9-10.GM.31 Explain derivations of the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone.	May use dissection arguments, Cavalieri's Principle, or informal limit arguments. Example: The area of a circle can be reduced by rearranging the sectors of two semi-circles to form a rough rectangle. Area :  $=r\cdot\frac{1}{2}\cdot$Circumference $=r\cdot\frac{1}{2}\cdot 2\pi r$ $=\pi r^2$
9-10.GM.32 Calculate the surface area for prisms, cylinders, pyramids, cones, and spheres to solve problems.	
9-10.GM.33 Know and apply volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems.	Example: Find the volume of the composite figure below: 
9-10.GM.34 Identify the shapes of two-dimensional cross-sections of three-dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects.	
9-10.GM.35* Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	
9-10.GM.36* Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; scaling a model).	Example: Learners design a soft drink package that minimizes surface area and cost. Example: Design an art sculpture composed of at least 4 solids. Calculate the amount of material used to build it.

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic concepts of probability.*	
Standards	**Clarification**
9-10.DPS.1* Represent data with plots on the real number line (dot plots, histograms, and box plots).	
9-10.DPS.2* Compare the center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets using statistics appropriate to the shape of the data distribution.	Learners may use technology to find the standard deviation.
9-10.DPS.3* Represent data on two quantitative variables on a scatter plot and describe how the variables are related. a. Fit a linear function to the data (with or without technology) if appropriate. b. Compute (using technology) and interpret the correlation coefficient of a linear fit. c. Interpret the meaning of the slope and y-intercept of the linear model in context. d. Interpolate and extrapolate the linear model to predict values.	
9-10.DPS.4* Distinguish between correlation and causation.	Example: It is noted that there is a high correlation between people who eat ice cream daily and their annual job salary. Does eating ice cream predict salary or vice-versa?
9-10.DPS.5* Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes or as unions, intersections, or complements of other events (“or,” “and,” “not”).	Example: Given a classroom of 30 learners, list the subset of learners in the room who are blonde and have blue eyes.
9-10.DPS.6* Recognize that event A is independent of event B if the probability of event A does not change in response to the occurrence of event B. Apply the formula P(A and B) = P(A)·P(B) given that events A and B are independent.	Understand that two events, A and B, are independent if the probability of A and B occurring together is the product of their chances and use this characterization to determine if they are independent.
9-10.DPS.7* Recognize that the conditional probability of an event A given B is the probability that event A will occur given the knowledge that event B has already occurred. Calculate the conditional probability of A given B and interpret the answer in context.	Example: A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test? Solution: P(A and B) = P(passed both) = 0.25 P(A) = P(passed the first test) = 0.42 Find P(B) given that P(A) is true: = P(A and B)/P(A) = 0.25/0.42 = 0.6 = 60% Therefore, 60% of those students who passed the first test also passed the second test.

9-10.DPS.8* Apply the formula P(A or B) = P(A) + P(B) – P(A and B) and interpret the answer in context.	
9-10.DPS.9* Determine the number of outcomes using permutations and combinations in context.	
9-10.DPS.10* Construct and interpret two-way frequency tables of data for two categorical variables. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.	Example: Collect data from a random sample of learners in your school on their favorite subject among mathematics, science, and English. Estimate the probability that a randomly selected student from your school will favor science, given that the student is in 10^{th} grade. Do the same for other subjects and compare the results. Example: Compare the chance of having lung cancer if you are a smoker with the possibility of being a smoker if you have lung cancer.

ELEVENTH AND TWELFTH GRADES

Math Attributes (MA) *Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement through participation in various relevant learning experiences.*		
Problem-Solving (P)	**Connections (C)**	**Reasoning and Proof (R)**
Analyze, execute, evaluate, and adapt approaches and solutions when problem-solving in novel situations.	*Create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.*	*Reason logically, citing relevant evidence to explain and critique what they see, think, and conclude through exploration, generalization and validation.*
9-12.MA.P Learners can analyze, execute, critique, and adapt approaches and solutions when problem-solving in novel situations.	**9-12.MA.C** Learners can create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.	**9-12.MA.R** Learners can reason logically, citing evidence to critique and explain what they see, think, and conclude through exploration, generalization, and validation.

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Number and Operations (NO) *Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.*	
Standards	**Clarification**
11-12.NO.1.Rewrite complex expressions involving radicals and rational exponents using the properties of exponents.	This standard is an extension of 9-10.NO.1. Example: Simplify: $\frac{\sqrt[3]{4xy^2}}{\sqrt[3]{32x^4y^4}}$ $=\sqrt[3]{\frac{1}{8x^3y^2}}$ $=\frac{1}{2x^{\frac{3}{3}}y^{\frac{2}{3}}}=\frac{1}{2xy^{\frac{2}{3}}}\cdot\frac{y^{\frac{1}{3}}}{y^{\frac{1}{3}}}$ $=\frac{\sqrt[3]{y}}{2xy}$

11-12.NO.2 Perform operations on complex radical expressions and simplify radicals to write equivalent expressions.	This standard is an extension of 9-10.NO.2. Operations include addition, subtraction, multiplication, and division (e.g., rationalizing the denominator and performing conjugation). Example: $\sqrt{63x^4}+2\sqrt{28x^3}-5x^2\sqrt{7}$ Solution: $=\sqrt{9}\sqrt{7}\sqrt{x^4}+2\sqrt{28x^3}-5x^2\sqrt{7}$ $=3x^2\sqrt{7}+4x\sqrt{7x}-5x^2\sqrt{7}$ $=-2x^2\sqrt{7}+4x\sqrt{7x}$
11-12.NO.3 Demonstrate that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.	Example: Evaluate $\sqrt{2}\cdot\sqrt{4}$ and identify which subset of the real number system the solution is in. Solution: $\sqrt{8} = 2\sqrt{2}$, which is irrational.
11-12.NO.4* Use units to understand problems and to guide the solution of multi-step problems (e.g., unit analysis). Choose and interpret units consistently in formulas. Choose and interpret the scale and the units in graphs and data displays.	This standard is an extension of 9-10.NO.3 and 9-10.NO.4. Example: Blood sugar level is measured in milligrams of glucose per deciliter of blood volume. If a person's blood sugar level measures 128 mg/dL, what is this in grams per liter?
11-12.NO.5* Choose a level of accuracy or precision appropriate to limitations on measurement when reporting quantities.	This standard applies to all high school mathematics. Example: When using a ruler, learners choose to report their measurements based on the precision of the ruler (e.g., to the nearest $^1/_{16}$ or the nearest $^1/_{32}$). Example: If you are playing soccer and you always hit the left goal post instead of scoring, then you are not accurate; you are precise. Example: When using a ruler, learners are able to measure accurately. Example: When calculating the cost of a road trip, learners are given the cost of gasoline to the thousandth place. When reporting the cost of the trip, learners determine what level of precision (to the hundredths place or to the thousandth place) is appropriate and why.

11-12.NO.6 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. Understand the hierarchal relationships among subsets of the complex number system.	Knowledge of complex numbers extends and reinforces student knowledge of the real number system. Example: $\sqrt{8}$ is a complex number because it can be written in the form $\sqrt{8}$ + 0i. $\sqrt{8}$ is also a real number since its imaginary coefficient is 0. $\sqrt{8}$ is also an irrational number because it cannot be written as a ratio of two integers.
11-12.NO.7 Use the definition $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	Knowledge of complex numbers extends and reinforces student knowledge of basic operations and properties of the real number system. Example: $(2 + 3i) + (4 - 5i) = 6 - 2i$ $(2+ 3i) - (4 - 5i) = -2 + 8i$ $(2 + 3i)(4 - 5i) = 8 - 10i + 12i - 15i^2$ $= 8 + 2i + 15$ $= 23 + 2i$
11-12.NO.8 Use conjugates to find quotients of complex numbers.	
11-12.NO.9 Apply the Fundamental Theorem of Algebra to determine the number of zeros for polynomial functions. Find all solutions to a polynomial equation.	This standard applies to multiple high school mathematics levels.
(+) 11-12.NO.10 Represent complex numbers on the complex plane in rectangular, trigonometric, and polar forms. Find the modulus (absolute value) of a complex number. Explain why the rectangular, trigonometric, and polar forms of a given complex number represent the same number.	
(+) 11-12.NO.11 Represent addition, subtraction, multiplication, conjugation, powers, and roots of complex numbers geometrically on the complex and/or polar plane; use properties of this representation for computation.	Example: $(1 - i\sqrt{3})^3 = -8$ because $(1 - i\sqrt{3})$ written in polar form is $[2, 300^\circ]$. Applying de Moivre's Theorem yields $[2, 300^\circ]^3 = [2^3, 300^\circ \cdot 3] = [8, 180^\circ]$ $a = 8\cos 180^\circ = -8$ $b = 8\sin 180^\circ = 0$ $a + bi = -8 + 0i$

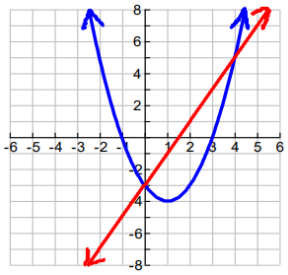
(+) 11-12.NO.12 Extend polynomial identities to the complex numbers.	Example: Rewrite x^2+4 as $(x+2i)(x-2i)$. Polynomial identities include but are not limited to: $(a+b)^2 = a^2 + 2ab + b^2$ $(a+b)(c+d) = ac + ad + bc + bd$ $a^2 - b^2 = (a+b)(a-b)$ $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ $x^2 + (a+b)x + ab = (x+a)(x+b)$
(+) 11-12.NO.13 Apply the Fundamental Theorem of Algebra to find all roots of a polynomial equation and determine the nature (i.e., integer, rational, irrational, real, complex) of the roots.	This standard applies to multiple high school mathematics levels.
(+) 11-12.NO.14 Recognize vector quantities as having both magnitude and direction, writing them in polar form.	
(+) 11-12.NO.15 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	
(+) 11-12.NO.16 Solve problems involving magnitude and direction that can be represented by vectors.	
(+) 11-12.NO.17 Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Know that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand that vector subtraction **v**-**w** is defined as **v**+(-**w**), where -**w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and using the components to perform vector subtraction.	
(+) 11-12.NO.18 Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction. Use the components to perform scalar multiplication (e.g., as $c(v_x, v_y) = (cv_x, cv_y)$). b. Compute the magnitude of a scalar multiple c**v** using $\|\|c\mathbf{v}\|\| = \|c\|\mathbf{v}$. c. Compute the direction of cv knowing that when $\|c\|\mathbf{v} \neq 0$, the direction of c**v** is either along **v** (for $c > 0$) or against **v** (for $c < 0$).	

(+) 11-12.NO.19 Represent data in a matrix. Perform operations (i.e., addition, subtraction, multiplication) on matrices of appropriate dimensions to solve problems and in context. Know that matrix multiplication is not commutative.	

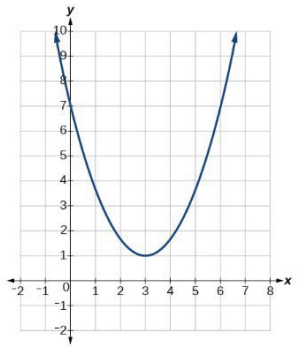
Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*	
Standards	**Clarification**
11-12.AR.1* Rearrange multi-variable formulas to highlight a quantity of interest.	In grades 11-12, the linear, exponential, or quadratic types of problems should draw from more complex situations than those addressed in grades 9-10. This standard is an extension of 9-10.AR.2. Example: Solve $A=P(1 + r)^t$ for r.
11-12.AR.2 Use the structure of an expression (to extend to polynomial and rational expressions) to identify ways to rewrite it.	Learners in grades 11-12 extend their focus to polynomial and rational expressions. This standard is an extension of 9-10.AR.1 Example: See $9a^2 - 4b^2$ as $(3a)^2 - (2b)^2$ and recognize it as a difference of squares that can be factored as $(3a - 2b)(3a + 2b)$. Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$, and further to $(x - y)(x + y)(x^2 + y^2)$.
11-12.AR.3* Interpret expressions that represent a quantity in context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	Learners in grades 11-12 extend their focus to polynomial and rational expressions. Example: Interpret $\frac{1}{2}h(b_1+b_2)$ as the product of the height of a trapezoid and the average of its base lengths.
11-12.AR.4* Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Use the properties of exponents to transform exponential expressions. c. Complete the square in a quadratic expression to produce an equivalent expression.	This standard is an extension of 9-10.AR.4. Example: Given a quadratic function explain the meaning of the zeros of the function. That is if $f(x) = 6x^2 - 11x - 10$, then f(x) can be factored into $f(x) = (2x - 5)(3x + 2)$. Therefore, $f\left(\frac{5}{2}\right)=0$ and $f\left(-\frac{2}{3}\right)=0$. Example: Find the center and radius of the circle whose equation is $x^2 + y^2 - 6x + 2y - 6=0$.

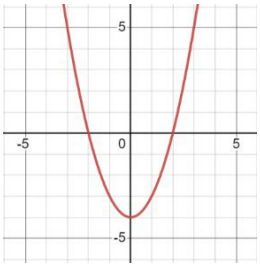
11-12.AR.5 Add, subtract, multiply, and divide rational expressions. Understand that rational expressions form a system analogous to rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.	Learners in grades 11-12 extend their understanding beyond the quadratic expressions.
11-12.AR.6 Rewrite simple rational expressions in different forms. Write a(x)/b(x) in the form of q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or technology for the more complicated examples.	Example: Use long division to rewrite: $\frac{3x^3 - 2x^2 + 4x - 3}{x^2 + 3x + 3}$ in the form: $(3x - 11) + \frac{28x + 30}{x^2 + 3x + 3}$
11-12.AR.7* Create equations and inequalities and use them to solve problems. Include equations arising from linear and quadratic equations and simple rational and exponential equations.	Learners in grades 11-12 use all available types of functions to create such equations, including root functions, but constrain to simple cases. This standard is an extension of 9-10.AR.3.
11-12.AR.8* Create equations in two or more variables to represent relationships between quantities. Graph equations on coordinate axes with proper labels and scales.	In grades 11-12, the linear, exponential, or quadratic types of problems should draw from more complex situations than those addressed in grades 9-10. This standard is an extension of 9-10.AR.4. Example: Every time Pinocchio lies, his nose grows about 20% of its size. Originally his nose is 2 inches long. Write an equation that models the situation and graph the model. How many lies would he have to tell before his nose is longer than 3 feet?
11-12.AR.9* Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context.	In grades 11-12, the linear, exponential, or quadratic types of problems should draw from more complex situations than those addressed in grades 9-10. This is an extension of 9-10.AR.8 Example: Willy Wonka's Chocolate Factory makes *Wonka Bars* and *The Everlasting Gobstopper*, among other amazing treats. Oompa Loompas and Fuzzy Fizzies work on each item. The Oompa Loompas spend 6 minutes making a *Wonka Bar* and 4 minutes mixing the ingredients for an *Everlasting Gobstopper.* There are enough Oompa Loompas for up to 6000 worker minutes per day. The Fuzzy Fizzies spend about 1 minute wrapping each *Wonka Bar* and 2 minutes wrapping each *Everlasting Gobstopper*. There are enough Fuzzy Fizzies for a maximum of 1200 worker minutes per day. Given the above constraints, find the feasible region for the number of *Wonka Bars* and *Everlasting Gobstoppers* that can be made per day.

	Solution: Oompa Loompas: (6 min/bar)(x bars) + (4 min/y gob) ≤ 6000 min. Fuzzy Fizzies: (1 min/bar)(x bars) + (2 min/gob)(y gob) ≤ 1200 min. using substitution, y = 150, x = 900 if the maximum number of hours are worked. Therefore, the feasible region for the number of *Wonka Bars* made in a day is $0 \leq x \leq 900$, and the feasible region for the number of *Everlasting Gobstoppers* is $0 \leq y \leq 150$.
(+) 11-12.AR.10 Derive the quadratic formula from the form $0 = ax^2 + bx + c$.	
11-12.AR.11 Solve quadratic equations with real coefficients that have solutions of the form a + bi and a - bi.	This standard is an extension of 9-10.AR.10. Example: $x^2 + 2x + 5 = 0$ $x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(5)}}{2(1)}$ $x = \frac{-2 \pm \sqrt{-16}}{2}$ $x = \frac{-2 \pm 4i}{2}$ $x = -1 \pm 2i$
11-12.AR.12 Solve simple rational and radical equations in one variable and identify extraneous solutions.	Example: Solve: $\sqrt{6-x} = x$ $\left(\sqrt{6-x}\right)^2 = x^2$ $6 - x = x^2$ $x^2 + x - 6 = 0$ $(x+3)(x-2) = 0$ $x = 2$ or $x = -3$ Check 2 and -3 in the original equation: $\sqrt{6-2} = \sqrt{4} = 2$ $\sqrt{6-(-3)} = \sqrt{9} = 3$ Because -3 does not satisfy the original equation, -3 is an extraneous solution. 2 is the only solution to the equation.
11-12.AR.13 Add, subtract, and multiply polynomials beyond quadratics. Understand that polynomials form a system comparable to integers, namely, they are closed under the operations of addition, subtraction, and multiplication.	Learners in grades 11-12 extend their understanding beyond quadratic polynomials. This standard is an extension of 9-10.AR.11.
11-12.AR.14 Identify zeros of polynomial equations when suitable factorizations are available. Use the zeros to construct a rough graph of the function defined by the polynomial.	

11-12.AR.15 Apply the Factor and Remainder Theorems to determine efficiently whether a linear expression is a factor of a polynomial equation. Apply the Remainder Theorem in context.	Pre-requisite knowledge for this standard includes an understanding of polynomial division and factoring. Example: The total production of eggs in billions in the United States can be modeled by the function $f(x) = 0.007x^3 - 0.149x^2 + 1.534x + 84.755$, where x is the number of years since 2000. Predict the total production of eggs in 2025. Solution: f(25) = 139.355 billion eggs
11-12.AR.16 Using graphs, technology, tables, or successive approximations, show that the solution(s) to the equation f(x) = g(x) are the x-value(s) that result in the y-values of f(x) and g(x) being the same.	Grades 11-12 will include combinations of linear, polynomial, rational, radical, absolute value, exponential and logarithmic functions. This standard is an extension of 9-10.AR.F.12. Example: Use a graphing calculator to find and justify the approximate solution(s) to the system below. $\begin{cases} f(x) = x + 4 \\ g(x) = 4 - x^2 \end{cases}$ Solutions: x = 0 and x = -1. f(0) = 4, g (0) = 4; f(-1) = 3, g(-1) = 3
11-12.AR.17 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	This standard is an extension of 9-10.AR.8 Example: Solve: $\begin{cases} y = x^2 - 2x - 3 \\ y = 2x - 3 \end{cases}$ Solutions: (0,-3), (4,5) 
(+) 11-12.AR.18 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).	Example: Solve using technology: $\begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & -1 \\ 5 & 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
(+) 11-12.AR.19 Solve a system of equations in three or more variables with matrices (using technology).	
(+) 11-12.AR.20 Apply the Binomial Theorem for the expansion of $(ax + by)^n$ in powers of x and y for a positive integer n and integers a and b.	

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

Standards	Clarification
11-12.AR.F.1* Write a function that describes a relationship between two quantities. a. Combine standard function types using arithmetic operations. b. Compose functions.	Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential and relate these functions to the model. Example: If T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
11-12.AR.F.2* Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	This standard is an extension of 9-10.AR.F.5 Example: Find the rate of change over the interval $0 \le x \le 3$ given the graph below.  Solution: The average rate of change of a function $y = f(x)$ over an interval $a \le x \le b$ is $\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$. Therefore, the estimated average rate of change for the function graphed above is $\frac{7 - 1}{0 - 3} = -2$.
11-12.AR.F.3* Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum, and symmetry of the graph, and interpret these in terms of context. b. Use the properties of exponents to interpret expressions for exponential functions.	This standard is an extension of 9-10.AR.F.6. Example: Identify the percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

11-12.AR.F.4* Identify the effect of transformations on the graph of a function by replacing f(x) with af(x), f(bx), f(x - h), and f(x) + k, for specific values of a, h, and k (both positive and negative). Find the value of a, b, h, and k given the graph of the function. Recognize even and odd functions from their graphs and equations.	Technology may be used to experiment with the effects of transformations on a graph. This standard is an extension of 9-10.AR.F.9. Learners in grades 11-12 will use transformations of functions to find models as learners consider increasingly more complex situations; note the effect of multiple transformations on a single graph and the common effect of each transformation across function types.
11-12.AR.F.5* Find inverse functions. a. Verify by composition that one function is the inverse of another. b. Recognize that the graph of a function and its inverse are reflection images over the line y = x. c. Produce an invertible function from a non-invertible function by restricting the domain.	Learners in grades 11-12 will extend to simple rational, simple radical, and simple exponential functions. This standard is an extension of 9-10.AR.F.10. Example: Find the inverse for each function: $f(x) = 2x^3$ or $f(x) = \frac{x+1}{x-1}$ for $x \neq 1$.
11-12.AR.F.6* Apply the inverse relationship between exponents and logarithms to solve problems.	
11-12.AR.F.7* Compare key features of two functions each represented in a different way (algebraically, graphically, numerically, in tables, or by verbal descriptions).	Grades 11-12 focus on using key features to guide the selection of the proper type of model function. This standard is an extension of 9-10.AR.F.7. Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. Example: Compare the intercepts of two functions, one represented graphically and the other symbolically.
11-12.AR.F.8* Use tables, graphs, verbal descriptions, and equations to interpret and sketch the key features of a function modeling the relationship between two quantities.	Key features include domain, range, intercepts; intervals where the function is increasing, decreasing, relative maximums and minimums; symmetries; end behavior; and periodicity. This standard is an extension of 9-10.AR.F.3. Example: Given $f(x)=x^2- 4$. Graph the function and identify the intercepts, intervals, where the function is increasing, decreasing, positive, negative, the relative maximum and minimum, and any symmetry. Solution: Intercepts: (-2, 0), (2, 0), (0,-4) Relative Minimum: (0, -4) Increasing: x > 0 Decreasing: x < 0 Positive: x < -2, x > 2 Negative: -2 < x < 2 Symmetric to the y-axis 

11-12.AR.F.9* Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	Grades 11-12 emphasize the selection of a model function based on behavior of data and content. This standard is an extension of 9-10.AR.F.4. Example: If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
11-12.AR.F.10* Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph square root, cube root, piecewise-defined, step, and absolute value functions. b. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. c. Graph exponential and logarithmic functions, showing intercepts and end behavior. d. Graph f(x) = sin x and f(x) = cos x as representations of periodic phenomena.	This standard addresses portions of 9-10.AR.F.3. Example: Solve the annual compound interest formula $A = P(1 + r)^t$ for t and draw a graph of time vs. amount for a given rate and principal amount, showing intercepts and end behavior. Compare this graph to the graph of amount vs. time.
(+) 11-12.AR.F.11* Analyze and graph functions expressed symbolically (by hand in simple cases and using technology for more complicated cases), identifying key features of the graph. a. (+) Graph rational functions, identifying domain, range, asymptote(s), removable and non-removable discontinuities, intercepts, behavior at the asymptote(s), and end behavior. b. (+) Graph trigonometric functions, showing period, midline, phase shift, and amplitude.	This standard is an extension of 9-10.AR.F.3.
11-12.AR.F.12* Compare the end behavior of linear, quadratic, and exponential functions using graphs and/or tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing as a linear or quadratic function.	
11-12.AR.F.13* Determine whether a linear, quadratic, polynomial, exponential, logarithmic, or trigonometric model fits a situation. Determine an appropriate mathematical model in context (with or without technology).	This standard is an extension of 9-10.AR.F.7.

11-12.AR.F.14* Write arithmetic and geometric sequences both recursively and with an explicit formula and convert between the two forms. Use sequences to model situations.	Example: Allen is training for a biking race and begins his workout regimen by biking 10 miles on day one and increasing his mileage by 2 miles per day for the next 15 days. Express the situation with a recursive and explicit formula. Solution: Explicit Formula: $a_n = 2n + 8$ Recursive Formula: $\begin{cases} a_1 = 10 \\ a_{n-1} + 2,\ n>1,\ n \in Z \end{cases}$
11-12.AR.F.15* Use properties of logarithms to express the solution to $ab^{ct} = d$ where a, c, and d are real numbers and b is a positive real number. Evaluate the logarithm using technology when appropriate.	Example: $3e^{2t} = 317$ $\ln e^{2t} = \ln\left(\frac{317}{3}\right)$ $2t = \ln\left(\frac{317}{3}\right)$ $t \approx 2.330$ Using a calculator and rounding t to the nearest thousandth: $t \approx 2.330$
11-12.AR.F.16 Extend right triangle trigonometry and apply knowledge of the unit circle to determine values of sine, cosine, and tangent for multiples of π/3, π/4, and π/6.	This standard is an extension of 9-10.GM.18 and 9-10.GM.20.
11-12.AR.F.17 Use the Pythagorean Identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to find sin (θ), cos (θ), or tan (θ) given sin (θ), cos (θ), or tan (θ) and the quadrant of the angle.	Example: Given θ is a Quadrant II angle and sin θ = 4/5, find cos θ using the Pythagorean Identity.
(+) 11-12.AR.F.18 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	Example: Find $\sin\frac{7\pi}{6}$
(+) 11-12.AR.F.19 Use the unit circle to express the values of sine, cosine, and tangent for π - x, π + x, and 2π - x in terms of their values for x, where x is any real number.	
(+) 11-12.AR.F.20 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	
(+) 11-12. AR.F.21 Create a trigonometric function to model periodic phenomena.	
(+) 11-12. AR.F.22 Restrict the domain of a trigonometric function to construct its inverse.	

(+) 11-12. AR.F.23* Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions and interpret them in context.	Example: A surveyor marks off points D, E, and F and records that the measure of angle D is 40.2 degrees, d = 100 m, and f = 500 m. Explain why there is a problem with the surveyor's measurements. Solution: Using the Law of Sines, $\sin \angle F \approx 3.227$, which is not possible. The surveyor has a measurement error. The surveyor has a measurement error.
(+) 11-12. AR.F.24 Know and apply the addition and subtraction formulas for sine, cosine, and tangent to solve problems.	

Geometry and Measurement (GM)

Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

Standards	Clarification
11-12.GM.1 Write the equation of a conic section given its special features. Convert between the standard form and general form equations of conic sections.	Conic sections include the circle, ellipse, parabola, and hyperbola. Key features include: • Circle – center, radius • Parabola – vertex, focus, directrix • Ellipse – center, foci, vertices, length of the major and minor axis • Hyperbola – center, foci, asymptotes
11-12.GM.2* Identify key features of a conic section given its equation. Apply properties of conic sections in context.	Identify key features of a conic section given its equation. Apply properties of conic sections in context.
11-12.GM.3 Determine and apply appropriate formulas to solve right and non-right triangle problems in context.	This standard is an extension of 9-10.GM.20.
(+) 11-12.GM.4 Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	

Data, Probability, and Statistics (DPS)

Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic concepts of probability.

Standards	Clarification
11-12.DPS.1* Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	

11-12.DPS.2* Use the mean and standard deviation of a data set to fit it to a normal distribution and estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate.	This standard is an extension of 9-10.DPS.2. Example: An example of a data set that does not fit to a normal distribution is the age at retirement. Most people retire in their mid-60s or older, with increasingly fewer retiring at increasingly earlier ages. This results in a skewed-left distribution.
11-12.DPS.3* Evaluate reports based on data. a. Identify and explain misleading use of data, recognize when claims based on data confuse correlation and causation. b. Recognize and describe how graphs and data can be distorted to support different points of view.	
11-12.DPS.4* Represent data on a scatter plot for two quantitative variables and describe how the variables are related. a. Fit a function to the data (with or without technology) and interpret the special features (e.g., meaning of a and b in the exponential function $y = ab^x$) of the function in context. b. Use functions fitted to data to solve problems in the context of the data.	Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. This standard is an extension of 9-10.DPS.3.
(+) 11-12.DPS.5* Informally assess the fit of a function by plotting and analyzing residuals.	Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.
(+) 11-12.DPS.6* Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	
(+) 11-12.DPS.7* Understand the process of making inferences about population parameters based on a random sample from that population.	Example: Suppose 50 fish are tagged in a pond. A fisherman catches 5 fish from the pond, and one has a tag. What conclusion can you draw about the fish population?
(+) 11-12.DPS.8* Decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation).	Example: A model says a spinning coin falls heads-up with a probability of 0.5. Would a result of 5 tails in a row cause you to question the model?
(+) 11-12.DPS.9* Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	Example: Design a simple study and explain the impact of sampling methods, bias ,and the phrasing of questions asked during data collection.
11-12.DPS.10* Determine when the order in counting matters and use permutations and combinations to compute probabilities of events accordingly. Determine probability situations as conditional, “or” (union), or “and” (intersection), and determine the probability of an event.	This standard is an extension of 9-10.DPS.7, 9-10.DPS.8, 9-10.DPS.9, and 9-10.DPS.10.

(+) 11-12.DPS.11* Use permutations and combinations to compute probabilities of compound events and solve problems.	Example: Given a football team of 60 athletes, what is the probability that the star quarterback and star linebacker are not chosen for drug testing? $\frac{{}_{58}C_2}{{}_{60}C_2} \approx 0.934 \approx 93.4\%$
(+) 11-12.DPS.12* Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space. Graph the corresponding probability distribution using the same graphical displays as for data distributions.	
(+) 11-12.DPS.13* Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	
(+) 11-12.DPS.14* Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. Evaluate and compare strategies on the basis of expected values.	Example: Find the expected winnings from a state lottery ticket or a game at a fast-food restaurant. Example: Compare a high-deductible versus a low-deductible automobile insurance policy using various but reasonable chances of having a minor or a major accident.
(+) 11-12.DPS.15* Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities are calculated; find the expected value.	Example: Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices; find the expected value.
(+) 11-12.DPS.16* Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.	Example: Find a current data distribution on the number of TV sets per household in the United States and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?

(+) 11-12.DPS.17* Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	Example: Tara and Brent decide to roll a pair of 6-sided dice to determine who has to clean out the garage. • If the sum is 7, then Tara has to clean out the garage. • If the sum is 3 or 4, then Brent has to clean out the garage. • If the sum is anything else, they roll again. Is this a fair way to decide who has to clean out the garage? Why or why not? Solution: P(sum of 7) = 6/36 P(sum of 3 or 4) = P(3) + P(4) - P(3 and 4) = 2/36 + 3/36 - 0/36 = 5/36 P(anything else) = 25/36 This is not a fair way to decide because Tara has a better chance of having to clean out the garage.
(+) 11-12.DPS.18* Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	

NORTH DAKOTA
MATHEMATICS K-12
STANDARDS PROGRESSIONS

Number and Operations (NO) *Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.*
Counting Forward
K.NO.CC.1 Count verbally in sequential order by ones and tens to 100, making accurate decuple transitions (ex.89 to 90). Count verbally forward from any given number within 100.
1.NO.CC.1 Count forward by ones and tens from any given point within 120.
2.NO.CC.1 Count forward from any given number within 1000.
Counting Backward
K.NO.CC.2 Count backward from 20 by ones and from a given number within 10.
1.NO.CC.2 Count backward by ones and tens from any given number within 120.
2.NO.CC.2 Count backward from any given number within 1000.
Number Identification and Writing
K.NO.CC.3 Identify and write any given numeral within 20.
1.NO.CC.3 Represent several objects with a written numeral up to 120.
2.NO.CC.3 Read and write numbers up to 1,000 using standard, word, and expanded forms.
3.NO.CC.1 Read and write numbers up to 10,000 using objects or visual representations including standard and expanded forms.
4.NO.CC.1 Read numbers to the millions place including word, standard and expanded form. Write numbers to the millions place including standard and expanded form.
5.NO.CC.1 Read, write, and compare decimals to the thousandths including standard and expanded forms.
Subitizing
K.NO.CC.4 Recognize and verbally label arrangements, without counting, for briefly shown collections up to 10 (e.g., "I saw 5." How do you know?" "I saw 3 and 2, that is 5.").
1.NO.CC.4 Recognize and verbally label arrangements, without counting, for briefly shown collections up to 20 (e.g., "I saw 16." "How did you know?" "I saw 10 and 6, that is 16.").
Counting Patterns
K.NO.CC.5 Count and tell how many objects up to 20 are in an arranged pattern or up to 10 objects in a scattered configuration. Represent a quantity of up to 20 with a numeral.
1.NO.CC.5 Skip count forward and backward by 5s and 10s from multiples and recognize the patterns of up to 10 skip counts.
2.NO.CC.4 Skip count forward and backward by 2s and 100s and recognize the patterns of skip counts.
9-10.DPS.9* Determine the number of outcomes using permutations and combinations in context.
(+) 11-12.DPS.11* Use permutations and combinations to compute probabilities of compound events and solve problems.
Place Value
K.NO.NBT.1 Compose and decompose numbers from 11 to 19 using a group of ten ones and some more ones using a model, drawing, or equation.
1.NO.NBT.1 Demonstrate that the two digits of a two-digit number represent a composition of some tens and some ones.
2.NO.NBT.1 Understand that the three digits of a three-digit number represent a composition of some hundreds, some tens, and some ones.
4.NO.NBT.1 Understand that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.
5.NO.NBT.1 Understand that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
8.NO.NS.3 Use scientific notation to represent very large or very small quantities. Interpret scientific notation generated by technology. Compare and order numbers in scientific and standard notation.

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Compare Numbers and/or Expressions
K.NO.NBT.2 Compare two numbers between 1 and 20 using words greater than, less than, or equal to.
1.NO.NBT.2 Compare two two-digit numbers using symbols >, <, and =. Justify comparisons based on the number of tens and ones.
2.NO.NBT.2 Compare two three-digit numbers using symbols >, <, and =. Justify comparisons based on the value of thousands, hundreds, tens, and ones.
3.NO.NBT.1 Compare two four-digit numbers using symbols, >, <, and =. Justify comparisons based on the value of thousands, hundreds, tens, and ones.
4.NO.NBT.2 Compare two numbers to the millions place and decimals to the hundredths place, using symbols >, <, and =. Justify comparisons based on the value of the digits.
5.NO.NBT.2 Compare two decimals to thousandths using symbols >, <, and =. Justify comparisons based on the value of the digits.
6.NO.NS.1 Explain and show the relationship between non-zero rational numbers and their opposites using horizontal and vertical number lines in authentic problems. Use rational numbers to represent quantities in authentic contexts and explain the meaning of 0 in certain situations.
6.NO.NS.2 Write, interpret, and explain statements of order for rational numbers on a number line and in authentic contexts.
7.NO.NS.1 Describe the absolute value of a number as its distance from zero on a number line.
8.NO.NS.1 Compare and classify real numbers within the real number system.
8.NO.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them on a number line diagram, and estimate the value of irrational expressions involving one operation.
8.NO.NS.3 Use scientific notation to represent very large or very small quantities. Interpret scientific notation generated by technology. Compare and order numbers in scientific and standard notation.
9-10.AR.6 Solve linear equations and inequalities (to include compound inequalities) in one variable.
9-10.AR.8 Graph the solution set to a two-variable system of linear equations. Create and graph the solution set to a two-variable system of linear inequalities in context.
9-10.AR.9 Solve absolute value equations and inequalities in one or two variables.
11-12.AR.9* Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context.
Rounding Numbers
3.NO.NBT.2 Apply place value understanding to round whole numbers to the nearest 10 or 100.
4.NO.NBT.3 Apply place value and/or understanding of numbers to round multi-digit whole numbers to any place.
5.NO.NBT.3 Apply place value understanding to round decimals to any place.
8.NO.NS.3 Use scientific notation to represent very large or very small quantities. Interpret scientific notation generated by technology. Compare and order numbers in scientific and standard notation.
Addition and Subtraction
1.NO.NBT.3 Add within 100 using a two-digit number and a one-digit number. Use concrete models, drawings, and strategies that reflect an understanding of place value.
2.NO.NBT.3 Add within 100 using place value strategies and/or the relationship between addition and subtraction.
1.NO.NBT.4 Subtract multiples of 10 within 100 using concrete models, drawings, and strategies that reflect an understanding of place value.
1.NO.NBT.5 Mentally add or subtract 10 to or from a given two-digit number and explain the reasoning used.
2.NO.NBT.4 Subtract within 100 using place value strategies and/or the relationship between addition and subtraction.
2.NO.NBT.5 Mentally add or subtract 10 or 100 to or from a given number between 100 and 900.
3.NO.NBT.3 Add and subtract within 1000 using place value strategies, algorithms, and/or the relationship between addition and subtraction.
4.NO.NBT.4 Add and subtract multi-digit whole numbers to the one million place using strategies flexibly, including the algorithm.

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Addition and Subtraction
5.NO.NBT.5 Use concrete models, drawings, place value strategies, properties of operations, and/or relationships to add, subtract, and multiply decimals to hundredths.
7.NO.O.1 Add, subtract, multiply, and divide integers using visual models and properties of operations in multi-step authentic and mathematical problems, including authentic problems.
7.NO.O.2 Add, subtract, multiply, and divide non-negative fractions in multi-step problems, including authentic problems.
7.NO.O.3 Add, subtract, multiply, and divide non-negative decimals to the hundredth place in multi-step problems using strategies or procedures, including authentic problems.
8.NO.O.2 Add, subtract, multiply, and divide rational numbers using strategies or procedures.
9-10.NO.2 Perform basic operations on simple radical expressions to write a simplified equivalent expression.
9-10.AR.11 Add, subtract, and multiply polynomials.
11-12.NO.3 Demonstrate that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.
11-12.AR.13 Add, subtract, and multiply polynomials beyond quadratics. Understand that polynomials form a system comparable to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.
(+) 11-12.NO.11 Represent addition, subtraction, multiplication, conjugation, powers, and roots of complex numbers geometrically on the complex and/or polar plane; use properties of this representation for computation.
(+) 11-12.NO.17 Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Know that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand that vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ is defined as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and using the components to perform vector subtraction. Represent vector subtraction graphically by connecting the tips of the appropriate order and using the components to perform vector subtraction.
(+) 11-12.NO.19 Represent data in a matrix. Perform operations (i.e., addition, subtraction, multiplication) on matrices of appropriate dimensions to solve problems and in context. Know that matrix multiplication is not commutative.
Multiplication and Division
3.NO.NBT.4 Multiply one-digit whole numbers by multiples of 10 within 100.
4.NO.NBT.5 Multiply a whole number up to four digits by a one-digit whole number and multiply two two-digit numbers. Show and justify the calculation using equations, rectangular arrays, and models.
5.NO.NBT.4 Multiply multi-digit whole numbers using strategies flexibly, including the algorithm.
5.NO.NBT.7 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
4.NO.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors using place value strategies. Show and justify the calculation by using equations, rectangular arrays, and models.
5.NO.NBT.5 Use concrete models, drawings, place value strategies, properties of operations, and/or relationships to add, subtract, and multiply decimals to hundredths.
5.NO.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and two-digit divisors using place value strategies. Show and justify the calculation using equations, rectangular arrays, and/or area models.

6.NO.O.1 Divide multi-digit whole numbers up to four-digit dividends and two-digit divisors using strategies or procedures.
7.NO.O.1 Add, subtract, multiply, and divide integers and positive rational numbers using visual models and properties of operations in multi-step problems, including authentic problems.
7.NO.O.2 Add, subtract, multiply, and divide non-negative fractions in multi-step problems, including authentic problems.
7.NO.O.3 Add, subtract, multiply, and divide non-negative decimals to the hundredth place in multi-step problems using strategies or procedures, including authentic problems.
8.NO.O.1 Evaluate mentally the square roots of perfect squares up to 225 and cube roots of perfect cubes up to 1000.
8.NO.O.2 Add, subtract, multiply, and divide rational numbers using strategies or procedures.
9-10.NO.2 Perform basic operations on radicals and simplify radicals to write equivalent expressions.
9-10.AR.11 Add, subtract, and multiply polynomials.
11-12.NO.4 Demonstrate that the sum or product of two rational numbers is rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.
11-12.AR.13 Add, subtract, and multiply polynomials beyond quadratics. Understand that polynomials form a system comparable to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.
(+) 11-12.NO.11 Represent addition, subtraction, multiplication, conjugation, powers, and roots of complex numbers geometrically on the complex and/or polar plane; use properties of this representation for computation.
(+) 11-12.NO.19 Represent data in a matrix. Perform operations (i.e., addition, subtraction, multiplication) on matrices of appropriate dimensions to solve problems and in context. Know that matrix multiplication is not commutative.
Fractions - Partition Shapes
1.NO.NF.1 Partition circles and rectangles into two and four equal shares using the language halves and fourths.
2.NO.NF.1 Partition circles and rectangles into two, three, or four equal shares. Describe the shares using the language of halves, thirds, fourths, half of, a third of, and a fourth of.
2.NO.NF.2 Recognize that identical wholes can be equally divided in different ways.
2.NO.NF.3 Recognize that partitioning shapes into more equal shares creates smaller shares.
3.NO.NF.1 Partition two-dimensional figures into equal areas and express the area of each part as a unit fraction of the whole. Describe using the language of sixths, eighths, a sixth of, and an eighth of.
Fractions
3.NO.NF.2 Represent and understand a fraction as a number on a number line.
3.NO.NF.3 Represent equivalent fractions using visual representations and number lines.
3.NO.NF.4 Recognize whole numbers as fractions and express fractions that are equivalent to whole numbers.
3.NO.NF.5 Compare fractions of the same whole having the same numerators or denominators, using symbols >, <, and = by reasoning about their size. (Fractions should be limited to denominators of 2, 3, 4, 6, and 8 and should not exceed the whole.)
4.NO.NF.1 Express equivalent fractions with a denominator of 10 and a denominator of 100 to generate a decimal notation.
4.NO.NF.2 Explain and demonstrate how a mixed number is equivalent to a fraction greater than one and how a fraction greater than one is equivalent to a mixed number using visual fraction models and reasoning strategies (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).
4.NO.NF.3 Generate equivalent fractions using numerical representations, visual representations, and number lines (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).
4.NO.NF.4 Demonstrate how equivalent fractions are generated by multiplying a fraction equivalent to 1 or the properties of multiplication (proper and improper fractions limited to 2, 3,4, 5, 6, 8, 10, 12, and 100).

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Fractions
4.NO.NF.5 Compare and order fractions having unlike numerators or denominators. Record comparisons using symbols >, <, = and justify using a visual fraction model (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).
5.NO.NF.1 Generate equivalent forms of commonly used fractions and decimals (e.g., halves, fourths, fifths, tenths).
6.NO.NS.1 Explain and show the relationship between non-zero rational numbers and their opposites using horizontal and vertical number lines in authentic and mathematical problems. Use rational numbers to represent quantities in authentic contexts and explain the meaning of 0 in certain situations.
6.NO.NS.2 Write, interpret, and explain statements of order for rational numbers on a number line and in authentic contexts.
7.NO.NS.2 Recognize common fractions and decimal equivalencies up to a denominator of 10. Convert a rational number to a decimal using technology.
Adding and Subtracting Fractions
4.NO.NF.6 Solve authentic word problems by adding and subtracting fractions and mixed numbers with like denominators (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).
5.NO.NF.3 Solve authentic word problems by adding and subtracting fractions and mixed numbers with unlike denominators using a visual fraction model and/or equation.
6.NO.O.2 Add and subtract fractions and decimals up to the hundredth place, including in authentic problems.
7.NO.O.2 Add, subtract, multiply, and divide non-negative fractions in multi-step problems, including authentic problems.
8.NO.O.2 Add, subtract, multiply, and divide rational numbers using strategies or procedures.
11-12.NO.4 Demonstrate that the sum or product of two rational numbers is rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.
11-12.AR.5 Add, subtract, multiply, and divide rational expressions. Understand that rational expressions form a system analogous to rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.
11-12.NO.6 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. Understand the hierarchal relationships among subsets of the complex number system.
Multiplying and Dividing Fractions
4.NO.NF.7 Solve problems by multiplying fractions and whole numbers using visual fraction models (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).
5.NO.NF.2 Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number and explain why multiplying a given number by a fraction less than one results in a product smaller than the given number.
5.NO.NF.4 Solve authentic word problems by multiplying fractions and mixed numbers using visual fraction models and equations.
6.NO.O.3 Apply multiplication and division of fractions and decimals to solve and interpret problems using visual models, including authentic problems.
7.NO.O.2 Add, subtract, multiply, and divide non-negative fractions in multi-step problems, including authentic problems.
8.NO.O.2 Add, subtract, multiply, and divide rational numbers using strategies or procedures.
9-10.NO.2 Perform basic operations of simple radical expressions to write a simplified equivalent expression.
11-12.NO.3 Demonstrate that the sum or product of two rational numbers is rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.

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Multiplying and Dividing Fractions
11-12.NO.2 Perform basic operations on advanced radicals and simplify radicals to write equivalent expressions.
11-12.AR.5 Add, subtract, multiply, and divide rational expressions. Understand that rational expressions form a system analogous to rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.
11-12.NO.6 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. Understand the hierarchal relationships among subsets of the complex number system.
Exponents
5.NO.NBT.7 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
6.AR.EE.1 Write, read, and evaluate numerical expressions, including expressions with whole number exponents and grouping symbols.
8.AR.EE.1 Explain the relationship between repeated multiplication and the properties of integer exponents. Apply a single exponent property to generate equivalent numeric and algebraic expressions that include numerical coefficients.
9-10.NO.1 Explain how the definition of rational exponents follows from extending the properties of integer exponents; rewrite simple expressions involving radicals and rational exponents using the properties of exponents.
9-10.NO.2 Perform basic operations on simple radical expressions to write a simplified equivalent expression.
9-10.AR.6* Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions.
9-10.AR.4* Create linear and exponential equations in two or more variables to represent relationships between quantities. Graph equations on coordinate axes with proper labels and scales.
9-10.AR.F.6* Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function. a. Use appropriate forms of linear, quadratic, and exponential functions to show zeros, extreme values, and symmetry (where applicable) and interpret these in context. b. Use the properties of an exponential function to classify it as growth or decay.
9-10.AR.F.8* Identify situations that can be modeled with linear, quadratic, and exponential functions. Justify the most appropriate model for a situation based on the rate of change over equal intervals. Include situations in which a quantity grows or decays.
11-12.NO.1.Rewrite complex expressions involving radicals and rational exponents using the properties of exponents.
11-12.NO.2 Perform basic operations on advanced radicals and simplify radicals to write equivalent expressions.
11-12.AR.7* Create equations and inequalities and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions.
11-12.AR.8* Create equations in two or more variables to represent relationships between quantities. Graph equations on coordinate axes with appropriate labels and scales.
11-12.AR.F.3* Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of context. b. Use the properties of exponents to interpret expressions for exponential functions.
11-12.AR.F.6* Apply the inverse relationship between exponents and logarithms to solve problems.
11-12.AR.F.15* Use properties of logarithms to express the solution to $ab^{ct} = d$ where a, c, and d are real numbers and b is a positive real number. Evaluate the logarithm using technology when appropriate.

Decimals
4.NO.NF.1 Express equivalent fractions with a denominator of 10 and a denominator of 100 to generate a decimal notation.
4.NO.NBT.2 Compare two numbers to the millions place and decimals to the hundredths place, using symbols >, <, and =. Justify comparisons based on the value of the digits.
5.NO.NBT.1 Understand that in a multi-digit whole number, a digit in one place represents ten times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
5.NO.CC.1 Read, write, and compare decimals to thousandths including standard form and expanded form.
5.NO.NBT.2 Compare two decimals to thousandths using symbols >, =, <. Justify comparisons based on the value of the digits.
5.NO.NBT.3 Apply place value understanding to round decimals to any place.
5.NO.NF.1 Generate equivalent forms of commonly used fractions and decimals (e.g., halves, fourths, fifths, tenths).
5.NO.NBT.5 Use concrete models, drawings, place value strategies, properties of operations, and/or relationships to add, subtract, and multiply decimals to hundredths.
5.NO.NBT.7 Explain patterns in the numbers of zeros of the product when multiplying a number by powers of 10. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
6.NO.O.2 Add and subtract fractions and decimals up to the hundredths place, including authentic problems.
7.NO.NS.2 Recognize common fractions and decimal equivalencies up to a denominator of 10. Convert a rational number to a decimal using technology.
7.NO.O.3 Add, subtract, multiply, and divide non-negative decimals to the hundredth place in multi-step problems using strategies or procedures, including authentic problems.
8.NO.NS.3 Use scientific notation to represent very large or very small quantities. Interpret scientific notation generated by technology. Compare and order numbers in scientific and standard notation.
Unit Size and Scale
6.NO.NS.2 Write, interpret, and explain statements of order for rational numbers on a number line diagram and in authentic contexts.
9-10.NO.3 Choose and interpret the scale and the origin in graphs and data displays.
9-10.NO.4 Define appropriate quantities and units for the purpose of descriptive modeling.
9-10.NO.5* Choose a level of accuracy or precision appropriate to limitations on measurement when reporting quantities
9-10.AR.4* Create linear and exponential equations in two or more variables to represent relationships between quantities. Graph equations on coordinate axes with appropriate labels and scales.
9-10.GM.14 Verify experimentally and justify the properties of dilations given by a center and a scale factor.
9-10.GM.26 Recognize that the radian measure of an angle is the ratio of the length of the arc to the length of the radius of a circle.
11-12.NO.4* Use units to understand problems and to guide the solution of multi-step problems (e.g., unit analysis). Choose and interpret units consistently in formulas. Choose and interpret the scale and the units in graphs and data displays.
11-12.NO.5* Choose a level of accuracy or precision appropriate to limitations on measurement when reporting quantities.
11-12.AR.8* Create equations in two or more variables to represent relationships between quantities. Graph equations on coordinate axes with appropriate labels and scales.

Complex Numbers
11-12.NO.6 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form of a + bi with a and b real. Understand the hierarchal relationships among subsets of the complex number system.
11-12.NO.7 Use the definition $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
11-12.NO.8 Use conjugates to find quotients of complex numbers.
11-12.NO.9 Apply the Fundamental Theorem of Algebra to determine the number of zeros for polynomial functions. Find all solutions to a polynomial equation.
11-12.AR.11 Solve quadratic equations with real coefficients that have solutions of the form a+bi and a-bi.
(+) 11-12.NO.10 Represent complex numbers on the complex plane in rectangular, trigonometric, and polar forms. Find the modulus (absolute value) of a complex number. Explain why the rectangular, trigonometric, and polar forms of a given complex number represent the same number.
(+) 11-12.NO.11 Represent addition, subtraction, multiplication, conjugation, powers, and roots of complex numbers geometrically on the complex and/or polar plane; use properties of this representation for computation.
(+) 11-12.NO.12 Extend polynomial identities to the complex numbers.
(+) 11-12.NO.13 Apply the Fundamental Theorem of Algebra to find all roots of a polynomial equation and determine the nature (i.e., integer, rational, irrational, real, complex) of the roots.

Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*
Operations and Algebraic Thinking (OA) *Learners will analyze patterns and relationships to generate and interpret numerical expressions.*
Operations – Basic Facts
K.AR.OA.1 Automatically add and subtract within 5.
1.AR.OA.1 Automatically add and subtract within 10.
2.AR.OA.1 Automatically add and subtract within 20.
3.AR.OA.1 Using mental strategies, multiply and divide basic facts within 100. Automatically multiply and divide up to 5 x 5 and 10s facts.
4.AR.OA.1 Automatically multiply and divide through 10 x 10.
5.AR.OA.1 Automatically multiply and divide through 12 x 12.
Adding On
K.AR.OA.2 For any number from 1 to 9, find the number that makes 10 when added to the given number, sharing the answer with a model, drawing, or equation.
1.AR.OA.2 For any number from 1 to 19, find the number that makes 20 when added to the given number, sharing the answer with a model, drawing, or equation.
Properties of Operations
2.AR.OA.2 Apply the properties of operations to solve addition and subtraction equations and justify thinking.
3.AR.OA.2 Apply the properties of operations to solve multiplication and division equations and justify thinking.
4.AR.OA.2 Identify and apply the properties of operations for addition, subtraction, multiplication, and division and justify thinking.
5.AR.OA.2 Analyze problems using the order of operations to solve and evaluate expressions while justifying thinking.
6.AR.EE.3 Identify when two expressions are equivalent. Apply the properties of operations to generate equivalent expressions.
7.AR.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions involving variables, integers, and/or non-negative fractions and decimals with an emphasis on writing equivalent expressions.
8.AR.EE.5 Solve linear equations with rational number coefficients and variables on both sides, including equations that require using the distributive property and/or combining and collecting like terms. Interpret the number of solutions. Give examples of linear equations in one variable with one solution, infinitely showing solutions or no solutions.
9-10.AR.5 Justify each step in solving a linear equation that may or may not have a solution.
Decompose Numbers and/or Expressions
K.AR.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way using verbal explanations, objects, or drawings.
1.AR.OA.3 Decompose numbers less than or equal to 20 in more than one way.
9-10.AR.1 Use the structure of an expression (i.e., quadratic and exponential) to identify ways to rewrite it.
9-1.AR.F.6* Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function. a. Use appropriate forms of linear, quadratic, and exponential functions to show zeros, extreme values, and symmetry (where applicable) and interpret these in context. b. Use the properties of an exponential function to classify it as growth or decay.
9-10.AR.F.11* Interpret the parameters in a linear, quadratic, or exponential function in context.
11-12.AR.1 Use the structure of an expression (to extend to polynomial and rational expressions) to identify ways to rewrite it.

NOTE: The new content or increased rigor in the standard content between grades is shown in blue type on the standards progression. High School symbols used: (*) indicates a modeling standard, and (+) indicates a "plus" standard used for advanced levels. **Assessment Boundary**: (+) Address advanced skills.

Decompose Numbers and/or Expressions
11-12.AR.3* Interpret expressions that represent a quantity in context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
11-12.AR.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, division, or technology for the more complicated examples.
11-12.AR.F.3* Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum, and symmetry of the graph, and interpret these in terms of context. b. Use the properties of exponents to interpret expressions for exponential functions.
(+)11-12.AR.20 Apply the Binomial Theorem for the expansion of $(ax + by)^n$ in powers of x and y for a positive integer n and integers a and b
Solve Problems Using Operations
K.AR.OA.4 Solve authentic word problems with addition by putting together or adding to within 10.
1.AR.OA.6 Use the +, -, and = symbols accurately in an equation.
1.AR.OA.4 Solve authentic word problems with addition, including three numbers and unknowns, within 20.
2.AR.OA.3 Solve one- and two-step authentic word problems with addition within 100, including the use of unknowns.
K.AR.OA.5 Solve authentic word problems with subtraction by taking apart or taking from within 10.
1.AR.OA.5 Solve authentic word problems with subtraction, including unknowns, within 20.
2.AR.OA.4 Solve one- and two-step authentic word problems with subtraction within 100, including the use of unknowns.
2.AR.OA.5 Use repeated addition to find the total number of objects arranged in a rectangular array.
3.AR.OA.3 Solve two-step authentic word problems using addition and subtraction within 1000, including equations with a letter as an unknown.
3.AR.OA.4 Use strategies and visual models to solve authentic word problems with multiplication within 100, including unknowns, using grouping models and equations.
3.AR.OA.5 Use strategies and visual models to solve authentic word problems with division within 100, including unknowns, using grouping models and equations.
4.AR.OA.3 Solve multi-step authentic word problems using the four operations, including problems with interpreted remainders. Represent problems using equations, including a symbol as an unknown.
Factor Pairs/Multiples
4.AR.OA.4 Find factor pairs and multiples within the range of 1-36 while classifying numbers as prime or composite.
5.AR.OA.4 Find factor pairs and multiples within the range of 1-100 while classifying numbers as prime or composite.
6.NO.O.4 Determine the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.
9-10.AR.1 Use the structure of an expression (i.e., quadratic and exponential) to identify ways to rewrite it.
9-10.AR.F.6* Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function. a. Use appropriate forms of linear, quadratic, and exponential functions to show zeros, extreme values, and symmetry (where applicable), and interpret these in context. b. Use the properties of an exponential function to classify it as growth or decay.
9-10.AR.F.11* Interpret the parameters in a linear, quadratic, or exponential function in context.
11-12.AR.2 Use the structure of an expression (to extend to polynomial and rational expressions) to identify ways to rewrite it.

NOTE: The new content or increased rigor in the standard content between grades is shown in blue type on the standards progression. High School symbols used: (*) indicates a modeling standard, and (+) indicates a "plus" standard used for advanced levels. **Assessment Boundary**: (+) Address advanced skills.

11-12.AR.3* Interpret expressions that represent a quantity in context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
11-12.AR.4* Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Use the properties of exponents to transform exponential expressions. c. Complete the square in a quadratic expression to produce an equivalent expression.
11-12.AR.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, division, or technology for the more complicated examples.
11-12.AR.F.3* Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of context. b. Use the properties of exponents to interpret expressions for exponential functions.
Linear Equations
7.AR.EE.1 Apply the properties of operations as strategies to add, subtract, factor, and expand linear expressions involving variables, integers, and/or non-negative fractions and decimals with an emphasis on writing equivalent expressions.
8.AR.EE.3 Explain the characteristics of a linear relationship, including identifying the slope and y-intercept in tables, graphs, equations, and descriptions.
8.AR.EE.4 Represent linear relationships using tables, graphs, equations, and descriptions when given a relationship in one of these forms.
8.AR.EE.5 Solve linear equations with rational number coefficients and variables on both sides, including equations that require using the distributive property and/or combining and collecting like terms. Interpret the number of solutions. Give examples of linear equations in one variable with one solution, many solutions, or no solutions.
9-10.AR.3* Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions.
9-10.AR.4 Create linear and exponential equations in two or more variables to represent relationships between quantities. Graph equations on coordinate axes with appropriate labels and scales.
9-10.AR.5 Justify each step in solving a linear equation that may or may not have a solution.
9-10.AR.6 Solve linear equations and inequalities (to include compound inequalities) in one variable.
9-10.AR.7* Solve a system of linear equations graphically and algebraically. Create and solve a system of linear equations in context and interpret the results.
11-12.AR.5 Add, subtract, multiply, and divide rational expressions. Understand that rational expressions form a system analogous to rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.
11-12.AR.7* Create equations and inequalities and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions.
11-12.AR.8 Create equations in two or more variables to represent relationships between quantities. Graph equations on coordinate axes with appropriate labels and scales.
11-12.AR.9 Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context.
11-12.AR.12 Solve simple rational and radical equations in one variable and identify extraneous solutions.
11-12.AR.F.14* Write arithmetic and geometric sequences both recursively and with an explicit formula and convert between the two forms. Use sequences to model situations.
11-12.AR.15 Apply the Factor and Remainder Theorems to determine efficiently whether a liner expression is a factor of a polynomial expression.

11-12.AR.16 Using graphs, technology, tables, or successive approximations, show that the solution(s) to the equation f(x) = g(x) is the x-value(s) that result in the y-values of f(x) and g(x) being the same.
11-12.AR.17 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
(+) 11-12.AR.18 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).
(+) 11-12.NO.13 Apply the Fundamental Theorem of Algebra to find all roots of a polynomial equation and determine the nature (i.e., integer, rational, irrational, real, complex) of the roots.
(+) 11-12.AR.19 Solve a system of equations in three or more variables with matrices (using technology).
(+) 11-12.AR.F.23* Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions and interpret them in context.
Equations/Expressions
1.AR.OA.6 Use the +, -, and = symbols accurately in an equation.
2.AR.OA.2 Apply the properties of operations to solve addition and subtraction equations and justify thinking.
3.AR.OA.2 Apply the properties of operations to solve multiplication and division equations and justify thinking.
3.AR.OA.3 Solve word two-step authentic word problems using addition and subtraction within 1000, including equations with a letter as an unknown.
3.AR.OA.4 Use strategies and visual models to solve authentic word problems with multiplication within 100, including unknowns, using grouping models and equations.
3.AR.OA.5 Use strategies and visual models to solve authentic word problems with division within 100, including unknowns, using grouping models and equations.
4.AR.OA.3 Solve multi-step authentic word problems using the four operations, including problems with interpreted remainders.
4.AR.OA.5 Interpret multiplication equations as a comparison. Represent multiplicative comparisons as multiplication equations.
5.AR.OA.2 Analyze problems using the order of operations to solve and evaluate expressions while justifying thinking.
5.AR.OA.3 Write simple expressions that record calculations with numbers. Interpret numerical expressions without evaluating them.
6.AR.EE.1 Write, read, and evaluate numerical expressions, including expressions with whole number exponents and grouping symbols.
6.AR.EE.2 Read and evaluate algebraic expressions, including expressions with whole number exponents and grouping symbols. Write algebraic expressions to represent simple and authentic situations.
6.AR.EE.4 Describe the concept of a solution of an equation or an inequality. Determine whether a given number is a solution to an equation or an inequality.
6.AR.EE.5 Write and solve equations of the form of x + p = q and px = q for cases in which p and q are non-negative whole numbers or decimals, including authentic problems.
7.AR.EE.2 Write and solve equations of the form px + q = r and p(x + q) = r , including in authentic problems.
8.AR.EE.1 Explain the relationship between repeated multiplication and the properties of integer exponents. Apply a single exponent property to generate equivalent numeric and algebraic expressions that include numerical coefficients.
8.AR.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a non-negative rational number.
8.AR.EE.6 Read, write, and evaluate numerical and algebraic expressions, including expressions involving absolute value. Solve and graph equations of the form |x| = r where r is a nonnegative rational number.
9-10.AR.1 Use the structure of an expression (i.e., quadratic and exponential) to identify ways to rewrite it.
9-10.AR.4 Create linear and exponential equations in two or more variables to represent relationships between quantities. Graph equations on coordinate axes with appropriate labels and scales.

11-12.AR.3 Interpret expressions that represent a quantity in context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
11-12.AR.2 Use the structure of an expression (to extend to polynomial and rational expressions) to identify ways to rewrite it.
Equivalent Expressions
6.AR.EE.3 Identify when two expressions are equivalent. Apply the properties of operations to generate equivalent expressions.
7.AR.EE.1 Apply the properties of operations as strategies to add, subtract, factor, and expand linear expressions involving variables, integers, and/or non-negative fractions and decimals with an emphasis on writing equivalent expressions.
8.AR.EE.1 Explain the relationship between repeated multiplication and the properties of integer exponents. Apply a single exponent property to generate equivalent numeric and algebraic expressions that include numerical coefficients.
9-10.NO.2 Perform basic operations on simple radical expressions to write a simplified equivalent expression.
9-10.AR.1 Use the structure of an expression (i.e., quadratic and exponential) to identify ways to rewrite it.
9-10.AR.2 Rearrange formulas to isolate a quantity or variable(s) of interest using the same reasoning as in solving equations.
9-10.AR.7* Rearrange multi-variable formulas to highlight a quantity of interest.
9-10.AR.F.6* Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function. a. Use appropriate forms of linear, quadratic, and exponential functions to show zeros, extreme values, and symmetry (where applicable) and interpret these in context. b. Use the properties of an exponential function to classify it as growth or decay.
11-12.NO.2 Perform operations on complex radical expressions to write a simplified equivalent expression.
11-12.AR.1* Rearrange multi-variable formulas to highlight a quantity of interest.
11-12.AR.2 Use the structure of an expression (to extend to polynomial and rational expressions) to identify ways to rewrite it.
11-12.AR.4* Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Use the properties of exponents to transform exponential expressions. c. Complete the square in a quadratic expression to produce an equivalent expression.
11-12.AR.5 Add, subtract, multiply, and divide rational expressions. Understand that rational expressions form a system analogous to rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.
11-12.AR.F.3* Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of context. b. Use the properties of exponents to interpret expressions for exponential functions.
11-12.GM.1 Write the equation of a conic section given its special features. Convert between the standard form and general form equations of conic sections.
11-12.GM.2* Identify key features of a conic section given its equation. Apply properties of conic sections in context.
(+) 11-12.NO.12 Extend polynomial identities to the complex numbers.

Inequalities
6.AR.EE.4 Describe the concept of a solution to an equation or an inequality. Determine whether a given number is a solution to an equation or an inequality.
6.AR.EE.6 Write a statement of inequality of the form x > c or the form x < c to represent a constraint or condition. Recognize that inequalities of the form x > c or the form x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
7.AR.EE.3 Write and solve one- and two-step inequalities where coefficients and solutions are integers and/or non-negative fractions and decimals, including authentic problems. Graph the solution set of the inequality and interpret it in the context of the problem.
8.AR.EE.7 Solve and graph inequalities in one variable with rational number coefficients and variables on both sides, including equations that require using the distributive property and/or combining like terms.
8.AR.EE.8 Graph linear inequalities in two variables on a coordinate plane. Interpret the possible solutions in the context of authentic problems.
9-10.AR.3* Create equations and inequalities in two variables and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions.
9-10.AR.6 Solve linear equations and inequalities (to include compound inequalities) in one variable.
9-10.AR.7* Solve a system of linear equations graphically and algebraically. Create and solve a system of linear equations in context and interpret the results.
9-10.AR.8 Graph the solution set to a two-variable system of linear inequalities. Create and graph the solution set to a two-variable system of linear inequalities in context.
9-10.AR.9 Solve absolute value equations and inequalities in one or two variables.
11-12.AR.7* Create equations and inequalities and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions.
11-12.AR.9* Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context.
Quadratic Equations
9-10.AR.10 Solve quadratic equations in one variable by inspection (e.g., for $x^2 = 49$) taking square roots, the quadratic formula, and factoring, as appropriate to the initial form of the equation.
11-12.NO.9 Apply the Fundamental Theorem of Algebra to determine the number of zeros for polynomial functions. Find all solutions to a polynomial equation.
11-12.AR.7* Create equations and inequalities and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions.
(+) 11-12.AR.10 Derive the quadratic formula from the form $0 = ax^2 + bx + c$.
(+)11-12.AR.11 Solve quadratic equations with real coefficients that have solutions of the form a + bi and a - bi.
11-12.AR.14 Identify zeros of polynomials when suitable factorizations are available. Use the zeros to construct a rough graph of the function defined by the polynomial.
11-12.AR.17 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
Ratio and Proportional Relationships
6.AR.RP.1 Describe the concept of a ratio relationship between two quantities using ratio language and visual models.
6.AR.RP.3 Make and use tables of equivalent ratios, tape diagrams, double number line diagrams, and equations to reason about ratios, rates, and unit rates.
7.AR.RP.2 Analyze the relationships between the dependent and independent variables of a proportional relationship using graphs and tables. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, k) where k is the unit rate.
7.AR.RP.3 Identify the constant of proportionality in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. Represent proportional relationships by an equation of the form y = kx, where k is the constant of proportionality, and describe the meaning of each variable (y, k, x) in the context of the situation.

7.AR.RP.4 Use proportional relationships to solve multi-step problems involving ratios, percents, and scale drawings of geometric figures, including authentic problems.
8.AR.EE.3 Explain the characteristics of a linear relationship, including identifying the slope and y-intercept in tables, graphs, equations, and descriptions.
8.AR.EE.4 Represent linear relationships using tables, graphs, equations, and descriptions when given a relationship in one of these forms.
9-10.GM.14 Verify experimentally and justify the properties of dilations given by a center and a scale factor.
9-10.GM.15 Use transformations to decide if two given figures are similar. Apply the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
9-10.GM.16 Prove similarity theorems about triangles.
9-10.GM.18 Recognize how the properties of similar right triangles allow the trigonometric ratios to be defined and determine the sine, cosine, and tangent of an acute angle in a right triangle.
9-10.GM.20* Solve applied problems involving right triangles using trigonometric ratios, the Pythagorean Theorem, and special right triangles (30°-60°-90° and 45°-45°-90°).
9-10.GM.25 Explain and use the formulas for arc length and area of sectors of circles.
9-10.GM.26 Recognize that the radian measure of an angle is the ratio of the length of the arc to the length of the radius of a circle.
9-10.GM.29 Determine the midpoint or endpoint of a line segment using coordinates. (+) Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
9-10.GM.36* Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; scaling a model).
11-12.GM.3 Determine and apply appropriate formulas to solve right and non-right triangle problems in context.
Unit Rate
6.AR.RP.2 Describe and calculate a unit rate when given a ratio relationship between two quantities using rate language and visual models.
6.AR.RP.5 Convert measurement units within and between measurement systems using ratio reasoning given conversion factors.
6.AR.RP.4 Calculate a percent of a quantity as a rate per 100. Solve problems involving finding the whole, given a part, and the percent.
7.AR.RP.1 Calculate unit rates associated with ratios of rational numbers, including ratios of lengths, areas, and other quantities measured in like or different units.
9-10.NO.3 Choose and interpret the scale and the units in graphs and data displays.
9-10.NO.4 Define appropriate quantities and units for the purpose of descriptive modeling.
9-10.AR.F.5 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
9-10.AR.F.6* Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function. a. Use appropriate forms of linear, quadratic, and exponential functions to show zeros, extreme values, and symmetry (where applicable) and interpret these in context. b. Use the properties of an exponential function to classify it as growth or decay.
11-12.NO.4* Use units to understand problems and to guide the solution of multi-step problems (e.g., unit analysis). Choose and interpret units consistently in formulas. Choose and interpret the scale and the units in graphs and data displays.
11-12.AR.F.2* Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Percents
6.AR.RP.4 Calculate a percent of a quantity as a rate per 100. Solve problems using ratio reasoning involving finding the whole when given a part and the percent.
7.AR.RP.4 Use proportional relationships to solve multi-step problems involving ratios, percents, and scale drawings of geometric figures, including authentic problems.
Patterns
K.AR.OA.6 Recognize, duplicate, complete, and extend repeating patterns in a variety of contexts (e.g., shape, color, size, objects, sounds, and movement).
1.AR.OA.7 Identify, create, complete, and extend patterns that are repeating, increasing, and decreasing in a variety of contexts.
2.AR.OA.6 Identify a group of objects from 0 to 20 as even or odd by showing even numbers as a sum of two equal parts.
3.AR.OA.6 Identify arithmetic patterns and explain them using the properties of operations.
4.AR.OA.6 Generate a number or shape pattern that follows a given rule while identifying apparent features of the pattern that were not explicit in the rule itself.
5.AR.OA.5 Generate two numerical patterns using two given rules and form ordered pairs consisting of corresponding terms from the two patterns. (Graphing on a coordinate plane).
9-10.AR.1 Use the structure of an expression (i.e., quadratic and exponential) to identify ways to rewrite it.
11-12.AR.2 Use the structure of an expression (to extend to polynomial and rational expressions) to identify ways to rewrite it.
Functional Relationships
8.AR.F.1 Defend whether a relation is a function from various representations using appropriate function language.
8.AR.F.2 Compare and contrast properties of two linear functions, each represented in a different way (algebraically, graphically, numerically in tables, and/or by descriptions).
8.AR.F.3 Compare and contrast linear and non-linear functions represented in different ways (algebraically, graphically, numerically in tables, and/or by descriptions).
8.AR.F.4 Model a linear relationship between two quantities by creating a table, graph, and equation. Interpret the rate of change and initial value of a linear function in terms of the situation it models.
8.AR.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph, including where the function is constant, increasing, or decreasing; linear or nonlinear; and discrete or continuous. Create a graph that exhibits the qualitative features of a function described.
9-10.AR.F.1 Determine whether a relationship is a function given a table, graph, or words, identifying x as an element of the domain and f(x) as an element in the range. Determine the domain and range of a function in context.
9-10.AR.F.2* Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of context.
9-10.AR.F.3* Sketch key features (to include intercepts, maximums, minimums, and lines of symmetry, where applicable) of linear, exponential, and quadratic functions modeling the relationship between two quantities using tables, graphs, written descriptions, and equations.
9-10.AR.F.4* Relate the domain of a linear, quadratic, or exponential function to its graph and, where applicable, to the quantitative relationship it describes.
9-10.AR.F.5* Calculate and interpret the average rate of change of a linear, quadratic, or exponential function (presented algebraically or as a table) over a specified interval. Estimate the rate of change from a graph.
9-10.AR.F.6* Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function. a. Use appropriate forms of linear, quadratic, and exponential functions to show zeros, extreme values, and symmetry (where applicable) and interpret these in terms of context. b. Use the properties of an exponential function to classify it as growth or decay.

9-10.AR.F.7* Compare key features of two linear, exponential, or quadratic functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
9-10.AR.F.8* Identify situations that can be modeled with linear, quadratic, and exponential functions. Justify the most appropriate model for a situation based on the rate of change over equal intervals. Include situations in which a quantity grows or decays.
9-10.AR.F.10 Find the inverse of a linear function and describe the relationship between the domain, range, and graph of the function and its inverse. Graph the inverse of a linear function.
9-10.AR.F.11* Interpret the parameters of a linear, quadratic, or exponential function in terms of context.
9-10.GM.2 Represent transformations in the plane. Describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (i.e., rigid versus non-rigid motion).
9-10.GM.26 Recognize that the radian measure of an angle is the ratio of the length of the arc to the length of the radius of a circle.
11-12.AR.F.1 Write a function that describes a relationship between two quantities. a. Combine standard function types using arithmetic operations. b. Compose functions.
11-12.AR.F.2* Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
11-12.AR.F.3* Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum, and symmetry of the graph; interpret these in terms of context. Use the properties of exponents to interpret expressions for exponential functions.
11-12.AR.F.4* Identify the effect of transformations on the graph of a function by replacing f(x) with af(x), f(bx), f(x - h), and f(x) + k, for specific values of a, h, and k (both positive and negative). Find the value of a, b, h, and k given the graph of the function. Recognize even and odd functions from their graphs and equations.
11-12.AR.F.5* Find inverse functions. a. Verify by composition that one function is the inverse of another. b. Recognize that the graph of a function and its inverse are reflection images over the line $y = x$. c. Produce an invertible function from a non-invertible function by restricting the domain.
11-12.AR.F.6* Apply the inverse relationship between exponents and logarithms to solve problems.
11-12.AR.F.7* Compare key features of two functions, each represented in a different way (algebraically, graphically, numerically, in tables, or by verbal descriptions).
11-12.AR.F.8* Use tables, graphs, verbal discussions, and equations to interpret and sketch the key features of a function modeling the relationship between two quantities.
11-12.AR.F.9* Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
(+) 11-12.AR.F.11* Analyze and graph functions expressed symbolically (by hand in simple cases and using technology for more complicated cases), identifying key features of the graph. a. (+) Graph rational functions, identifying domain, range, asymptote(s), removable and non-removable discontinuities, intercepts, behavior at the asymptote(s), and end behavior. b. (+) Graph trigonometric functions, showing period, midline, phase shift and amplitude.
11-12.AR.F.12* Compare the end behavior of linear, quadratic, and exponential functions using graphs and/or tables to show that a quantity
11-12.AR.F.13* Determine whether a linear, quadratic, polynomial, exponential, logarithmic, or trigonometric model fits the situation. Determine an appropriate mathematical model in context (with or without technology).
11-12.AR.F.14* Write arithmetic and geometric sequences both recursively and with an explicit formula and convert between the two forms. Use sequences to model situations.
11-12.AR.F.16 Extend right triangle trigonometry and apply knowledge of the unit circle to determine values of sine, cosine, and tangent for multiples of π /3, π/4 and π/6.

11-12.AR.F.17 Use the Pythagorean Identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
(+) 11-12.AR.F.18 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
(+) 11-12.AR.F.19 Use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
(+) 11-12.AR.F.20 Use the unit circle to explain the symmetry (odd and even) and the periodicity of trigonometric functions.
(+) 11-12.AR.F.21 Create a trigonometric function to model periodic phenomena.
(+) 11-12.AR.F.22 Restrict the domain of a trigonometric function to construct its inverse.
(+) 11-12.AR.F.23* Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions and interpret them in context.
(+) 11-12.AR.F.24 Know and apply the addition and subtraction formulas for sine, cosine, and tangent to solve problems.
Graphing Functions
8.AR.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph, including where the function is constant, increasing, or decreasing; linear or nonlinear; and discrete or continuous. Create a graph that exhibits the qualitative features of a function described.
9-10.AR.F.1 Determine whether a relationship is a function given a table, graph, or words, identifying *x* as an element of the domain and *f(x)* as an element in the range. Determine the domain and range of a function in context.
9-10.AR.F.3* Sketch key features (to include intercepts, maximums, minimums, and lines of symmetry, where applicable) of linear, exponential, and quadratic functions modeling the relationship between two quantities using tables, graphs, written descriptions, and equations.
9-10.AR.F.9 Identify the effect of transformations on the graph of a linear, absolute value, or quadratic function by replacing f(x) with af(x), f(x - h), and f(x) + k, for specific values of a, h, and k (both positive and negative). Find the value of a, h, and k given the graph of the function.
9-10.AR.F.10 Find the inverse of a linear function and describe the relationship between the domain, range, and graph of the function and its inverse. Graph the inverse of a linear function.
9-10.AR.F.12 Identify, using graphs or tables, the solution(s) to linear or exponential functions f(x) = g(x) as x-values that result in equivalent y-values.
11-12.AR.14 Identify zeros of polynomials when suitable factorizations are available. Use the zeros to construct a rough graph of the function defined by the polynomial.
11-12.AR.16 Identify, using graphs, technology, tables, or successive approximations, that the solution(s) to the equation f(x) = g(x) is the x-value(s) that result in the y-values of f(x) and g(x) being the same.
11-12.AR.F.4* Identify the effect of transformations on the graph of a function by replacing f(x) with af(x), f(bx), f(x - h), and f(x) + k, for specific values of a, h, and k (both positive and negative). Find the values of a, b, h, and k given the graph of the function. Recognize even and odd functions from their graphs and equations.
11-12.AR.F.8* Use tables, graphs, verbal descriptions, and equations to interpret and sketch the key features of a function modeling the relationship between two quantities.
11-12.AR.F.9* Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
11-12.AR.F.10* Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases. a. Graph square root, cube root, piecewise-defined, step, and absolute value functions. b. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. c. Graph exponential and logarithmic functions, showing intercepts and end behavior. d. Graph f(x) = sin x and f(x) = cos x as representations of periodic phenomena.

(+) 11-12.AR.F.11* Analyze and graph functions expressed symbolically (by hand in simple cases and using technology for more complicated cases), identifying key features of the graph. a. (+) Graph rational functions, identifying domain, range, asymptote(s), removable and non-removable discontinuities, intercepts, behavior at the asymptote(s), and end behavior. b. (+) Graph trigonometric functions, showing period, midline, phase shift, and amplitude.
11-12.AR.F.12* Compare the end behavior of linear, quadratic, and exponential functions using graphs and/or tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing as a linear or quadratic function.
Logarithms
11-12.AR.F.6 Apply the inverse relationship between exponents and logarithms to solve problems.
11-12.AR.F.10* Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases. a. Graph square root, cube root, piecewise-defined, step, and absolute value functions. b. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. c. Graph exponential and logarithmic functions, showing intercepts and end behavior. d. Graph f(x) = sin x and f(x) = cos x as representations of periodic phenomena.
11-12.AR.F.15 Use the properties of logarithms to express the solution to $ab^{ct} = d$ where a, c, and d are real numbers and b is a positive real number. Evaluate the logarithm using technology when appropriate.

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
Two-Dimensional Shapes
K.GM.G.1 Name shapes and identify them as two-dimensional (squares, circles, triangles, rectangles) regardless of their orientations or overall size.
K.GM.G.3 Compare and classify two-dimensional shapes to describe their similarities, differences, and attributes (squares, circles, triangles, rectangles).
1.GM.G.1 Name shapes and identify them as two-dimensional (trapezoids, rhombuses, pentagons, hexagons, octagons).
1.GM.G.3 Determine geometric attributes of two-dimensional and three-dimensional shapes.
2.GM.G.1 Identify two-dimensional shapes (parallelograms and quadrilaterals).
2.GM.G.3 Compose geometric shapes having specified geometric attributes, such as a given number of edges, angles, faces, vertices, and/or sides.
3.GM.G.1 In two-dimensional shapes, identify lines, angles (right, acute, obtuse), and perpendicular and parallel lines.
3.GM.G.2 Sort quadrilaterals into categories based on attributes.
4.GM.G.1 Identify, label, and draw points, lines, line segments, rays, and angles (right, acute, obtuse).
4.GM.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of specified size.
5.GM.G.1 Classify two-dimensional figures in a hierarchy based on properties.
9-10.GM.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment based on the undefined notions of point, line, and plane.
9-10.GM.9 Prove and apply theorems about lines and angles.
9-10.GM.10 Prove and apply theorems about triangles.
9-10.GM.11 Prove and apply theorems about parallelograms.
9-10.GM.34 Identify the shapes of two-dimensional cross-sections of three-dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects.
Three-Dimensional Shapes
K.GM.G.2 Name shapes and identify them as three-dimensional (cubes and spheres) regardless of their orientations or overall size.
1.GM.G.2 Name and identify solids as three-dimensional (cylinders, cones, triangular prisms, and rectangular prisms).
1.GM.G.3 Determine geometric attributes of two-dimensional and three-dimensional shapes.
2.GM.G.2 Identify two-dimensional shapes found within three-dimensional shapes.
2.GM.G.3 Compose geometric shapes having specified geometric attributes, such as a given number of edges, angles, faces, vertices, and/or sides.
6.GM.GF.3 Represent three-dimensional figures using nets made up of rectangles and triangles (right prisms and pyramids whose bases are triangles and rectangles). Calculate the surface area of prisms with rectangular and triangular bases using nets, including authentic problems.
Compose Shapes
K.GM.G.4 Compose a geometric shape by combining two or more simple shapes.
1.GM.G.4 Compose a geometric shape or solid by combining multiple two-dimensional shapes and/or three-dimensional solids.
2.GM.G.3 Compose shapes having specified geometric attributes, such as a given number of edges, angles, faces, vertices, and/or sides.
7.GM.GF.1 Draw triangles from given conditions using appropriate tools. Defend whether a unique triangle, multiple triangles, or no triangle can be constructed when given three measures of angles or sides.
9-10.GM.12 Make basic geometric constructions (e.g., segments, angles, bisectors, parallel and perpendicular lines) with a variety of tools and methods.
(+) 9-10.GM.13 Apply basic construction to create polygons such as equilateral triangles, squares, and regular hexagons inscribed in a circle.

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Symmetry
3.GM.G.3 Identify lines of symmetry in quadrilaterals.
4.GM.G.3 Draw lines of symmetry in two-dimensional figures.
Coordinate Plane
3.GM.G.1 In two-dimensional shapes, identify lines, angles (right, acute, obtuse), and perpendicular and parallel lines.
5.GM.G.2 Identify the x-coordinate and y-coordinate to graph and name points in the first quadrant of the coordinate plane.
5.GM.G.3 Form ordered pairs and graph points in the first quadrant of the coordinate plane to solve authentic word problems.
6.GM.GF.1 Identify and position ordered pairs of rational numbers in all four quadrants of a coordinate plane.
6.GM.GF.2 Draw polygons in the coordinate plane given coordinates for vertices. Determine the length of a side joining points with the same first or second coordinate, including authentic problems.
9-10.GM.27 Develop and verify the slope criteria for parallel and perpendicular lines. Apply the slope criteria for parallel and perpendicular lines to solve geometric problems using algebra.
9-10.GM.28 Verify simple geometric theorems algebraically using coordinates. Verify algebraically, using coordinates, that a given set of points produces a particular type of triangle or quadrilateral.
9-10.GM.29 Determine the midpoint or endpoint of a line segment using coordinates. (+) Find the point on a directed line segment between two given points that partitions the segments in a given ratio.
(+) 11-12.NO.10 Represent complex numbers on the complex plane in rectangular, trigonometric, and polar forms. Find the modulus (absolute value) of a complex number. Explain why the rectangular, trigonometric, and polar forms of a given complex number represent the same number.
(+) 11-12.NO.11 Represent addition, subtraction, multiplication, conjugation, powers, and roots of complex numbers geometrically on the complex and/or polar plane; use properties of this representation for computation.
(+) 11-12.NO.14 Recognize vector quantities as having both magnitude and direction, writing them in polar form.
(+) 11-12.NO.15 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
(+) 11-12.NO.16 Solve problems involving magnitude and direction that can be represented by vectors.
(+) 11-12.NO.17 Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Know that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand that vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ is defined as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and using the components to perform vector subtraction. Represent vector subtraction graphically by connecting the tips in the appropriate order and using the components to perform vector subtraction.
(+) 11-12.NO.18 Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction. Use the components to perform scalar multiplication (e.g., as $c(v_x, v_y) = (cv_x, cv_y)$). b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|c\mathbf{v}\| = |c|\mathbf{v}$. Compute the direction of cv knowing that when $|c|\mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$) or against $\mathbf{v}$ (for $c < 0$).
(+) 11-12.AR.F.18 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
(+) 11-12.AR.F.19 Use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

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Transformations
8.GM.GF.1 Perform single transformations to a figure on or off the coordinate plane and determine whether the figures are congruent or similar.
8.GM.GF.2 Describe the characteristics of transformations on the coordinate plane using transformation language.
8.GM.GF.3 Name the type of transformation(s) needed to map a pre-image to its image.
9-10.AR.F.9* Identify the effect of transformations on the graph of a linear, absolute value, or quadratic function by replacing f(x) with f(x) + k, f(x - h) and af(x), for specific values of a, h, and k (both positive and negative). Find the values of a, h, and k given the graph of the function.
9-10.GM.2 Represent transformations in the plane. Describe transformations as functions that take points as outputs. Compare transformations that preserve distance and angle to those that do not (i.e., rigid versus non-rigid motion).
9-10.GM.3 Describe the rotations and reflections of a triangle, rectangle, parallelogram, trapezoid, or regular polygon that map each figure onto itself or another figure.
9-10.GM.4 Develop or verify the characteristics of rotations, reflections, and translations in angles, circles, perpendicular lines, parallel lines, and line segments.
9-10.GM.5 Draw the image of a figure that has undergone a series of transformations [rotation(s), reflection(s), or translation(s)] of a geometric figure using a variety of methods (e.g., graph paper, tracing paper, or geometry software).
9-10.GM.6 Predict the effect of a specified rigid motion on a given figure using geometric descriptions of rigid motions. Determine whether two figures are congruent using the definition of congruence in terms of rigid motions.
9-10.GM.14 Verify experimentally and justify the properties of dilations given by a center and a scale factor.
9-10.GM.15 Use transformations to decide if two given figures are similar. Apply the meaning of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
9-10.GM.16 Prove similarity theorems about triangles.
9-10.GM.17 Apply knowledge of congruence and similarity criteria for triangles to solve problems and prove relationships in various geometric figures.
11-12.AR.F.4* Identify the effect of transformations on the graph of a function by replacing f(x) with af(x), f(bx), f(x-h), and f(x) + k, for specific values of a, h, and k (both positive and negative). Find the values of a, b, h, and k given the graph of the function. Recognize even and odd functions from their graphs and equations.
(+) 11-12.NO.18 Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction. Use the components to perform scalar multiplication (e.g., as $c(v_x, v_y) = (cv_x, cv_y)$). b. Compute the magnitude of a scalar multiple c**v** using $\|c\mathbf{v}\| = |c|\mathbf{v}$. Compute the direction of cv knowing that when $|c|\mathbf{v} \neq 0$, the direction of c**v** is either along **v** (for $c > 0$) or against **v** (for $c < 0$).
Congruence and Similarity
8.GM.GF.1 Perform single transformations to a figure on or off the coordinate plane and determine whether the figures are congruent or similar.
8.GM.GF.2 Describe the characteristics of transformations on the coordinate plane using transformation language.
9-10.GM.7 Use the definition of congruence, based on rigid motions, to show two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent.
9-10.GM.8 Prove two triangles are congruent using the congruence theorems.
9-10.GM.17 Apply knowledge of congruence and similarity criteria for triangles to solve problems and prove relationships in various geometric figures.

Measure Length
1.GM.M.1 Measure the length of an object as a whole number of same-size, non-standard units from end to end.
2.GM.M.1 Measure the length of an object using two different standard units of measurement. Describe how the two measurements relate to the size of the units chosen.
3.GM.M.1 Measure lengths using rulers marked with halves and fourths of an inch.
9-10.GM.25 Explain and use the formulas for arc length and area of sectors of circles.
9-10.GM.26 Recognize that the radian measure of an angle is the ratio of the length of the arc to the length of the radius of a circle.
Compare Objects
K.GM.M.1 Compare and order two objects with a common measurable attribute.
1.GM.M.2 Compare the lengths of three objects using a common measurable attribute.
2.GM.M.2 Estimate and measure to determine how much longer one object is than another, expressing the difference with a standard unit of measurement.
Units of Measurement
3.GM.M.2 Measure and estimate liquid volumes and masses of objects using standard units. Solve one-step authentic word problems involving masses or volumes given in the same units.
4.GM.M.1 Know the relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb., oz.; l, ml; hr., min., sec. Record measurement equivalents in a two-column table.
4.GM.M.3 Identify and use the appropriate tools, operations, and units of measurement, both customary and metric to solve problems involving time, length, weight, mass, and capacity.
4.GM.M.2 Generate simple conversions from a larger unit to a smaller unit to solve authentic problems within a single system of measurement, both customary and metric systems.
5.GM.M.1 Generate conversions among different-sized standard measurement units within a given measurement system, both customary and metric systems. Use these conversions in solving multi-step, authentic word problems.
6.AR.RP.5 Convert measurement units within and between measurement systems using ratio reasoning given conversion factors.
9-10.GM.36* Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; scaling a model).
9-10.NO.3 Choose and interpret the scale and the origin in graphs and data displays.
9-10.NO.4* Define appropriate quantities and units for the purpose of descriptive modeling.
9-10.NO.5 Choose a level of accuracy or precision appropriate to limitations on measurement when reporting quantities.
11-12.NO.4* Use units to understand problems and to guide the solution of multi-step problems (e.g., unit analysis). Choose and interpret units consistently in formulas. Choose and interpret the scale and the units in graphs and data displays.
11-12.NO.5* Choose a level of accuracy or precision appropriate to limitations on measurement when reporting quantities.
Time
K.GM.M.2 Tell time as it relates to daily life (today, yesterday, tomorrow, morning, afternoon, night).
1.GM.M.3 Tell and write time to the hour and half-hour (including o'clock and half past) using analog and digital clocks.
2.GM.M.3 Tell and write time to the nearest five minutes (including quarter after and quarter to) with a.m. and p.m. using analog and digital clocks.
3.GM.M.3 Tell and write time to the nearest minute and measure time intervals in minutes.
3.GM.M.4 Solve elapsed time authentic word problems on the hour and the half-hour, using a variety of strategies.

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Money
1.GM.M.4 Identify and tell the value of a dollar bill, quarter, dime, nickel, and penny.
1.GM.M.5 Count collections of coins (pennies, nickels, and dimes) relating to patterns of counting by 1s, 5s, and 10s up to one dollar.
2.GM.M.4 Count collections of money (quarters, dimes, nickels, and pennies) relating to patterns of counting by 1s, 5s, and 10s up to one dollar.
3.GM.M.5 Solve authentic word problems involving dollar bills, quarters, dimes, nickels, and pennies using the $ and ¢ symbols appropriately.
4.GM.M.4 Solve word authentic problems involving dollar bills, quarters, dimes, nickels, and pennies using the $ and ¢ symbols and decimal notation appropriately.
Angles/Triangles
3.GM.G.1 In two-dimensional shapes, identify lines, angles (right, acute, obtuse), and perpendicular and parallel lines.
4.GM.G.1 Identify, label, and draw points, lines, line segments, rays, and angles (right, acute, obtuse).
4.GM.M.7 Recognize angle measures as additive and solve addition and subtraction problems to find unknown angles on a diagram.
7.GM.GF.1 Draw triangles from given conditions using appropriate tools. Defend whether a unique triangle, multiple triangles, or no triangle can be constructed when given three measures of angles or sides.
7.GM.GF.2 Describe the angle-pair relationships: supplementary angles, complementary angles, vertical angles, and adjacent angles. Solve for an unknown angle in a figure by applying facts about these angles.
8.GM.GF.4 Describe the following angle-pair relationships: interior and exterior angles of triangles and angles formed when a transversal cuts parallel lines or intersecting lines. Solve for an unknown angle in a figure by applying facts about these angles.
8.GM.GF.5 Describe the relationship between the leg length and the hypotenuse length of a right triangle. Determine whether a triangle is a right triangle using this relationship.
8.GM.GF.6 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in two and three dimensions on and off a coordinate plane, including authentic problems.
9-10.GM.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment based on the undefined notions of point, line, and plane.
9-10.GM.9 Prove and apply theorems about lines and angles.
9-10.GM.10 Prove and apply theorems about triangles.
9-10.GM.18 Recognize how the properties of similar right triangles allow for trigonometric ratios to be defined and determine the sine, cosine, and tangent of an acute angle in a right triangle.
(+) 9-10.GM.19 Explain and use the relationship between the sine and cosine of complementary angles.
9-10.GM.20* Solve applied problems involving right triangles using trigonometric ratios, the Pythagorean Theorem, and special right triangles (30º, -60º, -90º, and 45º-45º-90º).
(+) 9-10.GM.21 Solve unknown sides and angles of non-right triangles using the Laws of Sines and Cosines.
(+) 9-10.GM.23 Construct the incenter and circumcenter of a triangle. Relate the incenter and circumcenter to the inscribed and circumscribed circles.
(+) 9-10.GM.24 Construct a tangent line from a point outside a given circle to the circle.
9-10.GM.26 Recognize that the radian measure of an angle is the ratio of the length of the arc to the length of the radius of a circle.
11-12.AR.F.16 Extend right triangle trigonometry and apply knowledge of the unit circle to determine values of sine, cosine, and tangent for multiples of π/3, π/4, and π/6.
11-12.AR.F.17 Use the Pythagorean Identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to find sin (θ), cos (θ), or tan (θ) given sin (θ), cos (θ), or tan (θ) and the quadrant of the angle.
11-12.GM.3 Determine and apply appropriate formulas to solve right and non-right triangle problems in context.

Perimeter
3.GM.M.6 Solve problems involving the perimeters of rectangles given the side lengths or when given the perimeter and unknown side length(s).
4.GM.M.5 Apply the area and perimeter formulas for rectangles, including connected rectangular figures, in problems.
5.GM.M.2 Find the area and perimeter of a rectangle, including connected rectangular figures, with fractional side lengths.
7.GM.AV.1 Describe the relationship between the circumference and diameter of a circle (pi). Apply given formulas to calculate the area and circumference of a circle, including in authentic problems.
9-10.GM.30* Compute perimeters of polygons and areas of triangles, parallelograms, trapezoids, and kites using coordinates.
Area/Surface Area
3.GM.M.7 Recognize area as an attribute of plane figures and understand concepts of area measurement.
3.GM.M.8 Find the area of a rectangle with whole-number side lengths by modeling with unit squares; show that area can be additive and is the same as found by multiplying the side lengths.
4.GM.M.5 Apply the area and perimeter formulas for rectangles, including connected rectangular figures, in problems.
5.GM.M.2 Find the area and perimeter of a rectangle, including connected rectangular figures, with fractional side lengths.
6.GM.AV.1 Derive the relationship of the areas of triangles using the area of rectangles. Calculate the areas of triangles and quadrilaterals in authentic and mathematical problems by composing and/or decomposing them into rectangles and triangles.
6.GM.GF.3 Represent three-dimensional figures using nets made up of rectangles and triangles (right prisms and pyramids whose bases are triangles and rectangles). Calculate the surface area of prisms with rectangular and triangular bases using nets. Apply these techniques in the context of solving authentic and mathematical problems.
7.GM.AV.2 Calculate areas of polygons by composing and/or decomposing them into rectangles and triangles, including in authentic problems. Solve problems involving the surface area of prisms and right pyramids using nets, including authentic problems.
9-10.GM.30* Compute perimeters of polygons and areas of triangles, parallelograms, trapezoids, and kites using coordinates.
(+) 11-12.GM.4 Derive the formula $A=\frac{1}{2}ab\sin C$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
Volume
5.GM.M.3 Recognize volume as an attribute of rectangular prisms and measure volume by counting unit cubes.
6.GM.AV.2 Describe the concept of volume of a right rectangular prism. Apply given formulas to calculate the volume of right rectangular prisms, including fractional edge lengths, including authentic problems.
7.GM.AV.3 Solve problems involving the volume of prisms and composite solids, including authentic problems.
8.GM.AV.1 Apply given formulas to solve problems involving the volume of cones, cylinders, and spheres, including authentic problems.
9-10.GM.32 Calculate the surface area for prisms, cylinders, pyramids, cones, and spheres to solve problems.
9-10.GM.33 Know and apply volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems.
9-10.GM.35* Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

Circle Measurements
7.GM.AV.1 Describe the relationship between the circumference and diameter of a circle (pi). Apply the given formula to calculate the area and circumference of a circle, including in authentic problems.
9-10.GM.22 Apply theorems about relationships between line segments and circles or angles and circles formed by radii, diameter, secants, tangents, and chords to find unknown lengths or angles.
9-10.GM.25 Explain and use the formulas for arc length and area of sectors of circles.
9-10.GM.26 Recognize that the radian measure of an angle is the ratio of the length of the arc to the length of the radius of a circle.
9-10.GM.31 Explain derivations of the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone.
11-12.GM.1 Write the equation of a conic section given its special features. Convert between the standard form and general form equations of conic sections.
11-12.GM.2* Identify key features of a conic section given its equation. Apply properties of conic sections in context.
(+) 11-12.AR.F.18 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions and making predictions; and understanding and applying basic concepts of probability.*
Data Collection
K.DPS.D.1 Sort and classify objects (up to 10) based on attributes and explain the reasoning used.
1.DPS.D.1 Collect, organize and represent data with up to three categories using picture and bar graphs.
2.DPS.D.1 Formulate questions and collect, organize, and represent data, with up to four categories using single unit scaled pictures and bar graphs.
3.DPS.D.1 Formulate questions to collect, organize, and represent data with more than four categories using scaled pictures and bar graphs.
4.DPS.D.1 Formulate questions to collect, organize, and represent data to reason with math and across disciplines.
6.DPS.D.1 Write a statistical question that can be answered using measures of center or variability of a data set.
7.DPS.D.1 Identify the strengths and weaknesses of a population sample, including possible bias in the process of the data collection.
(+) 11-12.DPS.6* Use data from a sample survey to estimate a population means or proportion; develop a margin of error through the use of simulation models for random sampling.
(+) 11-12.DPS.9* Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
Displaying Data
1.DPS.D.1 Collect, organize and represent data with up to three categories using picture and bar graphs.
2.DPS.D.1 Formulate questions and collect, organize, and represent data, with up to four categories using single unit scaled pictures and bar graphs.
3.DPS.D.1 Formulate questions to collect, organize, and represent data with more than four categories using scaled pictures and bar graphs.
4.DPS.D.1 Formulate questions to collect, organize, and represent data to reason with math and across disciplines.
2.DPS.D.2 Generate data and create line plots marked in whole number units.
3.DPS.D.2 Generate data and create line plots marked in whole numbers, halves, and fourths of a unit.
4.DPS.D.2 Generate data and create line plots to display a data set of fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.
5.DPS.D.1 Generate data and create line plots to display a data set of fractions of a unit (1/2, 1/4, 1/8). Use grade-level operations for fractions to solve problems involving information presented in line plots.
6.DPS.D.4 Display numerical data in plots on a number line, including dot plots and histograms. Describe any overall patterns in data, such as gaps, clusters, and skews.
9-10.NO.3 Choose and interpret the scale and the units in graphs and data displays.
9-10.NO.5 Choose a level of accuracy or precision appropriate to limitations on measurement when reporting quantities.
9-10.DPS.1* Represent data with plots on the real number line (dot plots, histograms, and box plots).
9-10.DPS.3* Represent data on two quantitative variables on a scatter plot and describe how the variables are related. a. Fit a linear function to the data (with or without technology) if appropriate. b. Compute (using technology) and interpret the correlation coefficient of a linear fit. c. Interpret the meaning of the slope and y-intercept of the liner model in context. d. Interpolate and extrapolate the linear model to predict values.
9-10.DPS.10* Construct and interpret two-way frequency tables of data for two categorical variables. Use the two-way table as a sample space to decide if events are independent and approximate conditional probabilities.

11-12.NO.4* Use units as a way to understand problems and to guide the solution of multi-step problems (e.g., unit analysis). Choose and interpret units consistently in formulas. Choose and interpret the scale and the units in graphs and data displays.
11-12.DPS.4 Represent data on a scatter plot for two quantitative variables and describe how the variables are related. a. Fit the function to the data (with or without technology) and interpret the special features (e.g., meaning of a and b in the exponential function $y = ab^x$) of the function in context. b. Use functions fitted to data to solve problems in the context of the data.
Data Analysis
1.DPS.D.2 Analyze data by answering descriptive questions.
2.DPS.D.3 Analyze data and interpret the results to solve one-step comparison problems using information from the graphs.
3.DPS.D.3 Analyze data and make simple statements to solve one- and two-step problems using information from the graphs.
4.DPS.D.2 Generate data and create line plots to display a data set of fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.
4.DPS.D.3 Utilize graphs and diagrams to represent and solve word problems using the four operations involving whole numbers, benchmark fractions, and decimals.
5.DPS.D.1 Generate data and create line plots to display a data set of fractions of a unit (1/2, 1/4, 1/8). Use grade-level operations for fractions to solve problems involving information presented in line plots.
5.DPS.D.2 Utilize graphs and diagrams to represent, analyze, and solve authentic problems using information presented in one or more tables or line plots, including whole numbers, fractions, and decimals.
6.DPS.D.2 Calculate measures of center (median and mean) and variability (range and mean absolute deviation) to answer a statistical question. Identify mode(s) if they exist.
6.DPS.D.3 Identify outliers by observation and describe their effect on measures of center and variability. Justify which measures would be appropriate to answer a statistical question.
6.DPS.D.4 Display numerical data in plots on a number line, including dot plots and histograms. Describe any overall patterns in data, such as gaps, clusters, and skews.
7.DPS.D.2 Analyze and draw inferences about a population using single and multiple random samples by using given measures of center and variability for the numerical data set.
8.DPS.D.1 Interpret scatter plots for bivariate measurement data to investigate patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
8.DPS.D.2 Draw a trend line on a given scatter plot with a linear association and justify its fit by describing the closeness of the data points to the line.
8.DPS.D.3 Solve authentic problems in the context of bivariate measurement data by interpreting the slope and intercept(s) and making predictions using a linear model.
8.DPS.D.4 Construct and interpret a two-way table summarizing bivariate categorical data collected from the same subjects.
9-10.DPS.2* Compare the center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets using statistics appropriate to the shape of the data distribution.
9-10.DPS.3* Represent data on two quantitative variables on a scatter plot and describe how the variables are related. a. Fit a linear function to the data (with or without technology) if appropriate. b. Compute (using technology) and interpret the correlation coefficient of a linear fit. c. Interpret the meaning of the slope and y-intercept of the liner model in context. d. Interpolate and extrapolate the linear model to predict values.
9-10.DPS.4* Distinguish between correlation and causation.

9-10.DPS.10* Construct and interpret two-way frequency tables of data for two categorical variables. Use the two-way table as a sample space to decide if events are independent and approximate conditional probabilities.
11-12.DPS.1* Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
11-12.DPS.2 Use the mean and standard deviation of a data set to fit it to a normal distribution and estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate.
11-12.DPS.3 Evaluate reports based on data. a. Evaluate articles, reports, or websites based on data published in the media by identifying sources of the data, the design of the study, and the way the data are analyzed and displayed. b. Identify and explain misleading use of data and recognize when claims based on data confuse correlation and causation. c. Recognize and describe how graphs and data can be distorted to support different points of view.
11-12.DPS.4 Represent data on a scatter plot for two quantitative variables and describe how the variables are related. a. Fit the function to the data (with or without technology) and interpret the special features (e.g., meaning of *a* and *b* in the exponential function $y = ab^x$) of the function in context. b. Use functions fitted to data to solve problems in the context of the data.
(+) 11-12.DPS.5* Informally assess the fit of a function by plotting and analyzing residuals.
(+) 11-12.DPS.6* Use data from a sample survey to estimate a population means or proportion; develop a margin of error through the use of simulation models for random sampling.
5(+) 11-12.DPS.7* Understand the process of making inferences about population parameters based on a random sample from that population.
(+) 11-12.DPS.8* Decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation).
Probability
7.DPS.P.1 Develop a probability model to find probabilities of theoretical events and contrast probabilities from an experimental model.
7.DPS.P.2 Develop a probability model to find theoretical probabilities of independent compound events.
9-10.DPS.5* Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes or as unions, intersections, or complements of other events ("or," "and," "not").
9-10.DPS.6* Recognize that event A is independent of event B if the probability of event A does not change in response to the occurrence of event B. Apply the formula P(A and B) = P(A)·P(B) given that events A and B are independent.
9-10.DPS.7* Recognize the conditional probability of an event A given B is the probability that event A will occur given the knowledge that event B has already occurred. Calculate the conditional probability of A given B and interpret the answer in context.
9-10.DPS.8* Apply the formula P(A or B) = P(A) + P(B) – P(A and B) and interpret the answer in context.
9-10.DPS.9* Determine the number of outcomes using permutations and combinations in context.
9-10.DPS.10* Construct and interpret two-way frequency tables of data for two categorical variables. Use the two-way table as a sample space to decide if events are independent and approximate conditional probabilities.
11-12.DPS.10 Determine when the order in counting matters and use permutations and combinations to compute probabilities of events accordingly. Determine probability situations as conditional, "or" (union), or "and" (intersection), and determine the probability of an event.
(+) 11-12.DPS.11* Use permutations and combinations to compute probabilities of compound events and solve problems.

(+) 11-12.DPS.12* Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space. Graph the corresponding probability distribution using the same graphical displays as for data distributions.
(+) 11-12.DPS.13* Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
(+) 11-12.DPS.14* Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. Evaluate and compare strategies on the basis of expected values.
(+) 11-12.DPS.15* Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
(+) 11-12.DPS.16* Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.
(+) 11-12.DPS.17* Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
(+) 11-12.DPS.18* Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

MATHEMATICAL TERMS

Absolute Value – the distance a number is from zero on the number line.

Example: |-52| =52 and |52| = 52.

Accuracy – a measure of correctness. Note: This definition is specific to the high school standard.

Adjacent – having a common vertex and a common side.

Algebraic Expression – a mathematical statement that uses a combination of symbols representing numbers, variables, and arithmetic operations.

Algorithm – a process, routine, or set of rules to be followed in calculations.

Alternate Exterior Angles – pair of angles on opposite sides of a transversal and outside the two intersecting lines.

Alternate Interior Angles – pair of angles on opposite sides of a transversal and inside the two intersected lines.

Area – the space covered by any two-dimensional geometric shape.

Array – an arrangement of objects, pictures, or numbers in columns or rows.

Association – a relationship between two numerical variables described by their form, direction, strength, and outliers.

Attribute – a characteristic or property of an object.

Authentic Problems – suggest four principles to create real-life problems: 1. Problems where the solution has a real purpose; 2. Select contexts which have relevance to learners and their worlds; 3. The problem should need to be solved using mathematics; and 4. The problem-solving process and solution should foster discussion.

Automatic – without conscious thought or attention.

Bar Graph – a graph that represents categorical data using rectangular bars.

Base (Numbers) – the number multiplied when using an exponent.

Base (Geometry) – the side of a polygon or the face of a solid that is perpendicular to the height.

Bias – a phenomenon that occurs when a model or data set is unrepresentative of the population.

Bivariate Data – data on each of two variables, where each value of one of the variables is paired with a value of the other variable. Example: a list of heights and weights for each player on a football team.

Box Plot – a method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.

Cardinality – the number of elements in a given mathematical set.

Categorical Data – data that can be divided into specific groups.

Causation – the capacity of one variable to influence another.

Cavalieri's Principle – 2D: Suppose two regions in a plane are included between two parallel lines in that plane. If every line parallel to these lines intersects both regions in line segments of equal length, then the two regions have equal areas. 3D: Suppose two regions in three-space(solids) are included between two parallel planes. If every plane parallel to these two planes intersects both regions in cross-sections of equal area, then the two regions have equal volumes.

Chord – a line segment connecting two points on a curve.

Circumference – the distance around the edge of a circle. It is a type of perimeter.

Cluster – a grouping of neighboring values in a distribution of data that occurs more often.

Coefficient – any number multiplied by a variable. Example: 2x + 3, the 2 is the coefficient.

Collecting Like Terms – using additive inverses to combine like terms on opposite sides of an equal or inequality sign.

Combining Like Terms – adding or subtracting the numerical coefficients of like terms.

Compose – to put together parts or elements.

Composite Solid – a solid made of two or more solids.

Complementary Angles – two angles that add up to 90°.

Compound Event – the probability of more than one outcome.

Condition – an assumption on which rests the validity or effect of something else, a circumstance.

Conditional Probability – the probability of an event (A), given that another (B) has already occurred.

Cone – a three-dimensional solid that has a circular base joined to a point (vertex) by a curved side.

Congruent – two planes or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Constant – a fixed value.

Constant of Proportionality – the constant value of the ratio of two proportional quantities x and y; usually written y = kx, where k is the factor of proportionality

Constraint – limitation, a condition which must be satisfied.

Continuous – structures or sets of values that do not have any breaks or holes.

Coordinate/Ordered Pair – two numbers written in a fixed order to show position on a graph.

Coordinate Plane – a two-dimensional surface formed by two perpendicular number lines (axes).

Correlation – a mutual relationship between two or more things.

Corresponding Angles – the angles which occupy the same relative position at each intersection where a transversal crosses two other lines.

Counting (Natural) Numbers- any number used for counting items. These numbers do not include zero, negative numbers, fractions, or decimals.

Counting On – a strategy for finding the number of objects in a group without having to count every member of the group. Example: When using the counting on strategy for the problem $4 + 3$, one would start with four and then add three by saying “4, 5, 6, 7.” One can find the total by counting on – pointing to the top book and saying “eight,” followed by “nine, ten, eleven.”

Cube Root – one of three identical factors of a given number.

Cylinder – a three-dimensional solid consisting of two parallel circular bases joined by a curved surface.

Decompose – separate into parts or basic elements.

Decuple – crossing over to the next ten.

Dependent Variable – a variable (often denoted by *y*) whose value depends on another.

Diameter – the distance from one point on a circle through the center to another point on the circle.

Dilation – a transformation that moves each point along a ray through the point emanating from a fixed center and multiples distances from the center by a common scale factor.

Discrete – structures of sets of values that are countable or otherwise distinct and separable.

Domain – the set of all possible inputs for a function; the set of possible values for the independent variable.

Dot Plot – a chart that shows data points as dots on a graph. Dot plots are similar to a histogram or bar graph but use dots instead of rectangular bars.

Elapsed Time – the amount of time that has passed between the beginning and end of an event.

Equal – the same in all aspects.

Equality – a relationship between two quantities (or more generally, two mathematical expressions) asserting that the quantities have the same value or that the expressions represent the same mathematical object.

Equation – a statement that the values of two mathematical expressions are equal (indicated by the sign =).

Equivalent – two or more expressions that represent the same value but are not identical.

Expand – to multiply each term in the bracket by the expression outside the bracket.

Expanded Form – a multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten (e.g., 643 = 600 + 40 + 3).

Expected Value – for a random variable, the weighted average of its possible values, with weights given by respective probabilities.

Experimental Event – based on actual experiments and recordings of the event.

Exponent – a quantity representing the power to which a given number or expression is to be raised, usually expressed as a raised symbol beside the number or expression. (e.g., 3 in $2^3 = 2 \times 2 \times 2$).

Expression – numbers, symbols, and operators such as + and *x*) grouped together that show the value of something; does not have an equal sign.

Exterior Angle – the angle between a side of a polygon and an extended adjacent side.

Extrapolation – estimating an unknown value based on extending a known sequence of values or facts.

Factor- a number or quantity that divides another number or quantity with no remainder.

First Quartile – for a data set with a median *M,* the first quartile is the median of the data value less than M

Fluency (Computational) – having efficient, flexible, and accurate methods for computing.

Fluency (Procedural) – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.

Fraction – a number expressible in the form $\frac{a}{b}$ where a and b are integers.

Function – a relation from a set of inputs to a set of outputs where each input is related to exactly one output.

Gap – missing areas in a data set.

Geometric Figure – any combination of points, lines, or planes.

Grouping Symbol – symbols used to show that a particular collection of numbers and meaningful operations are grouped together and considered as one quantity.

Greatest Common Factor (GCF) – the largest whole number that divides evenly into two or more whole numbers.

Height – the length perpendicular to the base.

Histogram – a bar graph that shows the frequency for numerical data within equivalent intervals.

Hypotenuse – the side opposite the right angle in a triangle, which is the longest side.

Image – the new position of a point, a line, a line segment, or a figure after a transformation.

Independent Variable – the variable that is stable and unaffected by the other variables measured.

Inequality – a comparison of two values showing if one is less than, greater than, or simply not equal to another value.

Initial Value – the output value when the input/independent variable value is 0.

Input – a value that is substituted into an expression or equation.

Integer – a numerical value in a number system that is not fractional; these numbers include natural numbers, their opposites, and zero.

Intercept – the point where a line crosses an axis.

Interior Angle – the inner of two angles formed where two sides of a polygon come together.

Interpolation – determining a value from the existing values in a data set.

Interquartile Range – a measure of variation in a set of numerical data based on splitting the data into quartiles. The interquartile range defines the difference between the third and first quartile.

Inverse – operation or function that undoes another operation or function.

Irrational Number – a real number that cannot be made by dividing two integers.

Kite – a quadrilateral with two pairs of consecutive congruent sides.

Least Common Multiple (LCM) – the smallest common non-zero number which is a multiple of two or more numbers.

Leg – one of the sides of the triangle.

Like Terms – terms that have identical variable parts and the same exponents.

Line Plots – a type of graph that shows data as symbols (e.g., dots, crosses, or check marks) above a number line.

Linear Expression – an algebraic expression with the highest exponent of a variable being 1.

Linear – a mathematical relationship that can be graphically represented as a straight line.

Linear Model – an equation that describes a relationship between two quantities that show a constant rate of change.

Linear Pair – two adjacent angles that add up to180°.

Linear Relationship – any relationship between two variables creating a straight line when graphed on a coordinate plane.

Line of Reflection – a line that acts as a mirror for a figure or a pre-image in a plane.

Mathematical Proof – a carefully reasoned argument for verifying a conjecture that would meet the standards of the broader mathematics community.

Mathematics in Context – mathematics in context emphasizes the dynamic, active nature of mathematics and the way mathematics enables learners to make sense of their world.

Mean – a measure of center in a set of numerical data computed by adding the values in a list and dividing by the number of values in the list.

Mean Absolute Deviation – a measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values.

Measurement Data – data in which the values result from measuring.

Median – middle value in a given set of numbers or data.

Midline – in the graph of a trigonometric function, the horizontal line is halfway between its maximum and minimum values.

Multiplicative Comparison – comparing two quantities showing that one quantity is a specified number of times larger or smaller than the other quantity.

Net – a pattern that can be cut and folded to make a model of a solid shape.

Nonlinear – a mathematical relationship that is not a straight line when represented graphically.

Nonnegative Rational Numbers – the positive rational numbers and zero.

Number Line – a pictorial representation of numbers on a straight line.

Numerical Expression – a statement that involves only numbers and one or more mathematical operations.

One-to-one Correspondence – when counting objects, number names are said in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

Opposite Numbers – numbers that are in opposite positions on the number line.

Order of Operations – a set of rules to be followed in a particular sequence while solving an expression.

Outlier – a data point that lies outside the overall pattern in a distribution.

Output – the amount produced by a mathematical process.

Parallel – lines, planes, surfaces, or objects that are always side by side and have the same distance continuously between them.

Parallelogram – a quadrilateral with two pairs of parallel lines.

Parameter – a constant or variable in a mathematical expression that distinguishes various specific cases. (Example: In the equation y = mx + b , m and b are parameters which specify the particular straight line represented by the equation.).

Partition – divide into pieces.

Percent – parts per hundred

Percent Rate of Change – the rate of change expressed as a percent.

Perfect Cube – a product of three identical factors.

Perfect Square – a product of two identical factors.

Polygon – a closed two-dimensional shape with three or more sides made up of line segments (not curves) that do not cross each other.

Perimeter – the distance around a two-dimensional shape.

Perpendicular Lines – two lines that meet or intersect at right angles.

Pi $(\pi\pi)$ **–** a mathematical constant that is the ratio of a circle's circumference to its diameter.

Pictograph – a representation of data using images or symbols.

Point of Rotation – the central point around which an object is rotated.

Population – a pool of individuals from which a statistical sample is drawn for a study.

Precision – the amount of information conveyed by a number (in terms of the number of digits).

Pre-Image – the image or a graph or figure before it has been transformed.

Prism – a three-dimensional solid object with two identical ends and flat sides.

Probability – the likelihood something will occur.

Probability Distribution – the set of probable values of a random variable with a probability assigned to each.

Probability Model – a mathematical representation of a random phenomenon that is defined by its sample space, events within the sample space, and probabilities associated with each event.

Proportional Relationship – a relationship in which two quantities vary directly with each other (if one increases, the other increases; if one decreases, the other decreases). In a direct proportional relationship, a dependent variable (y) is equal to a constant (k) times an independent variable (x). Formula: y = kx .

Pyramid – a three-dimensional shape in which the sides are triangles that meet at the top and the base is a polygon.

Pythagorean Theorem – the sum of the squares on the legs of a right triangle is equal to the square on the hypotenuse.

Quadrilateral – a polygon with four edges (sides) and four vertices (corners).

Radius – describes the distance from any point on the circle to a fixed point, called the center. A radius is half the diameter.

Random – happening or chosen by chance or without method or conscious decision.

Random Sample – a sampling method in which the researcher randomly selects a subset of participants from a population.

Random Variable – a rule that assigns a numerical value to each outcome in a sample space.

Range (function) – the set of all possible outputs for a function; the set of possible values for the dependent variable.

Range (data and statistics) – the difference between the lowest and highest values.

Rate – a comparison of two related quantities.

Rate of Change – measure of how one variable changes in relation to another variable over a period of time.

Formula: Rate of Change $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

Radian – the ratio of the length of the arc to the length of the radius of a circle.

Ratio – the quantitative relation between two amounts showing the number of time one value contains or contained within the other.

Rational Expression – a quotient of two polynomials with a non-zero denominator.

Rational Number – a number that can be expressed as a fraction or a ratio of two integers.

Real Number – the union of both rational and irrational numbers which can be both positive and negative and are denoted by the symbol "ℝ".

Reciprocal – the inverse of a value or number.

Rectangle – a quadrilateral with four right angles.

Reflection – a transformation in which a shape or geometric figure is mirrored across a line or plane.

Relation (Between Two Sets) – a collection of ordered pairs containing one object from each set.

Relative Frequency – the ratio of the number of times an event occurs to the total number of possible outcomes.

Repeating Decimal – a decimal number with a digit (or group of digits) that repeats forever.

Residual – the difference between the observed value of a quantity and its predicted value.

Rhombus – a quadrilateral whose four sides are all the same length.

Right Triangle – a triangle in which one interior angle is 90°.

Rigid Motion – one or more transformations where the image and pre-image are congruent.

Rotation – the circular motion of a shape or geometric figure around a fixed point a certain number of degrees.

Same Side Exterior Angles – a pair of angles on the same side of a transversal and outside the two intersecting lines.

Same Side Interior Angles – a pair of angles on the same side of the transversal and inside the two intersecting lines.

Sample (Statistics) – a subset of a larger population selected to estimate characteristics of the whole.

Sample Space – a list of individual outcomes to be considered in a probability model for a random process.

Scale - the ratio of the length in a drawing or model to the length of the actual object.

Scale Drawing – a drawing showing an object with accurate sizes reduced or enlarged by a certain scale.

Scatter Plot – a graph in which the values of two variables are plotted along two axes in the coordinate plane, the pattern of the resulting points reveals correlations between the variables.

Scientific Notation – a way to represent numbers too large or too small to conveniently be written in decimal form. This representation uses a number from 1 up to (but not including) 10 times an integer power of 10.

Shares – groups, sets, parts, or partitions.

Similar – two figures which are proportional to each other.

Skew – data that creates an asymmetrical curve on a graph.

Slope – the ratio between the vertical change and horizontal change of a line.

Standard Algorithm – a step-by-step procedure specifying how to solve a problem.

Solution – the value(s) which make the equation true when substituted for a variable in an equation.

Solution Set – the set of values that satisfy a given set of equations or inequalities.

Sphere – a three-dimensional solid in which every point on the surface is the same distance from the center.

Square – quadrilateral with four sides of equal length and four right angles.

Square Root – one of two identical factors of a given number.

Standard Deviation – the measure of how dispersed the data is in relation to the mean.

Substitution – the process of replacing a variable with an equivalent expression.

Supplementary Angles – two angles that add up to 180°.

Surface Area – the total area of the surface of a three-dimensional object.

Tape Diagram – a rectangular visual model resembling a piece of tape used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Term – single numbers, variables, or the product of a number and variables.

Terminating Decimal – a decimal with a finite number of digits.

Theoretical Event – an expected probability based on knowledge of the situation.

Third Quartile – the number halfway between the middle number and the highest number (Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15). See also: median, first quartile, interquartile range.

Translation – a rigid transformation that moves every point of the figure, shape, or space by the same distance in a given direction without changing the size or shape of the original figure.

Transformation – a process that manipulates a polygon or other two-dimensional object on a plane or coordinate system (rotations, reflection, translation, dilation).

Transversal – a line that crosses at least two other lines.

Trapezoid – a quadrilateral with at least one pair of parallel sides

Trend Line – a line on a graph showing the general direction that a group of points appear to follow.

Two-Way Tables – a table used to organize data based on two categorical variables.

Uniform Probability Model – a probability model which assigns equal probability to all outcomes. See also: probability model.

Unit Rate – a ratio between two separate but related measurements with the second term reduced to a value of one.

Variable – a symbol, usually a letter, which represents an arbitrary or unknown element that can change or take on other values.

Vertical Angles – a pair of angles that are opposite each other and share the same vertex when two-line segments intersect. These angles are always equal.

Vertex – a point where two or more lines or edges meet.

Visual Fraction Model – a tape diagram, number line diagram, or area model.

Volume – a measure of space that a three-dimensional object occupies.

Whole Numbers – numbers starting at 0 and counting up forever. Whole numbers do not include negative numbers, fractions, or decimals.

Y-Intercept – the point where a line or curve crosses the y-axis of a graph.

APPENDIX A

MATHEMATICAL PROPERTIES AND ORDER OF OPERATIONS

Table 1. Properties of Operations

Property	Definition	Example
Associative Property of Addition	The mathematical property states that the grouping of the addends can be changed, resulting in the same sum.	$(a + b)+ c=a + (b + c)$
Commutative Property of Addition	The mathematical property states that the addends can be reversed and result in the same sum.	$a + b = b + a$
Additive Identity Property of 0	The mathematical property states adding 0 to any number does not change its value.	$a + 0 = 0 + a = a$
Existence of Additive Inverses	The mathematical property states that the sum of a number and its negative (the additive inverse) is always 0.	$a + (-a) = (-a) + a = 0$
Associative Property of Multiplication	The mathematical property states that the grouping of the factors can be changed, resulting in the same product.	$(a \times b) \times c = ax(b \times c)$
Commutative Property of Multiplication	The mathematical property states that the factors can be reversed and result in the same product.	$a \cdot b = b \cdot a$
Multiplicative Identify Property of 1	The mathematical property states that multiplying any number by 1 does not change its value.	$1 \cdot a = a \cdot 1 = a$
Existence of Multiplicative Inverses	The mathematical property states that any number not equal to zero multiplied by its multiplicative inverse will equal 1. Also known as **reciprocals**.	$a \times \frac{1}{a} = \frac{1}{a} \times a = 1$ $a \neq 0$
Distributive Property of Multiplication Over Addition	The mathematical property states that when a factor is multiplied by the sum/addition of two terms, it is essential to multiply each of the two numbers by the factor and finally perform the addition operation.	$a \times (b + c) = a \times b + a \times c$

a, b, and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Table 2. Properties of Equality

Property	Definition	Example
Reflexive Property of Equality	The mathematical property which states that a number is always equal to itself.	a = a
Symmetric Property of Equality	The mathematical property states that both sides of an equation are the same.	If a = b, then b = a
Transitive Property of Equality	The mathematical property states that if two values are equal, and either of those values is equal to a third value, all the values must be equal.	If a = b and b = c, then a = c
Addition Property of Equality	The mathematical property states that when the same number is added to both sides of an equation, the equation does not change.	If a = b, then a + c = b + c
Subtraction Property of Equality	The mathematical property states that when the same number is subtracted from both sides of an equation, the two sides will still be equal.	If a = b, then a - c = b - c
Multiplication Property of Equality	The mathematical property that states if both sides of an equation are multiplied by the same number, the expressions on both sides of the equation remain equal.	If a= b, then a x c = b x c
Division Property of Equality	The mathematical property states that dividing both sides of an equation by the same number does not affect the equation.	If a = b and $c \neq 0$, then $a \div c = b \div c$
Substitution Property of Equality	The mathematical property states that if two quantities are equal, then either can replace the other in any equation or expression.	If a = b, then b may be substituted for a in any expression containing a.

a, b, and c stand for arbitrary numbers in rational, real, or complex number systems.

Table 3. Properties of Inequality

Property	Definition	Example
Transitive Property of Inequality	The mathematical property states that if a, b, and c are three quantities, and if a is related to b by the same rule, and b is related to c by the same rule, then a is related to c by the same rule.	If $a < b$ and $b < c$, then $a < c$ If $a > b$ and $b > c$, then $a > c$
Reversal Property of Inequality	The mathematical property states that an inequality (the entire expression) can be reversed without affecting the validity of the expression.	If $a > b$, then $b < a$ If $a > b$, then $b < a$
Law of Trichotomy Property of Inequality	The mathematical property states that when any two numbers are compared, there are only three possible outcomes: *a<b* or *a=b* or *a>b.*	Only one of the following is true: $a < b$ or $a = b$ or $a > b$
Addition & Subtraction Property of Inequality	The mathematical property states that if a number is added or subtracted to one side of an inequality, the same number must also be added or subtracted to the opposite side so that the inequality remains satisfied.	If $a < b$, then $a + b < b + c$ If $a > b$, then $a + b > b + c$ If $a < b$, then $a - b < b - c$ If $a > b$, then $a - b > b - c$
Multiplication Property of Inequality	The mathematical property states that if both sides of an inequality are multiplied or divided by the same positive number, it will result in an equivalent inequality. When multiplied by a negative number, the inequality reverses.	If $a < b$, and c is **positive**, then $ac < bc$ If $a < b$, and c is **negative**, then $ac > bc$ If $a > b$, and c is **positive**, then $a \div c > b \div c$ If $a > b$, and c is **negative**, then $a \div c < b \div c$
Additive Inverse Property of Inequality	The mathematical property states if the additive inverse of both sides of the inequality is taken, the direction of the inequality symbol reverses.	If $a < b$, then $-a > -b$ If $a > b$ then $-a < -b$
Multiplicative Inverse Property of Inequality	The mathematical property states if given that the values on either side of the equation are both positive or both negative, the multiplicative inverse (reciprocal) is taken on both sides, and the direction of the inequality symbol is reversed.	If $a < b$ then $\frac{1}{a} > \frac{1}{b}$ If $a > b$ then $\frac{1}{a} < \frac{1}{b}$
Non-Negative Property of Squares	The mathematical property states a square of a number is greater than or equal to 0.	$a^2 \geq 0$
Square Root Property of Inequality	The mathematical property states that the square root of a number will not change the inequality when both numbers are greater than or equal to 0.	If $a \leq b$ then $\sqrt{a} \leq \sqrt{b}$ (For $a, b \geq 0$)

Table 4. Order of Operations

Step 1	**( ) { } [ ] | |**	**PARENTHESES AND GROUPING SYMBOLS**
Step 2	**X^2 and $\sqrt{x}$**	**EXPONENTS AND RADICALS**
Step 3	**X and ÷**	**MULTIPLY AND DIVIDE**
Step 4	**+ and -**	**ADD AND SUBTRACT**

APPENDIX B

RECOMMENDED AUTOMATICITY FOR MATHEMATICS CONTENT STANDARDS

Grade	Recommended Automaticity
K	Add and subtract within 5 (K.AR.OA.1)
1	Add and subtract within 10 (1.AR.OA.1)
2	Add and subtract within 20 (2.AR.OA.1)
3	Multiply and divide up to 5 x 5 and 10's facts (3.AR.OA.1)
4	Multiply and divide up to 10 x 10 (4.AR.OA.1)
5	Multiply and divide up to 12 x 12 (5.AR.OA.1)
6	GCF of two whole numbers up to 100 and LCM of two whole numbers up to 12 (6.NO.O.4)
7	Common fraction and decimal equivalencies up to a denominator of 10 (7.NO.NS.2)
8	Roots of perfect squares up to 225 and roots of perfect cubes up to 1000 (8.NO.O.1)

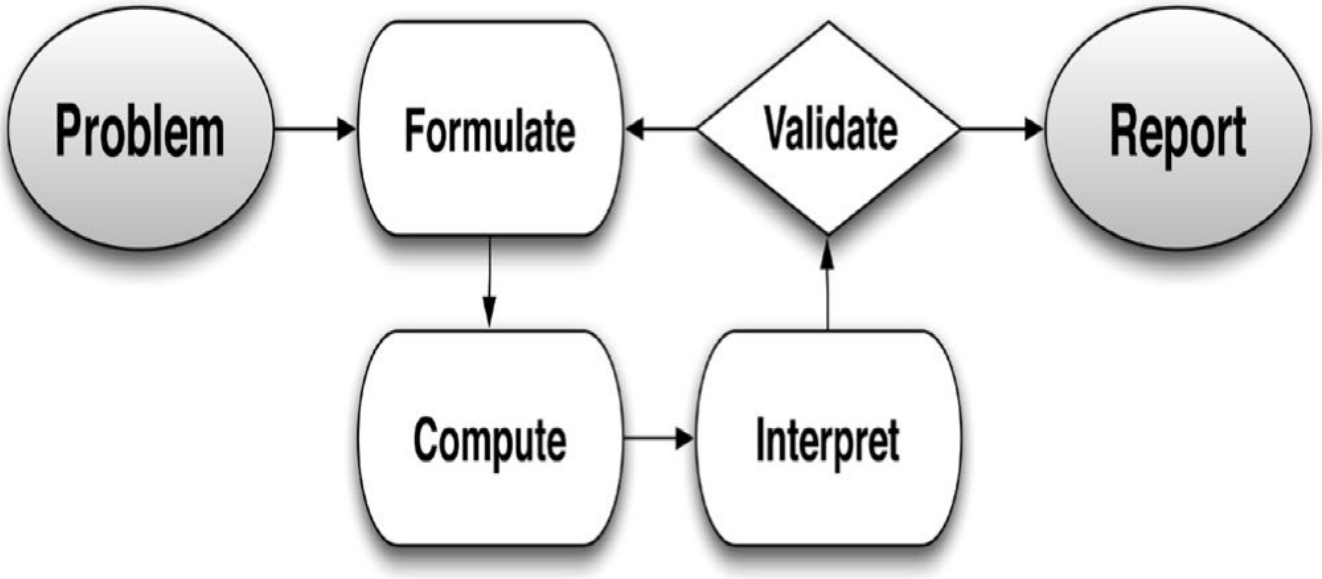
APPENDIX C

MODELING MATHEMATICAL CONCEPTS

Many of the Mathematics Content Standards include modeling mathematical concepts, which help learners apply their understanding in authentic situations. Mathematical modeling follows a process that goes beyond traditional word problems. This process requires learners to translate authentic situations into mathematical structures, make predictions, and perform operations on those structures. Learners evaluate and interpret the results in context while also analyzing the validity of their model.

The modeling standards describe five different actions that learners take over the course of a complete modeling task:

1. Identifying essential variables in a situation
2. Formulating models and making predictions from those variables
3. Computing operations using those models
4. Interpreting the results of those operations
5. Validating the conclusions of those results



APPENDIX D

CLASSIFICATION OF QUADRILATERALS

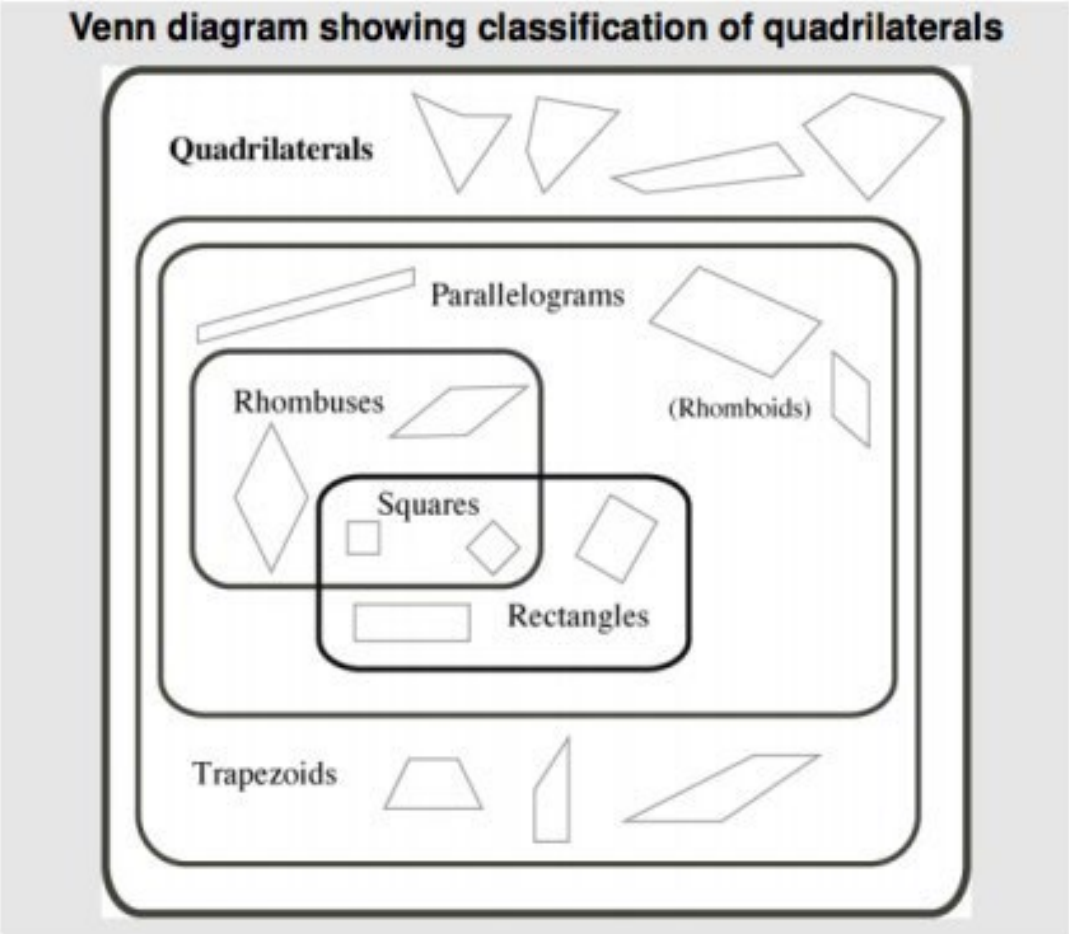


Figure	Defining Characteristic
Quadrilateral	A polygon with 4 sides
Trapezoid	A quadrilateral with **at least** 1 pair of parallel opposite sides
Parallelogram	A quadrilateral with 2 pairs of parallel sides
Rectangle	A quadrilateral with 4 right angles
Rhombus	A quadrilateral with 4 congruent sides
Square	A quadrilateral with 4 congruent sides and 4 right angles.

NOTE: Rhomboids are parallelograms that are not rhombuses or rectangles. This example uses the exclusive definitions of a trapezoid.

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