NORTH DAKOTA DEPARTMENT OF
PUBLIC INSTRUCTION

NORTH DAKOTA MATHEMATICS CONTENT STANDARDS PROGRESSIONS GUIDE GRADES K-12

AUGUST 2024

North Dakota Department of Public Instruction
Kirsten Baesler, State Superintendent
600 East Boulevard Avenue, Dept. 201
Bismarck, North Dakota 58505-0440
www.nd.gov/dpi

Contents

INTRODUCTION

The teachers who wrote the 2023 Mathematics Content Standards collaborated with the North Dakota Department of Public Instruction to design a standards resource document showing the connections between the standards. This guide is intended to provide a resource for districts and teachers as they plan curriculum and instruction.

The guide provides the following information for each standard:

Preceding Standard(s)
Standards that should be taught before the identified standard. Sometimes, these come from the prior grade level. In other cases, the standard may go further back. This information is helpful in identifying where prerequisite standards were taught.

Related Standard(s)
Standards within the grade level that use skills/concepts within the identified standard and support or are supported by the identified standard. In a classroom, these standards may be taught simultaneously or next to each other because they build upon each other.

Successive Standard(s)
Standards that are taught at the following level or that build directly upon the skills/concepts within the identified standard.

Prerequisite Skills
This area contains important skills and concepts without which the learner may struggle with the identified standard. The North Dakota Standards Based Learning Proficiency Scales were used to assist with generation of prerequisite skills for the priority standards identified by that group.

Key Vocabulary
The vocabulary words identified are terms teachers and/or learners need to understand to either teach or become proficient with the identified standard. This area also highlights mathematical notations and representations needed for the identified standard. Terms to be used by teachers for instruction are noted with a (T). The North Dakota Standards Based Learning Proficiency Scales were used to assist with generation of vocabulary for the priority standards identified by that group.

Instructional Notes and Common Learner Misconceptions/Errors
The category includes instructional notes (IN) to assist teachers. This area contains common misconceptions and errors (ME) learners make when working on the identified standard. Artificial Intelligence (AI) was used to assist with wording and generation of common errors.

Courses
This information aligns with the work to align the course codes to the standards. It shows potential courses in which the standards may be taught. This area aligns with the work done to align the courses in the North Dakota Course Code document with the standards.

HOW TO READ THIS DOCUMENT

EIGHTH GRADE — Grade

Operations (O) — Category *Learners will expand their computational fluency to create connections and solve problems within and across concepts.*		
8.NO.O.1 Evaluate mentally the square roots of perfect squares up to 225 and cube roots of perfect cubes up to 1000. — Sub-category		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
6.AR.EE.1 7.NO.O.1	8.NO.NS.1 8.NO.NS.2 8.AR.EE.2 8.GM.GF.6	9-10.NO.1 9-10.NO.2 9-10.AR.10 — Standard
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/Errors (ME)**
• Identify the symbol for numbers squared (^{2}) and numbers cubed (^{3}). • Know how to square a number. • Know how to find the area of a square and the volume of a cube.	• square roots • perfect squares • cube roots • perfect cubes • radical • radicand • index • base • exponent Notation • square root ($\sqrt{}$) and cube root ($\sqrt[3]{}$)	• This is the first time learners are evaluating radicals so connections should be made to area of squares and volume of cubes. (IN) • Learners may double the number instead of raising it to the 2nd power. (ME) • Learners may multiply the base by 3 instead of raising it to the 3rd power. (ME)
Courses		
Mathematics 8 Prealgebra		

MATH STANDARDS PROGRESSIONS COMMITTEE

Brent Aasby *Fargo Public Schools*	*Rebecca Jordan* *McKenzie Co. Public Schools*	*Tara Morse* *Glenburn Public Schools*
Michelle Bertsch *Fargo Public Schools*	*Megan Kratcha* *West Fargo Public Schools*	*Nicole Seyfried* *West Fargo Public Schools*
Kimberly Carpenter *Jamestown Public Schools*	*Kayla Lee* *West Fargo Public Schools*	*Elsie Sheldon* *Bismarck Public Schools*
Samantha Jones *Minot Public Schools*	*Jill Leier* *West Fargo Public Schools*	

MATH ATTRIBUTES

The math attributes contained in these standards summarize the mathematical practices found in the 2017 North Dakota Mathematics Content Standards and align with the 2022 North Dakota Learning Continuum. These attributes will help learners solve authentic problems while connecting concepts, providing supporting evidence, explaining the reasoning and efficiency of strategies used, and proving the accuracy of solutions. The three attributes identified will be used by learners throughout their education and future careers. The attributes are arranged by grade span. The chart below shows the progression of the Math Attributes.

Math Attributes (MA)

Learners will practice and demonstrate broad, transferable, and enduring skills necessary for advancement through participation in various relevant learning experiences.

Problem-Solving (P)	Connections (C)	Reasoning and Proof (R)
Analyze, execute, evaluate, and adapt approaches and solutions when solving novel situations.	*Create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.*	*Reason logically, citing relevant evidence to explain and critique what they see, think, and conclude through exploration, generalization, and validation.*
K-2.MA.P Learners can identify and use strategies to problem-solve situations and determine an appropriate solution.	**K-2.MA.C** Learners can use prior knowledge and experiences to explain their thinking.	**K-2.MA.R** Learners can use prior knowledge and experiences to explain their thinking.
3-5.MA.P Learners can develop and carry out a logical plan to problem-solve situations, reflect on the reasonableness of solutions, and explore alternate strategies with guidance.	**3-5.MA.C** Learners can make connections and summarize related ideas using supporting evidence.	**3-5.MA.R** Learners can reason logically based on experience and knowledge, citing evidence to support their reasoning and conclusions.
6-8.MA.P Learners can analyze information and formulate a flexible, systematic plan to problem-solve authentic situations and reflect on the reasonableness of the solution, making revisions when necessary.	**6-8.MA.C** Learners can create connections within and across concepts and provide examples of how they relate to other learning and ideas using supporting evidence	**6-8.MA.R** Learners can reason logically, citing evidence to evaluate and explain what they see, think, and conclude through exploration and justification.
9-12.MA.P Learners can analyze, execute, critique, and adapt approaches and solutions when problem-solving in novel situations.	**9-12.MA.C** Learners can create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences.	**9-12.MA.R** Learners can reason logically, citing evidence to critique and explain what they see, think, and conclude through exploration, generalization, and validation.
Lifelong MA.P Learners can integrate their cumulative knowledge and life experiences to discern and prioritize information in authentic situations, consider and apply alternative methods of resolution, and evaluate the relevance, efficacy, and accuracy of solutions.	**Lifelong MA.C** Learners can apply connections and develop generalizations within and across concepts to execute effective decision-making or generate new ideas.	**Lifelong MA.R** Learners can reason logically to discern the validity of information and synthesize it to formulate, investigate, and critique claims and evidence.

(2022 North Dakota Learning Continuum)

KINDERGARTEN

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

Counting and Cardinality (CC)

Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

K.NO.CC.1 Count verbally in sequential order by ones and tens to 100, making accurate decuple transitions (e.g., 89 to 90). Count verbally forward from any given number within 100.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
Early Learning Standards Goal P.MATH.1: Child knows number names and the count sequence (up to 10).	K.NO.CC.3 K.NO.CC.5	K.NO.CC.2 K.NO.CC.5 K.NO.NBT.1 1.NO.CC.1 1.NO.CC.3 1.NO.CC.5

Prerequisite Skills	Key Vocabulary	Common Misconceptions (and Instructional Notes (IN)
• Say number names in sequence within 10. • Count forward to 30 verbally by ones. • Recognize tens and ones. • Recognize and name the tens families (10s, 20s, 30s, etc.). • Count verbally to 50 by tens. • Count forward from any given number within 30. • Name the number word after within 100.	• count • ones • tens • decuple • forward • backward • number	• Learners who know number names in standard order may not be proficient in their understanding of addition or subtraction. (IN) • Some learners might need to review basic understanding of number words after completing more advanced tasks. (IN) • Learners may have difficulty with the different structures of teens and twenties. (ME) • Learners may have difficulty progressing into the next decade, e.g., after 29. (ME) • Learners might confuse the pronunciation of teens and decuples, e.g., fifty instead of fifteen. (ME) • Learners who can say the sequence of forward number word sequence might not be able to identify the number word after. (ME) • Learners who have difficulty solving the number word after tasks might use a dropping-back strategy. This involves saying a forward number word sequence to find the number word before. (ME)

Courses
Mathematics K

KINDERGARTEN

Counting and Cardinality (CC)
Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

K.NO.CC.2 Count backward from 20 by ones and from a given number within 10.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.NO.CC.1 (Knowledge within 20)	K.NO.CC.3 K.NO.CC.5	K.NO.NBT.1 1.NO.CC.2 1.NO.CC.5
Prerequisite Skills	**Key Vocabulary**	**Common Misconceptions (Teacher and Learner) and Instructional Notes (IN)**
• Count forward by ones verbally to 20. • Count backward by ones verbally from 10. • Name the number word within 10.	• count • backward • ones	• Learners do not need to be fully proficient in K.NO.CC.1 to be successful in K.NO.CC.2. Forward counting sequentially within 20 would support backward counting from 20. (IN) • Learners who can recite forward number word sequences might have difficulty reciting backward number word sequences with the teens. (ME) • Learners might omit a word in the backward number sequence that they do not omit in the forward number sequence. (ME) • Learners might use a dropping-back strategy. This involves saying a forward number word sequence to find the number word before. (ME) • Learners might confuse the number word before with the number word after. (ME)

Courses
Mathematics K

KINDERGARTEN

Counting and Cardinality (CC)
Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

K.NO.CC.3 Identify and write any given numeral within 20.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
Early Learning Standards Goal P.MATH.5: Child associates a quantity with written numerals up to five and begins to write numbers.	K.NO.CC.1 K.NO.CC.2 K.F.13 (ELA) Write uppercase and lowercase letters accurately.	K.NO.CC.5 K.NO.NBT.1 1.NO.CC.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Differentiate between a letter and a numeral. • Hold a writing utensil. • Trace numerals. • Recognize numerals within 10. • Identify numerals within 10. • Write numerals within 10. • Identify numbers sequentially within 20. • Write numbers sequentially within 20.	• numeral	• In a progression, learners may identify and write numerals sequentially prior to being able to identify and write any numeral within 20. (IN) • Although standards K.F.13 and K.NO.CC.3 are not directly related, the skills within a progression would be similar to accurately forming numerals. (IN) • Recognition includes providing numerals for learners to select from, whereas identification would involve producing the name for the numeral they are given. (IN) • Learners might mix up letters and numbers. (ME) • Learners might be able to recognize but not identify numerals. This means they can choose the correct numeral when shown choices but not identify it in isolation. (ME)

Courses
Mathematics K

KINDERGARTEN

Counting and Cardinality (CC) *Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.*
K.NO.CC.4 Recognize and verbally label arrangements, without counting, for briefly shown collections up to 10 (e.g., “I saw 5.” How do you know?” “I saw 3 and 2, that is 5.”).

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
Early Learning Standards Goal P.MATH.2: Child recognizes the number of objects in a small set. **Early Learning Standards Goal P.MATH.2:** Quickly recognizes the number of objects in a small set (referred to as "subitizing").	K.NO.CC.5 K.AR.OA.1	K.NO.NBT.1 1.NO.CC.4
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Know and say number names within 10. • Instantly recognize, without counting, small quantities up to 5 in an arranged pattern and say the number.	• label (T) • arrangements (T) • collections (T) • subitizing (T)	• Recognize arrangements without counting. Use scattered arrangements for combinations up to 7. Structured arrangements such as ten frames (utilizing 5+ and double patterns) can be utilized for combinations up to 10. (IN) • This is referred to as subitizing. (IN) • Learners may see groups of numbers like the example in the standard, but they are not counting each item by one. (IN) • Learners might have difficulty recognizing the number of objects if arranged in an unfamiliar or irregular pattern. (ME)

Courses
Mathematics K

KINDERGARTEN

Counting and Cardinality (CC)
Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

K.NO.CC.5 Count and tell how many objects up to 20 are in an arranged pattern or up to 10 objects in a scattered configuration. Represent a quantity of up to 20 with a numeral.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
Early Learning Standards Goal P.MATH.3: Understands that number words refer to quantity. May point to or move objects while counting objects to 10 and beyond (one-to-one correspondence). Understands that the last number represents how many objects are in a group (cardinality). K.NO.CC.1 K.NO.CC.3	K.NO.CC.1 K.NO.CC.2 K.NO.CC.4 K.NO.NBT.2 K.AR.OA.1 K.DPS.D.1	K.NO.NBT.1 K.NO.NBT.2 K.AR.OA.1 K.AR.OA.4 K.AR.OA.5 1.NO.CC.5 1.AR.OA.4 1.AR.OA.5

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Know and say number names within 20. • One-to-one correspondence. • Cardinality - understand that the last number name said tells the number of objects counted, regardless of their arrangement or order in which they were counted. • Determine the quantity of a collection within 20. • Given a number from 1-20, count out that number of objects.	• count • pattern • scattered (T) • numeral • quantity	• Standard K.NO.CC.4 does not need to be mastered before instruction begins on standard K.NO.CC.5 because learners are counting their quantities compared to subitizing a collection. (IN) • Learners might lose their place when counting objects. (ME) • Learners may not understand that the last number counted represents the total number of objects. (ME) • Learners might omit a number word or incorrectly coordinate number words and items. (ME) • Learners might make coordination errors which include saying one word for more than one item or saying more than one word for one item. (ME)

Courses
Mathematics K

KINDERGARTEN

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

K.NO.NBT.1 Compose and decompose numbers from 11 to 19 using a group of ten ones and some more ones using a model, drawing, or equation.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.NO.CC.1 K.NO.CC.2 K.NO.CC.3 (if using an equation) K.NO.CC.4 K.NO.CC.5	K.AR.OA.3	K.NO.NBT.2 1.NO.CC.4 1.NO.NBT.1 1.AR.OA.2 1.AR.OA.4 1.AR.OA.5

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Count forward and backward by ones. • Identify numerals. • Write numerals. • Identify ones and tens places. • Compose numbers up to 10 using a model, drawing, or equation. • Decompose numbers up to 10 using a model, drawing, or equation. • Represent numbers from 11-19 using individual objects without using a group of 10.	• compose (T) • decompose (T) • group • tens • ones • place value (T) • model • equation	• Learners need to practice building quantities by one before they move to composing and decomposing quantities using the structure of numbers. (IN) • Learners may not conceptually understand the value of the ten represented by a ten or a bundle of ten sticks. They might see all representations as one. (ME) • Learners might lose track when counting the ones after identifying the group of ten. (ME)

Courses
Mathematics K

KINDERGARTEN

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

K.NO.NBT.2 Compare two numbers between 1 and 20 using words greater than, less than, or equal to.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
Early Learning Standards Goal P.MATH.4.48-60: Counts to determine and compare number amounts even if the larger group's objects are smaller in size, such as buttons, compared with the smaller group's objects that are larger in size, such as markers. Uses numbers related to order or position. K.NO.CC.5 K.NO.NBT.1	K.NO.CC.5 K.DPS.D.1	1.NO.NBT.2 1.DPS.D.2

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Count objects. • One-to-one correspondence. • Understands terms greater than, less than, and equal to. • Compare groups of objects using terms greater than, less than, and equal to. • Compare two numbers between 1 and 10 using terms greater than, less than, and equal to.	• compare • greater than • less than • equal to	• In a progression, learners will use groups of objects for comparison prior to comparing numerals. (IN) • Learners' understanding of quantity and numeral comparison can begin in a smaller range as they develop proficiency within 20. (IN) • Learners are learning the meaning of comparing quantities and numerals through the language of greater than, less than, or equal to but are not using the symbols to show comparison. (IN) • Learners might get the terminology greater than and less than mixed up. (ME) • Learners may overly rely on rote counting sequences and inaccurately compare the quantities. (ME)

Courses
Mathematics K

KINDERGARTEN

Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*		
Operations and Algebraic Thinking (OA) *Learners will analyze patterns and relationships to generate and interpret numerical expressions.*		
K.AR.OA.1 Automatically add and subtract within 5.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
Early Learning Standards Goal P.MATH.6: Child understands addition as adding to and subtraction as taking away from. K.NO.CC.5	K.NO.CC.4 K.NO.CC.5 K.AR.OA.2 K.AR.OA.3 K.AR.OA.4 K.AR.OA.5	1.NO.NBT.3 1.AR.OA.1
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Represent addition and subtraction in different ways. • Count forwards from 0 to 5. • Count backwards from 5 to 0. • Compose and decompose numbers less than or equal to 5 in pairs more than one way (verbal explanation, objects, drawing). • Count on from a larger number to add. • Count back from a larger number to subtract. • Develop a flexible understanding of vertical and horizontal orientation.	• add • subtract • sum • automatically	• Learners are fluent in their understanding of numbers within 5, not just of 5. Their addition and subtraction of all numbers within 5 is without conscious thought or attention; thus, they are not counting by ones. (IN) • Timed tests are not the only way to assess the automaticity of learners' combinations within five. (IN) • Learners may be fluent in addition and not make the connection with subtraction. (IN) • While using counting to subtract, discrete learners may use a counting sequence different than an interval counter. This is not wrong; it is just a different way to solve it. For example, given 5 - 2, a discrete counter will say, "5, 4, so the answer is 3." An interval counter will say, " 5...4, 3, so the answer is 3. (IN) • Learners may assume that 1 + 4 has a different answer than 4 + 1 because the numbers are in a different order. (ME)
Courses		
Mathematics K		

KINDERGARTEN

Operations and Algebraic Thinking (OA) *Learners will analyze patterns and relationships to generate and interpret numerical expressions.*
K.AR.OA.2 For any number from 1 to 9, find the number that makes 10 when added to the given number, sharing the answer with a model, drawing, or equation.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
Early Learning Standards Goal **P.MATH.6:** Child understands addition as adding to and subtraction as taking away from.	K.AR.OA.1 K.AR.OA.3 K.AR.OA.4 K.AR.OA.5	1.NO.NBT.3 1.AR.OA.1 1.AR.OA.2 1.AR.OA.3 2.AR.OA.1
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Use one-to-one correspondence. • Demonstrate understanding of addition as adding to. • Demonstrate understanding of quantity of 10. • Find different ways to make numbers within 5. • Add and subtract within 5 (vertically and horizontally). • Use visual models to support ways of composing and decomposing 10. • Use counting to support ways of composing and decomposing 10.	• added to • model • equation • combination	• Learners may use objects or drawings to find numbers from 1 to 9 to make 10. (IN) • Learners will find less difficulty when the unknown number is 1 or 2. (IN) • Learners who hesitate to respond may be quickly counting to solve. (IN) • Learners who can find combinations and partitions of 10 without counting gain an important foundation for strategies that do not involve counting by ones. (IN) • Learners may not understand that numbers can be broken down and combined in different ways to make a total. (ME)

Courses
Mathematics K

KINDERGARTEN

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

K.AR.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way using verbal explanations, objects, or drawings.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
Early Learning Standards Goal P.MATH.6: Child understands addition as adding to and subtraction as taking away from.	K.NO.NBT.1 K.AR.OA.1 K.AR.OA.2 K.AR.OA.4 K.AR.OA.5	1.AR.OA.1 1.AR.OA.2 1.AR.OA.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Use one-to-one correspondence. • Demonstrate understanding of subtraction as taking away from. • Demonstrate understanding of quantity of 10. • Find different ways to make numbers within 5. • Add and subtract within 5 (vertically and horizontally). • Use visual models to support ways of composing and decomposing 10. • Use counting to support ways of composing and decomposing 10.	• decompose (T) • less than • equal to • pairs	• Learners may rely on the count-by-one strategy to solve. If so, carefully use visual-spatial models to provide support to help them keep track of their count. (ME) • Learners might omit possible pairs or may not recall number pairs. (ME) • When counting, learners may skip numbers or count the same number twice. (ME) • Learners might confuse addition and subtraction. (ME) • Learners may not understand that decomposition involves breaking a number into two or more parts that add up to the original number. (ME)

Courses
Mathematics K

KINDERGARTEN

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

K.AR.OA.4 Solve authentic word problems with addition by putting together or adding to within 10.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
Early Learning Standards Goal P.MATH.6: Child understands addition as adding to and subtraction as taking away from. **Early Learning Standards Goal P.MATH.6.48-60:** Solves addition problems by joining objects together and subtraction problems by separating, using manipulatives and fingers to represent objects.	K.AR.OA.1 K.AR.OA.2 K.AR.OA.3 K.AR.OA.5	1.AR.OA.1 1.AR.OA.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Represent addition and subtraction in a variety of ways. • Demonstrate understanding of addition as adding to or joining objects together. • Add within 5 (vertically and horizontally). • Use counting to solve addition within 10. • Add combinations of numbers within 10. • Solve word problems with addition by adding within 5.	• addition (putting together, adding to)	• At the kindergarten level, objects, drawings, and verbal explanations can be used for teaching as well as learner evidence. (IN) • Build proficiency in solving word problems through the context of the problem rather than underlining or focusing on keywords or numbers. (IN) • Learners who have difficulty with addition within 10 may struggle to connect reading and meaning from a word problem. (ME) • Learners may not be able to visualize the word problems. If so, go back to familiar materials that are consistent and provide needed support. (ME)

Courses
Mathematics K

KINDERGARTEN

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

K.AR.OA.5 Solve authentic word problems with subtraction by taking apart or taking from within 10.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
Early Learning Standards Goal P.MATH.6: Child understands addition as adding to and subtraction as taking away from. **Early Learning Standards Goal P.MATH.6.48-60:** Solves addition problems by joining objects together and subtraction problems by separating, using manipulatives and fingers to represent objects.	K.AR.OA.1 K.AR.OA.2 K.AR.OA.3 K.AR.OA.4	1.NO.NBT.4 1.AR.OA.1 1.AR.OA.5

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Represent addition and subtraction in a variety of ways. • Demonstrate understanding of subtraction as taking away from or separating. • Subtract within 5 (vertically and horizontally). • Use counting to solve subtraction within 10. • Subtract combinations of numbers within 10. • Solve word problems with subtraction by taking from/taking apart within 5.	• subtraction (taking from, taking apart)	• At the kindergarten level, objects, drawings, and verbal explanations can be used for teaching as well as learner evidence. (IN) • Build proficiency in solving word problems through the context of the problem rather than underlining or focusing on keywords or numbers. (IN) • Learners who have difficulty with subtraction within 10 may struggle to connect reading and meaning from a word problem. (ME) • Learners might not be able to visualize the word problems. If so, go back to familiar materials that are consistent and provide needed support. (ME)

Courses
Mathematics K

KINDERGARTEN

Operations and Algebraic Thinking (OA) *Learners will analyze patterns and relationships to generate and interpret numerical expressions.*
K.AR.OA.6 Recognize, duplicate, complete, and extend repeating patterns in a variety of contexts (e.g., shape, color, size, objects, sounds, movements).

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
Early Learning Standards Goal P.MATH.7: Child understands simple patterns. **Early Learning Standards Goal P.MATH.7.48-60:** Creates, identifies, extends, and duplicates simple repeating patterns in different forms, such as with objects, numbers, sounds, and movements.		1.AR.OA.7
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Recognize simple patterns (color, shape, size, objects, sounds, movements). • Recognize simple repeating patterns. • Duplicate simple repeating patterns. • Complete simple repeating patterns. • Extend simple repeating patterns.	• pattern • repeating pattern • duplicate (T) • complete • extend	• Finger use will be an important aspect of sound patterns. When building temporal sequences, children may use their fingers to track the pattern. They may need to recount to determine the items in the sequence. (IN) • The ability to match numbers to finger patterns assists learners in keeping track of counts in additive or subtractive situations. (IN) • Learners might miss an element or step in the pattern sequence, causing errors when completing or extending the pattern. (ME)

Courses
Mathematics K

KINDERGARTEN

Geometry and Measurement (GM)
Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties; represent and solve problems using a coordinate plane.

K.GM.G.1 Name shapes and identify them as two-dimensional (squares, circles, triangles, rectangles) regardless of their orientations or overall sizes.

Preceding Standard(s	Related Standard(s)	Successive Standard(s)
Early Learning Standards Goal P.MATH.9: Child identifies, describes, compares, and composes shapes.	K.GM.G.3 K.GM.G.4 1.NO.NF.1	1.GM.G.1 2.GM.G.1 2.GM.G.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Recognize two-dimensional shapes (square, circle, triangle, rectangle).	• two-dimensional shapes • square • circle • triangle • rectangle	• Learners might use informal language to name shapes. (IN) • Learners may think rectangles and squares are the same because they both have four sides. (ME)

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties; represent and solve problems using a coordinate plane.

K.GM.G.2 Name shapes and identify them as three-dimensional (cubes and spheres) regardless of their orientations or overall sizes.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
Early Learning Standards Goal P.MATH.9: Child identifies, describes, compares, and composes shapes.		1.GM.G.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify and name two-dimensional shapes (square, circle, triangle, rectangle). • Recognize three-dimensional shapes (cube, sphere).	• three-dimensional shape • cube • sphere	• Learners may begin to use informal language to describe three-dimensional shapes and gradually build academic vocabulary. (IN) • Learners might confuse two-dimensional and three-dimensional shapes. (ME)

Courses
Mathematics K

KINDERGARTEN

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.

K.GM.G.3 Compare and classify two-dimensional shapes to describe their similarities, differences, and attributes (squares, circles, triangles, rectangles).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
Early Learning Standards Goal P.MATH.9: Child identifies, describes, compares, and composes shapes. K.GM.G.1	K.GM.G.1	1.GM.G.3 2.GM.G.3 3.GM.G.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify and name two-dimensional shapes (square, circle, triangle, rectangle). • Understand terms similar and different. • Identify attributes of two-dimensional shapes, e.g., number of sides, number of corners, size of sides.	• two-dimensional shape • square • circle • triangle • rectangle • similar • difference • attribute	• Learners may use informal language to describe the attributes of two-dimensional shapes. (IN) • Learners might overlook important attributes and focus on only one characteristic. (ME)

Courses
Mathematics K

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.

K.GM.G.4 Compose a geometric shape by combining two or more simple shapes.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
Early Learning Standards Goal P.MATH.9: Child identifies, describes, compares, and composes shapes.	K.GM.G.1	1.GM.G.4 2.GM.G.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify and name two-dimensional shapes (square, circle, triangle, rectangle).	• combine • compose • two-dimensional shape • square • circle • triangle • rectangle	• Composition of shapes is the combination of two or more two-dimensional shapes. (IN) • Composition includes age-appropriate drawing, building, and/or creating. (IN) • Learners may struggle with aligning the shapes resulting in gaps or overlaps. (ME) • Learners might not fully enclose the intended shape leaving gaps of missing portions where shapes connect. (ME) • Learners may struggle with the proportion and/or orientation of the new shape. (ME)

Courses
Mathematics K

KINDERGARTEN

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.

K.GM.M.1 Compare and order two objects with a common measurable attribute.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
Early Learning Standards Goal P.MATH.8: Child measures objects by their various attributes using standard and non-standard measurements. Child uses differences in attributes to make comparisons.		1.GM.M.2 2.GM.M.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify and describe measurable attributes. • Describe several measurable attributes of a single object.	• compare • order • measurable attribute (length, weight, height)	• Learners may need assistance to learn how to line objects up end to end to compare them. (IN) • Learners might have difficulty distinguishing between measurable and non-measurable attributes. (ME)

Courses
Mathematics K

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

K.GM.M.2 Tell time related to daily life (today, yesterday, tomorrow, morning, afternoon, night).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
Early Learning Standards Goal P.SS.1: Child demonstrates a basic understanding of the past, present, and future and how things, people, and places change over time.		1.GM.M.3 2.GM.M.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Develop an understanding of terms today, yesterday, tomorrow, morning, afternoon, and night. • Develop an understanding of the relationship between terms. • Develop an understanding of the relationship between the times of day to a.m. and p.m. • Develop an understanding of the days of the week and the order of the days of the week.	• today • yesterday • morning (a.m.) • afternoon (p.m.) • night (p.m.) • days of the week	• Learners may get the terms confused (yesterday, today, tomorrow, and a.m., p.m.). (ME) • Learners might get the order of the days of the week mixed up. (ME)

Courses
Mathematics K

KINDERGARTEN

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*		
Data (D) *Learners will represent and interpret data.*		
K.DPS.D.1 Sort and classify objects (up to 10) based on attributes and explain the reasoning used.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
Early Learning Standards Goal P.SCI.3: Child compares and categorizes observable phenomena. **Early Learning Standards Goal P.SCI.3.48-60:** With increasing independence, a child can sort objects into groups based on more complex attributes, such as weight, sound, or texture. Uses measurement tools to assess the properties of and compare observable phenomena.	K.NO.CC.5 K.NO.NBT.2	1.DPS.D.1 1.DPS.D.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify attributes. • Identify common attributes in groups of objects. • Compare objects based on common attributes.	• sort • attributes • classify	• This standard focuses on sorting and classifying objects based on common attributes. The range of objects is limited to 10. This standard does not have learners counting or totaling datasets. (IN) • Learners might mix different attributes causing incorrect sorts. (ME) • Learners may start sorting by one attribute and switch to another leading to mixed groups. (ME) • Learners might miscount the number of objects in a group. (ME) • Learners may leave some objects unsorted. (ME)
Courses		
Mathematics K		

FIRST GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

Counting and Cardinality (CC)

Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

1.NO.CC.1 Count forward by ones and tens from any given point within 120.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.NO.CC.1	1.NO.CC.2 1.NO.NBT.1 1.GM.M.5	1.NO.NBT.5 2.NO.CC.1
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Count forward by ones to 100 verbally. • Count forward by tens to 100 verbally, starting at 10. • Count forward by ones from any number within 100 verbally. • Recognize tens and ones.	• forward • tens • ones • count	• Learners tend to have difficulty with the different structures of teens and twenties. (ME) • Learners may have difficulty progressing into the next decade, e.g., after 29. (ME) • Learners might confuse the pronunciation of teens and decuples, e.g., fifty instead of fifteen. (ME) • Learners who can say the sequence of forward number word sequence might not be able to identify the number word after. (ME) • Some learners might need to drop back to solve a number word after task. (ME) • Learners who can say the sequence through 99, 100, 101, etc., might say 200 after 109. (ME) • Learners may errantly use the word "and" when naming numerals beyond 100. (ME)

Courses
Mathematics 1

FIRST GRADE

Counting and Cardinality (CC)
Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

1.NO.CC.2 Count backward by ones and tens from a given number within 120.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.NO.CC.2	1.NO.CC.1	1.NO.NBT.4 1.NO.NBT.5 2.NO.CC.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Count forward by ones verbally to 120. • Count backward by ones verbally from 20. • Count backward from any given number within 10. • Name the number word before within 120.	• backward • ones • tens • count	• Learners who can recite forward number word sequences might have difficulty reciting backward number word sequences with the teens. (ME) • Learners might omit a word in the backward number sequence that they do not omit in the forward number sequence. (ME) • Learners might use a dropping-back strategy. This involves saying a forward number word sequence to find the number word before. (ME) • Learners might say the next lowest decuple 52, 51, 40, 49, 48 or omit a decuple 52, 51, 49, 48. (ME) • Learners might confuse the number word before with the number word after. (ME) • Learners may errantly include the word "and" when naming numerals beyond 100. (ME)

Courses
Mathematics 1

FIRST GRADE

Counting and Cardinality (CC)
Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

1.NO.CC.3 Represent several objects with a written numeral up to 120.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.NO.CC.1 K.NO.CC.3 K.NO.CC.5	1.NO.NBT.1 1.NO.NBT.2 1.NO.NBT.5	2.NO.CC.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Count forward by ones to 120 verbally. • Count objects using one-to-one correspondence up to 120. • Identify written numerals to 100. • Write individual digits (0-9). • Write written numerals to 100.	• numeral • objects	• Learners do not need to count, draw, or build items by one individually to represent them with a written numeral, e.g., ten frames, sticks, and bundles, base ten blocks, or a place on a number line. (IN) • Learners may reverse teen numerals with decade numbers, e.g., 16, 61, etc. (ME) • Learners might struggle with the formation of the numeral when writing it. (ME) • When counting, learners might miscount the number of objects or skip objects. (ME) • Learners may forget to represent zero when no objects are present, leading to omissions in the written numeral. (ME)

Courses
Mathematics 1

FIRST GRADE

Counting and Cardinality (CC)
Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

1.NO.CC.4 Recognize and verbally label arrangements, without counting, for briefly shown collections up to 20 (e.g., “I saw 16.” How do you know?” “I saw 10 and 6, that is 16.”).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.NO.CC.4 K.NO.NBT.1	1.AR.OA.2 1.AR.OA.3 1.AR.OA.4 1.AR.OA.5	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Instantly recognize without counting small quantities up to 5 in an arranged pattern and say the number. • Subitize quantities to 10.	• collections (T) • label • pattern	• Learners are to recognize without counting. Structured arrangements such as ten frames (utilizing 10+ and double patterns) can be utilized for combinations up to 20. (IN) • This skill is referred to as subitizing. (IN) • Learners may see groups of numbers like the example in the standard, but they are not counting each item by one. (ME) • Learners may have difficulty recognizing the number of objects if arranged in an unfamiliar or irregular pattern. (ME)

Courses

Mathematics 1

FIRST GRADE

Counting and Cardinality (CC)
Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

1.NO.CC.5 Skip count forward and backward by 5s and 10s from multiples and recognize the patterns of up to 10 skip counts.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.NO.CC.1 K.NO.CC.2 K.NO.CC.5	1.AR.OA.7 1.GM.M.5 2.AR.OA.5 3.NO.NBT.4	1.NO.NBT.4 2.NO.CC.4 2.NO.NBT.5 2.GM.M.3 2.GM.M.4 3.AR.OA.6
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify multiples of five within 50 on a number chart or number line. • Identify multiples of ten within 100 on a number chart or number line. • Skip count by fives. • Skip count by tens.	• skip count (fives, tens) • forward • backward • multiple (T) • pattern	• Standard K.NO.CC.5 can support this standard by utilizing configurations like a dice pattern or five and ten frames to support skip counting. (IN) • Skip counting builds verbal words for numerical patterns but does not mean that learners are able to multiply. (IN) • Learners might start from the incorrect number in the sequence causing errors in skip counting. (ME) • Learners may count inconsistently, at times by ones and others by fives or tens. (ME)

Courses
Mathematics 1

FIRST GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

1.NO.NBT.1 Demonstrate that the two digits of a two-digit number represent a composition of some tens and some ones.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.NO.NBT.1	1.NO.CC.1 1.NO.CC.3 1.NO.NBT.3 1.NO.NBT.5 2.NO.NBT.3 2.AR.OA.2 2.AR.OA.3 2.AR.OA.4	1.NO.NBT.2 1.NO.NBT.3 1.NO.NBT.4 2.NO.NBT.1 2.NO.NBT.3 2.NO.NBT.4
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Compose and decompose numbers from 11-19. • Identify tens and ones. • Represent each digit of a two-digit number on a place value mat. • Group tens and ones.	• tens • ones • digit • two-digit number	• Learners can use concrete models to build an understanding of tens and ones and represent the composition of tens and ones (e.g., sticks/ bundles, ten frames, and base ten blocks). (IN) • This standard highly supports standard 1.NO.NBT.2, in which learners are comparing two two-digit numerals using <, >, and = and justifying the comparison of the value of tens and ones. (IN) • Learners tend to get the < and > signs confused. (ME) • Learners may reverse digits or confuse tens and ones. (ME)

Courses
Mathematics 1

FIRST GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

1.NO.NBT.2 Compare two two-digit numbers using symbols >, <, and =. Justify comparisons based on the value of tens and ones.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.NO.NBT.2 1.NO.NBT.1	1.NO.CC.3 1.AR.OA.6 1.DPS.D.2	2.NO.NBT.1 2.NO.NBT.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify tens and ones. • Understand the concept of greater than, less than, and equal to. • Identify symbols >, <, and =. • Compare two one-digit numbers using symbols >,<, and =. • Identify the value of tens and ones using concrete models, materials, or in number form.	• tens • ones • two-digit number • symbols >, <, and = • greater than • less than • equal to • compare • value	• Teaching quantities using the vocabulary of greater than, less than, and equal to will lend to understanding tens and ones before moving to the symbolic level of comparing two two-digit numerals with symbols of <, >, or =. (IN) • Ordering numbers may be included for number comparison. (IN) • Learners tend to get the < and > signs confused. (ME) • Learners may only focus on the ones place and ignore the tens place (when comparing 37 and 45, look only at the 7 and 5 and assume 37 is greater.) (ME)

Courses
Mathematics 1

FIRST GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

1.NO.NBT.3 Add within 100 using a two-digit number and a one-digit number. Use concrete models, drawings, and strategies that reflect an understanding of place value.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.AR.OA.1 K.AR.OA.2 1.NO.NBT.1	1.NO.NBT.1 1.NO.NBT.5 1.AR.OA.1 1.AR.OA.4 1.AR.OA.6 2.NO.NBT.3	2.NO.NBT.3 2.AR.OA.2 2.AR.OA.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Add numbers to 10 automatically (vertically and horizontally) • Identify tens and ones. • Add two one-digit numbers using place value understanding (use models, drawings, strategies). • Show the value of tens or ones in more than one way. • Develop a flexible understanding of vertical and horizontal orientation.	• two-digit number • one-digit number • add • place value (T) • concrete model (T) • strategy	• In a progression of skills, adding a two-digit number and a multiple of 10 will support adding a two-digit and one-digit number. (IN) • Learners may not understand the concept and will create a three-digit numeral from the two-digit and one-digit numerals. (ME) • Learners might treat the two-digit number as two separate one- digit numbers, adding the ones but ignoring the tens. (ME) • Learners may reverse the numbers (e.g., write 17 instead of 71). (ME)

Courses
Mathematics 1

FIRST GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

1.NO.NBT.4 Subtract multiples of 10 within 100 using concrete models, drawings, and strategies that reflect an understanding of place value.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.AR.OA.5 1.NO.CC.2 1.NO.CC.5 1.NO.NBT.1	1.NO.NBT.5 1.AR.OA.5 1.AR.OA.6 2.NO.NBT.3	2.NO.NBT.4 2.AR.OA.2 2.AR.OA.4
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Subtract numbers within ten automatically (vertically and horizontally). • Identify tens and ones. • Model subtraction using concrete models or materials.	• subtract • place value (T) • concrete model (T) • multiple (T) • strategy	• In a progression of skills, adding a two-digit number and a multiple of 10 will happen before subtracting a two-digit and a multiple of 10. (IN) • Learners might subtract digits without considering their place value. (ME) • Learners may try to subtract larger numbers from smaller numbers. (ME) • Learners might skip tens. (ME)

Courses
Mathematics 1

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

1.NO.NBT.5 Mentally add or subtract 10 to or from a given two-digit number and explain the reasoning used.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.NO.CC.1 1.NO.CC.2	1.NO.CC.3 1.NO.NBT.1 1.NO.NBT.3 1.NO.NBT.4 1.AR.OA.7	2.NO.NBT.3 2.NO.NBT.4 2.NO.NBT.5 2.AR.OA.3 2.AR.OA.4
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify tens and ones. • Add a multiple of 10 and a multiple of 10. • Add a multiple of 10 and a two-digit number using models, drawings, and/or strategies. • Subtract 10 from a multiple of ten using models, drawings, and/or strategies.	• add • subtract • two-digit number • mental math (T) • digit (T)	• Teachers may want to begin instruction with models or drawings to build the ability to think abstractly about numbers when adding and subtracting 10 from a given two-digit numeral. This will benefit the explanation of reasoning from learners. (IN) • Some learners might perceive solving mentally as counting quickly in their heads (hiding their fingers). (ME) • Learners may not understand that adding or subtracting tens affects the tens digit, not the ones digit. (ME)

Courses
Mathematics 1

FIRST GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

1.NO.NF.1 Partition circles and rectangles into two and four equal shares using the language halves and fourths.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
	K.GM.G.1 2.NO.NF.3	2.NO.NF.1 2.NO.NF.2

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify circles and rectangles. • Understand the concept of equal shares. • Understand the concept of half and fourth.	• partition (circles, rectangles) • equal shares • half • fourth	• It is appropriate to start with instruction on making equal shares of shapes, even if learners are not able to identify or name them. (IN) • There is an age-appropriate margin of error when students are beginning to explore fractional partitioning. (IN) • Learners may need time to execute this action-based standard to learn how to partition equal shares that use the whole shape without focusing on fractional notation and tricks of perfection. (IN) • Some learners may create equal parts but not use the entire whole. (ME) • Some learners may use the entire whole but not create equal parts. (ME)

Courses
Mathematics 1

FIRST GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

Operations and Algebraic Thinking (OA)

Learners will analyze patterns and relationships to generate and interpret numerical expressions.

1.AR.OA.1 Automatically add and subtract within 10.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.AR.OA.1 K.AR.OA.2 K.AR.OA.3 K.AR.OA.4 K.AR.OA.5	1.NO.NBT.3 1.AR.OA.2 1.AR.OA.3 1.AR.OA.4 1.AR.OA.5 1.AR.OA.6	2.AR.OA.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Count forwards from 0 to 10. • Count backwards from 10 to 0. • Compose and decompose numbers less than or equal to 10 in pairs in more than one way (explanations, objects, drawings). • Understand the concept of addition and subtraction. • Add and subtract automatically within 5 (vertically and horizontally).	• add • subtract • automatically	• Learners are fluent in their understanding of numbers within 10, not just of 10. Their addition and subtraction of all numbers within 10 is without conscious thought or attention; thus, they are not counting by ones. (IN) • Automaticity is built through strategy practice and exposure but is practiced through rehearsal once learners own their understanding. (IN) • As defined in the 2023 standards, automaticity means to add and subtract without conscious thought or attention. (IN) • Learners may rely too heavily on counting instead of recalling facts from memory. (ME) • Learners might reverse digits when writing numbers (e.g., write 6 for 9). (ME) • Learners might mix up addition and subtraction facts. (ME)

Courses
Mathematics 1

FIRST GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

1.AR.OA.2 For any number from 1 to 19, find the number that makes 20 when added to the given number, sharing the answer with a model, drawing, or equation.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.NO.NBT.1 K.AR.OA.2 K.AR.OA.3	1.NO.CC.4 1.AR.OA.1 1.AR.OA.3 1.AR.OA.4 1.AR.OA.5 1.AR.OA.7 2.AR.OA.6	2.AR.OA.1 2.AR.OA.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Represent addition and subtraction in different ways. • Use visual models to support ways of composing and decomposing 20. • Use counting to support ways of composing and decomposing 20. • Find the number that makes 10 when added to a given number (0-10).	• equation • model • number	• Learners may use objects or drawings to find numbers from 1 to 19 to make 20. (IN) • Learners may skip numbers or lose track of their count when applying the counting strategy. (ME) • Learners might mix up addition and subtraction. (ME) • Learners may not adjust both the tens and ones digits appropriately. (ME)

Courses
Mathematics 1

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

1.AR.OA.3 Decompose numbers less than or equal to 20 into pairs in more than one way.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.AR.OA.2 K.AR.OA.3	1.NO.CC.4 1.AR.OA.1 1.AR.OA.2 1.AR.OA.4 1.AR.OA.5 1.AR.OA.7	2.AR.OA.1 2.AR.OA.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Represent addition and subtraction in different ways. • Decompose numbers less than or equal to 10 in more than one way. • Use visual models to support ways of composing and decomposing 20. • Use counting to support ways of composing and decomposing 20.	• decompose numbers (T) • less than • equal to • pairs	• Learners may try to use their ten fingers to solve decomposition problems within 20 and will be unsuccessful. (ME) • Learners might omit possible pairs or may not recall number pairs. (ME) • When counting, learners may skip numbers or count the same number twice. (ME) • Learners might confuse addition and subtraction. (ME)

Courses
Mathematics 1

FIRST GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

1.AR.OA.4 Solve authentic word problems with addition, including three numbers and unknowns, within 20.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.NO.CC.5 K.NO.NBT.1 K.AR.OA.4	1.NO.CC.4 1.NO.NBT.3 1.AR.OA.1 1.AR.OA.2 1.AR.OA.3 1.AR.OA.5 1.AR.OA.6	2.AR.OA.1 2.AR.OA.2 2.AR.OA.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Add automatically to 10 (vertically and horizontally). • Use counting strategies to add. • Understand the concept of the unknown. • Add two numbers within 20 (vertically and horizontally). • Add three numbers within 20 (vertically and horizontally). • Add combinations of three numbers and unknowns within 20. • Solve authentic word problems within 10 using the addition strategy.	• addition • unknown • solve	• Learners use objects, drawings, verbal explanations, and numerical representations to show or explain their thinking. (IN) • Build proficiency with solving word problems through the context of the problem rather than underlining or focusing on keywords or numbers. (IN) • Learners may struggle when a third addend is introduced, despite proficiency with two addends within a word problem. (ME) • Learners might not be able to visualize the word problems. If so, go back to familiar materials that are consistent and provide needed support. (ME) • Learners who have difficulty with addition within 20 may struggle to connect reading and meaning from a word problem. (ME) • Learners may not label their answers. (ME)

Courses
Mathematics 1

FIRST GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

1.AR.OA.5 Solve authentic word problems with subtraction, including unknowns, within 20.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.NO.CC.5 K.NO.NBT.1 K.AR.OA.5	1.NO.CC.4 1.NO.NBT.4 1.AR.OA.1 1.AR.OA.2 1.AR.OA.3 1.AR.OA.4 1.AR.OA.6	2.AR.OA.1 2.AR.OA.2 2.AR.OA.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Subtract automatically within 10 (vertically and horizontally). • Use counting strategies to subtract. • Understand the concept of the unknown. • Subtract combinations of numbers within 20. • Subtract using a number and an unknown within 20. • Solve subtraction word problems within 10.	• subtraction • unknown • solve	• Students use objects, drawings, verbal explanations, and numerical representations to show or explain their thinking. (IN) • Build proficiency with solving word problems through the context of the problem rather than underlining or focusing on keywords or numbers. (IN) • Learners might not be able to visualize the word problems. If so, go back to familiar materials that are consistent and provide needed support. (ME) • Learners who have difficulty with subtraction within 20 may struggle to connect reading and meaning from a word problem. (ME) • Learners may not label their answers. (ME)

Courses
Mathematics 1

FIRST GRADE

Operations and Algebraic Thinking (OA) *Learners will analyze patterns and relationships to generate and interpret numerical expressions.*
1.AR.OA.6 Distinguish and use the +, -, and = symbols accurately in an equation.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
	1.NO.NBT.2 1.NO.NBT.3 1.NO.NBT.4 1.AR.OA.1 1.AR.OA.4 1.AR.OA.5 2.AR.OA.3 2.AR.OA.4	2.NO.NBT.3 2.NO.NBT.4 2.AR.OA.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify the +, -, and = symbols and their functions. • Distinguish between the +, -, and = symbols. • Understand the concept of an equation. • Understand how the +, - and = symbols are used to write equations. • Solve equations using models or drawings starting or ending with the answer. • Write equations using models or drawings starting or ending with the answer. • Compare both sides of an equal sign to determine if an equation is true or false.	• equation • symbols +, -, and = • compare • true • false	• The equal sign indicates a balance of quantities on either side of it. (IN) • The equal sign will present differently when given an equation in vertical notation. (IN) • Learners may confuse the symbols. (ME) • Learner might place the symbols incorrectly in an equation. (ME)

Courses
Mathematics 1

FIRST GRADE

Operations and Algebraic Thinking (OA) *Learners will analyze patterns and relationships to generate and interpret numerical expressions.*
1.AR.OA.7 Identify, create, complete, and extend patterns that are repeating, increasing, and decreasing in a variety of contexts.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
K.AR.OA.6	1.NO.CC.5 1.NO.NBT.5 1.AR.OA.2 1.AR.OA.3 2.NO.CC.4	2.AR.OA.5 2.AR.OA.6 3.AR.OA.6 4.AR.OA.6
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Recognize patterns (color, shape, size, objects, sounds, movements, number). • Recognize repeating, increasing, and decreasing patterns. • Duplicate repeating, increasing, and decreasing patterns. • Complete repeating, increasing, and decreasing patterns. • Extend repeating, increasing, and decreasing patterns. • Create repeating, increasing, and decreasing patterns.	• repeating pattern • increasing pattern • decreasing pattern	• This is an action-based standard where learners will need time within patterning. (IN) • Learners might not identify the core unit of the pattern leading to inconsistencies. (ME) • Learners may eliminate or add elements to the pattern sequence. (ME) • When working with increasing or decreasing patterns, learners might not apply the correct increment. (ME)

Courses
Mathematics 1

FIRST GRADE

Geometry and Measurement (GM)
Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.

1.GM.G.1 Name shapes and identify them as two-dimensional (trapezoids, rhombuses, pentagons, hexagons, octagons).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.GM.G.1	1.GM.G.3 1.GM.G.4	2.GM.G.1 2.GM.G.2 3.GM.G.1
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify two-dimensional shapes (square, circle, triangle, rectangle). • Name two-dimensional shapes (square, circle, triangle, rectangle). • Identify two-dimensional shapes (trapezoids, rhombuses, pentagons, hexagons, octagons). • Name two-dimensional shapes (trapezoids, rhombuses, pentagons, hexagons, octagons).	• two-dimensional shape (squares, circles, triangles, rectangles, trapezoids, rhombuses, pentagons, hexagons, octagons)	• Learners may begin with the use of informal language to name shapes and gradually build academic vocabulary. (IN) • Learners might confuse shapes with similar features (e.g., squares, rectangles, rhombuses) (ME)

Courses
Mathematics 1

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.

1.GM.G.2 Name and identify solids as three-dimensional (cylinders, cones, triangular prisms, and rectangular prisms).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.GM.G.2	1.GM.G.3 1.GM.G.4	2.GM.G.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify three-dimensional shapes (cube, sphere). • Name three-dimensional shapes (cube, sphere). • Identify three-dimensional shapes/solids (cylinders, cones, triangular prisms, rectangular prisms). • Name three-dimensional shapes/solids (cylinders, cones, triangular prisms, rectangular prisms).	• solid • three-dimensional shape (cube, sphere, cylinder, cone, triangular prism, rectangular prism)	• Learners may begin with the use of informal language to name shapes and gradually build academic vocabulary. (ME) • Learners might confuse two-dimensional and three-dimensional shapes. (ME) • Learners may confuse shapes with similar features. (ME)

Courses
Mathematics 1

FIRST GRADE

Geometry (G) *Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.*
1.GM.G.3 Determine geometric attributes of two- dimensional and three-dimensional shapes (squares, circles, triangles, rectangles, trapezoids, rhombuses, pentagons, hexagons, octagons, cubes, spheres, cylinders, cones, triangular prisms, and rectangular prisms).

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
K.GM.G.3	1.GM.G.1 1.GM.G.2 1.GM.G.4	2.GM.G.1 2.GM.G.2 2.GM.G.3 3.GM.G.1 3.GM.G.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify attributes of two-dimensional shapes, e.g., number of sides, number of corners/vertices, size of sides. • Identify attributes of squares, circles, triangles, rectangles, trapezoids, rhombuses, pentagons, hexagons, and octagons. • Identify attributes of three-dimensional solids, e.g., number of faces, number of corners, number of edges. • Identify attributes of cubes, cylinders, spheres, cubes, triangular prisms, and rectangular prisms.	• geometric attribute • two-dimensional shape (square, circle, triangle, rectangle, trapezoid, rhombus, pentagon, hexagon, octagon) • three-dimensional shape (cube, sphere, cylinder, cone, triangular prism, rectangular prism)	• Part of determining geometric attributes is identifying defining and nondefining attributes, e.g., color, size, orientation, etc. (IN) • Learners may confuse defining and nondefining attributes. (ME) • Learners might confuse two-dimensional and three-dimensional shapes. (ME)

Courses
Mathematics 1

FIRST GRADE

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.

1.GM.G.4 Compose a geometric shape or solid by combining multiple two-dimensional shapes and/or three-dimensional solids (squares, circles, triangles, rectangles, trapezoids, rhombuses, pentagons, hexagons, octagons, cubes, spheres, cylinders, cones, triangular prisms, and rectangular prisms).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.GM.G.4	1.GM.G.1 1.GM.G.2 1.GM.G.3	2.GM.G.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Recognize simple shapes. • Compose a geometric shape by combining two or more simple shapes. • Identify and name two-dimensional shapes. • Identify and name three-dimensional shapes.	• geometric shape • solid • two-dimensional shape (square, circle, triangle, rectangle, trapezoid, rhombus, pentagon, hexagon, octagon) • three-dimensional solid (cube, sphere, cylinder, cone, triangular prism, rectangular prism)	• Composition includes age-appropriate drawing, building, and/or creating. (IN) • Learners may not overlap shapes correctly, leaving gaps or unintended overlaps. (ME) • Learners might ignore the faces of solids leading to misfits. (ME) • Learners may not understand how the surfaces of three-dimensional shapes interact, leading to incorrect combinations of different types of solids. (ME)

Courses
Mathematics 1

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

1.GM.M.1 Measure the length of an object as a whole number of same-size, non-standard units from end to end.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
	1.GM.M.2	2.GM.M.1 3.GM.M.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Count objects up to 20 in an arranged pattern. • Understand the concept of measurement. • Recognize non-standard unit measures, e.g., paperclips, cubes, popsicle sticks. • Utilize the same non-standard unit from end to end. • Begin measuring from an initial starting point. • Line units up that span the object without gaps or overlaps. • Count total units for the length of an object.	• length • measurable unit (T) • end to end	• Learners might move the starting point after they have started measuring. (ME) • Learners may overlap or have gaps when using the non-standard units. (ME)

Courses
Mathematics 1

FIRST GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

1.GM.M.2 Compare the lengths of three objects using a common measurable attribute.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.GM.M.1	1.GM.M.1	2.GM.M.2
Prerequisite Skills	**Key Vocabulary (Teacher and Learner or Both)**	**Common Misconceptions (Teacher and Learner) and Instructional Notes**
• Measure length in non-standard units.	• length • measurable attribute	• Learners might not line up the starting points of each object for a fair comparison. (ME)

Courses

Mathematics 1

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

1.GM.M.3 Tell and write time to the hour and half-hour (including o-clock and half past) using analog and digital clocks.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.GM.M.2		2.GM.M.3 3.GM.M.3 3.GM.M.4
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Skip count by fives. • Identify the difference between a digital and analog clock. • Identify hour and minute hands on an analog clock. • Understand the movement of the hour hand in relationship to the movement of the minute hand. • Understand the concept of telling time to the hour (o'clock). • Understand the concept of writing time to hour. • Understand the concept of half a circle. • Understand hand placement for time to half an hour. • Understand the concept of telling time to half-hour (half past). • Understand the concept of writing time to half-hour. • Match analog to digital time. • Match time written in word form to digital and analog time.	• hour (o'clock) • half-hour (half past) • hour hand • minute hand • analog clock • digital clock	• Learners might confuse the hour and minute hands on an analog clock. (ME) • Learners may not understand that the hour hand is halfway between two numbers at the half-hour mark. (ME) • Learners may reverse the hour and minute when writing time, e.g., write 3:30 as 30:3. (ME)

Courses

Mathematics 1

FIRST GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

1.GM.M.4 Identify and tell the value of a dollar bill, quarter, dime, nickel, and penny.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
		1.GM.M.5 2.GM.M.4
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the concept of money. • Identify a dollar bill, quarter, dime, nickel, and penny. • Identify the value of a penny, nickel, dime, quarter, and dollar bill.	• penny • nickel • dime • quarter • dollar • value	• Learners may get the coins mixed up (both in identification and in value). (ME)

Courses
Mathematics 1

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

1.GM.M.5 Count collections of coins (pennies, nickels, and dimes) relating to counting patterns by 1s, 5s, and 10s up to one dollar.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.GM.M.4	1.NO.CC.1 1.NO.CC.5	2.GM.M.4 3.GM.M.5 4.GM.M.4
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Count forward and backward by ones to 100. • Skip count forward and backward by five to 100. • Skip count forward and backward by tens to 100. • Identify a dollar bill, quarter, dime, nickel, and penny. • Identify the value of a penny, nickel, dime, quarter, and dollar bill. • Transition counting pattern based on the value of a coin. • Combine counting skills and understanding of value to count coins.	• penny • nickel • dime • dollar • counting pattern (ones, fives, tens) • coin	• Learners may get coin identity and value confused. (ME) • Learners might skip or double count a coin. (ME) • Learners may have difficulty switching counting between counting by tens (dimes) and fives (nickels). (ME)

Courses
Mathematics 1

FIRST GRADE

Data, Probability, and Statistics (DPS)
Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.

Data (D)
Learners will represent and interpret data.

1.DPS.D.1 Collect, organize, and represent data with up to three categories using picture and bar graphs.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.DPS.D.1	1.DPS.D.2	2.DPS.D.1 2.DPS.D.2 3.DPS.D.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify a picture graph. • Identify a bar graph. • Count forward and backward by ones to represent datasets. • Sort objects by attributes. • Identify similarities and differences in objects or characteristics. • Record data horizontally or vertically on a graph using one-to-one correspondence.	• data • categories • picture graph • bar graph	• Learners might have difficulty reading a picture or bar graph to find the data and compare it. (ME) • When collecting data, learners might mix up categories or select categories that overlap. (ME) • Learners may have difficulty grouping similar items together into categories and counting items accurately. (ME) • Learners might have difficulty drawing bars or pictures proportional to the quantities represented. (ME) • Learners might have difficulty keeping items aligned and organized on the graph. (ME) • Learners may not label the axes and categories on the graph. (ME)

Courses
Mathematics 1

FIRST GRADE

Data (D)
Learners will represent and interpret data.

1.DPS.D.2 Analyze data by answering descriptive questions.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.NO.NBT.2 K.DPS.D.1	1.NO.NBT.2 1.DPS.D.1	2.DPS.D.3 3.DPS.D.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify a picture graph. • Identify a bar graph. • Count forward and backward by ones to represent datasets. • Sort objects by attributes. • Identify information given on a graph. • Find the total number of data points. • Find how many are in each category. • Compare datasets. • Find how many more or less are in one category than in another.	• data • analyze • compare • total/how many altogether • how many more/greater than • how many less/less than	• Learners might need support with a comparison of datasets and answering descriptive questions of “How many more?” and “How many less?” (IN) • Learners may make errors counting the number of items or data points on the graph. (ME) • Learners may not recognize the bars on bar graphs or images on picture graphs as representations of data. (ME)

Courses

Mathematics 1

SECOND GRADE

Number and Operations (NO)
Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

Counting and Cardinality (CC)
Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

2.NO.CC.1 Count forward from any given number within 1000.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.NO.CC.1	2.NO.CC.2 2.NO.CC.4 2.NO.NBT.1	3.NO.CC.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Count forward to 120 from any given number. • Recognize the pattern within numerals and correctly transition across tens and hundreds.	• count forward (B) • number (B) • number word after (B)	• Learners do not need to count from 0 to 1000 to demonstrate proficiency in this standard. (IN) • Learners may errantly include the word “and” when naming numerals beyond 100. (ME) • Learners might skip or repeat numbers. (ME) • Learners may lose track of their place in the count. (ME) • Learners might have difficulty transitioning between decades. (ME)

Courses
Mathematics 2

Counting and Cardinality (CC)
Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

2.NO.CC.2 Count backward from any given number within 1000.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.NO.CC.2	2.NO.CC.1 2.NO.CC.4	3.NO.CC.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Count backward from 120 from any given number. • Recognize the pattern within numerals and correctly transition across tens and hundreds.	• count backward (B) • number (B) • number word before (B)	• Learners do not need to count from 0 to 1000 to demonstrate proficiency in this standard. (IN) • Learners might errantly include the word “and” when naming numerals beyond 100. (ME) • Learners might skip or repeat numbers. (ME) • Learners may lose track of their place in the count. (ME) • Learners might have difficulty transitioning between decades. (ME)

Courses
Mathematics 2

SECOND GRADE

Counting and Cardinality (CC) *Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.*		
2.NO.CC.3 Read and write numbers up to 1000 using standard, word, and expanded forms.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
1.NO.CC.3	2.NO.NBT.1	3.NO.CC.1
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the concept of standard, word, and expanded form. • Understand place value (ones, tens, hundreds, thousands).	• standard form/numerical form • word form/written form • expanded form	• Spelling is not assessed. (IN) • Learners might write numbers inaccurately, such as 1060 for 160. (ME) • Learners may struggle with decomposing numbers into expanded form. (ME) • Learners may struggle to recall the word form of larger numbers with irregular names. (ME)
Courses		
Mathematics 2		

Counting and Cardinality (CC) *Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.*		
2.NO.CC.4 Skip count forward and backward by 2s and 100s and recognize the patterns of skip counts.		
Preceding Standard(s)	**Grade Level Related Standard(s)**	**Successive Standard(s)**
1.NO.CC.5	1.AR.OA.7 2.NO.CC.1 2.NO.CC.2 2.AR.OA.5	3.AR.OA.1 3.AR.OA.6
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Skip count by fives forward and backward to 1000. • Skip count by tens forward and backward to 1000. • Skip count by twos forward and backward to 10. • Skip count by 100s forward and backward to 1000. • Skip count by twos to 20. • Describe why some numbers are even and some are not.	• skip count (2s, 100s) • forward • backward • multiple (T) • pattern	• Skip counting builds verbal words for numerical patterns but does not mean that learners are able to multiply. (IN) • Start from any multiple and move forward or backward by 2s or 100s, e.g., 20, 18, 16, etc., within the range of 2-20 and 100-1,000. (IN) • Learners might start by stress counting (1, **2**, 3, **4**, 5, **6**, etc.) as they build proficiency in skip counting. (ME) • Learners may struggle with decade transitions when they are skip counting. (ME)
Courses		
Mathematics 2		

SECOND GRADE

Counting and Cardinality (CC)
Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

2.NO.NBT.1 Understand that the three digits of a three-digit number represent a composition of some hundreds, some tens, and some ones.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.NO.NBT.1 1.NO.NBT.2	2.NO.CC.1 2.NO.CC.3 2.NO.NBT.2 2.NO.NBT.3 2.NO.NBT.4 2.NO.NBT.5 2.AR.OA.3 2.AR.OA.4	3.NO.NBT.1 3.NO.NBT.2 4.NO.NBT.1 4.NO.NBT.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify ones, tens, and hundreds using place-value materials. • Demonstrate the two digits of a two-digit number represent a composition of some tens and some ones. • Represent each digit's value in a three-digit number.	• hundreds • tens • ones • digit	• Learners can use concrete models to build an understanding of hundreds, tens, and ones and represent the composition of hundreds, tens, and ones, e.g., sticks/ bundles, ten frames, and base ten blocks. (IN) • This standard highly supports standard 2.NO.NBT.2, in which learners are comparing two two-digit numerals using <, >, and = and justifying the comparison of the value of hundreds, tens, and ones. (IN) • Learners might not understand the composition based on value and count everything by 1s, 10s, or 100s. (ME)

Courses
Mathematics 2

SECOND GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

2.NO.NBT.2 Compare two three-digit numbers using symbols >, <, and =. Justify comparisons based on the value of hundreds, tens, and ones.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.NO.NBT.2	2.NO.NBT.1	3.NO.NBT.1 4.NO.NBT.4
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify hundreds, tens, and ones. • Understand the concept of greater than, less than, and equal to. • Identify symbols >, <, and =. • Identify the value of hundreds, tens, and ones using concrete models, materials, or in number form.	• symbols >, <, and = • hundreds • tens • ones	• Teaching quantities using the vocabulary of greater than, less than, and equal to will lend to an understanding of hundreds, tens, and ones before moving to the symbolic level of comparing two three-digit numerals with symbols of <, >, or =. (IN) • Ordering numbers may be included for number comparisons. • Learners may confuse the < and > symbols. (ME) • Learners may compare digits individually without considering their place value. (e.g., Compare 285 and 542. Learners might assume 285 is greater because 8 is larger than 4, ignoring the hundreds.) (ME) • Learners might transpose or reverse digits. (ME)

Courses
Mathematics 2

SECOND GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

2.NO.NBT.3 Add within 100 using place value strategies and/or the relationship between addition and subtraction.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.NO.NBT.1 1.NO.NBT.3 1.NO.NBT.5 1.AR.OA.6	1.NO.NBT.1 1.NO.NBT.3 1.NO.NBT.4 2.NO.NBT.1 2.NO.NBT.4 2.NO.NBT.5 2.AR.OA.1 2.AR.OA.2 2.AR.OA.3 2.AR.OA.4	3.NO.NBT.3 3.AR.OA.3 4.NO.NBT.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Add numbers to 20 automatically (vertically and horizontally). • Identify hundreds, tens, and ones. • Add within 100 using a two-digit and a one-digit number. • Write two-digit numbers in expanded form. • Use strategies to compose and decompose numbers using place value. • Add numbers without regrouping. • Add numbers with regrouping.	• addition • place value • relationship • subtraction	• The teaching of the standard algorithm is not part of the standard until standard 3.NO.NBT.3. (IN) • Learners might have difficulty counting forward by 10 from any given number, which forces them to count by ones to solve addition equations within 100. (ME) • Learners may struggle with adding across a ten, which makes addition across a decade difficult. (ME) • Learners may struggle to identify the decade before and the decade after, which gives them an incorrect answer as they add across a decade. (ME) • Learners may not regroup when the sum of the digits in the tens or ones exceeds 9. (ME)

Courses
Mathematics 2

SECOND GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

2.NO.NBT.4 Subtract within 100 using place value strategies and/or the relationship between addition and subtraction.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.NO.NBT.1 1.NO.NBT.4 1.NO.NBT.5 1.AR.OA.6	2.NO.NBT.1 2.NO.NBT.3 2.NO.NBT.5 2.AR.OA.1 2.AR.OA.2 2.AR.OA.3 2.AR.OA.4	3.NO.NBT.3 3.AR.OA.3 4.NO.NBT.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Count backward from a given number. • Subtract numbers within 20 (vertically and horizontally) automatically. • Write two-digit numbers in expanded form. • Identify hundreds, tens, and ones. • Use strategies to compose and decompose numbers using place value. • Subtract numbers without regrouping. • Subtract numbers with regrouping.	• subtraction • place value • relationship • addition	• The teaching of the standard algorithm is not part of the standard until standard 3.NO.NBT.3 (IN) • Learners might have difficulty decrementing (counting backward by 10 from any given number), which forces them to count by ones to solve subtraction equations within 100. (ME) • When regrouping in subtraction, the learners may try to subtract the larger digit from the smaller digit rather than regroup. (ME) • When regrouping in subtraction, learners may not remember to take the ten away when adding it to the one. (ME)

Courses
Mathematics 2

SECOND GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

2.NO.NBT.5 Mentally add or subtract 10 or 100 to or from a given number between 100 and 900.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.NO.CC.5 1.NO.NBT.5	2.NO.NBT.1 2.NO.NBT.3 2.NO.NBT.4 2.AR.OA.3 2.AR.OA.4	3.NO.NBT.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify hundreds, tens, and ones. • Add a multiple of 10 and a two-digit number using models, drawings, and/or strategies. • Subtract 10 from a multiple of ten using models, drawings, and/or strategies. • Add a multiple of 100 and a three-digit number using models, drawings, and/or strategies. • Subtract 100 from a multiple of one hundred using models, drawings, and/or strategies.	• add • subtract • mentally	• Teachers may want to begin instruction with models or drawings to build the ability to think abstractly about numbers when adding and subtracting 10 or 100 from a given three-digit numeral. This will benefit the explanation of reasoning from learners. (IN) • Some learners perceive solving mentally as counting quickly in their heads (hiding their fingers). (ME) • Learners may reverse digits. (ME) • Learners might have difficulty regrouping when crossing tens boundaries. (ME) • Learners may not add or subtract the correct digit (e.g. add 10 to 380 and getting the answer 381 instead of 390.)

Courses
Mathematics 2

SECOND GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

2.NO.NF.1 Partition circles and rectangles into two, three, or four equal shares. Describe the shares using the language of halves, thirds, fourths, half of, a third of, and a fourth of.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.NO.NF.1	2.NO.NF.2 2.NO.NF.3	3.NO.NF.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify circles and rectangles. • Identify the given whole. • Understand the concept of equal shares. • Identify if a given circle or rectangle is partitioned into equal shares. • Understand the concept of half, third, and fourth. • Identify if a given partitioned shape is in halves, thirds, or fourths.	• partition (circle, rectangle) • half, half of • third, third of • fourth, fourth of • equal shares	• There is an age-appropriate margin of error when learners are beginning to explore fractional partitioning. (IN) • Learners may need time to execute this action-based standard to learn how to partition equal shares that use the whole shape without focusing on fractional notation and tricks of perfection. (IN) • It is recommended to start with partitioning rectangles to build a conceptual understanding. (IN) • Partitioning circles can pose a challenge for learners to see the connection based on the curves and varied sizes of parts. (IN) • Learners need support describing the shares with appropriate academic language. (IN) • Learners might have difficulty dividing shapes into equal shares. (ME) • Some learners may create equal parts but not use the entire whole. (ME) • Some learners may use the entire whole but not create equal parts. (ME)

Courses
Mathematics 2

SECOND GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

2.NO.NF.2 Recognize that identical wholes can be equally divided in different ways.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.NO.NF.1	2.NO.NF.1 2.NO.NF.3 3.NO.NF.2	3.NO.NF.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the concept of equal shares. • Identify if a given shape is partitioned into equal shares. • Identify if a whole shape is used during partitioning. • Identify identical wholes partition differently.	• whole • equal • divided	• Learners might have difficulty dividing shapes into equal shares. (ME) • Learners might have difficulty envisioning different ways a shape may be divided equally. (ME)

Courses
Mathematics 2

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

2.NO.NF.3 Recognize that partitioning shapes into more equal shares creates smaller shares.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
	1.NO.NF.1 2.NO.NF.1 2.NO.NF.2	3.NO.NF.1 3.NO.NF.3 3.NO.NF.5
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the concept of equal shares. • Identify if a given shape is partitioned into equal shares.	• partition (shape) • equal shares	• Learners might have difficulty dividing shapes into equal shares. (ME) • Learners may not understand that a fourth is less than a half, a third is less than a half, and a third is more than a fourth because they see denominators are larger numbers, so they interpret the fraction as larger. (ME)

Courses
Mathematics 2

SECOND GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

Operations and Algebraic Thinking (OA)

Learners will analyze patterns and relationships to generate and interpret numerical expressions.

2.AR.OA.1 Automatically add and subtract within 20.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.AR.OA.2 1.AR.OA.1 1.AR.OA.2 1.AR.OA.3 1.AR.OA.4 1.AR.OA.5	2.NO.NBT.3 2.NO.NBT.4 2.AR.OA.2 2.AR.OA.3 2.AR.OA.4 3.NO.NBT.4 3.AR.OA.1	3.NO.NBT.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the concept of addition and subtraction. • Add and subtract numbers within 10 automatically (vertically and horizontally). • Use models to demonstrate addition and subtraction within 20. • Use mental strategies flexibly to demonstrate addition and subtraction within 20.	• addition • subtraction • automatically	• Learners are fluent in their understanding of numbers within 20, not just of 20. Their addition and subtraction of all numbers within 20 is without conscious thought or attention; thus, they are not counting by ones. (IN) • Automaticity is built through strategy practice and exposure but is practiced through rehearsal once learners own their understanding. (IN) • As defined in the 2023 standards, automaticity is to add and subtract without conscious thought or attention. • Learners might be fluent in their automaticity of addition but not subtraction. (IN) • Learners may rely too heavily on counting instead of recalling facts from memory. (ME) • Learners might reverse digits when writing numbers (e.g., write 6 for 9). (ME) • Learners might mix up addition and subtraction facts. (ME)

Courses
Mathematics 2

SECOND GRADE

Operations and Algebraic Thinking (OA) *Learners will analyze patterns and relationships to generate and interpret numerical expressions.*
2.AR.OA.2 Apply the properties of operations to solve addition and subtraction equations within 100 and justify thinking.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
1.NO.NBT.3 1.NO.NBT.4 1.AR.OA.2 1.AR.OA.3 1.AR.OA.4 1.AR.OA.5 1.AR.OA.6	1.NO.NBT.1 2.NO.NBT.3 2.NO.NBT.4 2.AR.OA.1 2.AR.OA.3 2.AR.OA.4 3.AR.OA.2	3.AR.OA.6 4.AR.OA.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Add and subtract numbers within 20 automatically (vertically and horizontally). • Identify the +, -, and = symbols. • Distinguish between the +, -, and = symbols. • Understand the concept of an equation. • Understand how the +, - and = symbols are used to write equations. • Develop an understanding of the commutative property of addition. • Develop an understanding of the relationship between addition and subtraction. • Develop an understanding of the identity property of 0. • Develop an understanding of associative property.	• properties of operations (associative, commutative, additive property of 0) (T-name specific B-skills within properties of operations) • addition • subtraction • equation	• Learners are not expected to name the properties of operations in second grade; however, they will apply the properties within addition and subtraction equations to justify their thinking. (IN) • Learners might not do the portion of the problem in parentheses first when using the associative property. (ME) • Learners might mistakenly assume the commutative property applies to subtraction. (ME)

Courses
Mathematics 2

SECOND GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

2.AR.OA.3 Solve one- and two-step authentic word problems with addition within 100, including the use of unknowns.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.NO.NBT.3 1.NO.NBT.5 1.AR.OA.4	1.NO.NBT.1 1.AR.OA.6 2.NO.NBT.3 2.NO.NBT.4 2.NO.NBT.5 2.AR.OA.1 2.AR.OA.2	3.AR.OA.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Add and subtract within 20 automatically (vertically and horizontally). • Understand place value (ones, tens, hundreds). • Understand the concept of an unknown. • Solve an addition problem.	• addition • unknown • word problem	• Learners use objects, drawings, verbal explanations, and numerical representations to show or explain their thinking. (IN) • Build proficiency in solving word problems through the context of the problem rather than underlining or focusing on keywords or numbers. (IN) • When solving two-step problems, learners may not complete the second step of the problem. (ME) • Learners may not label their answers. (ME) • Learners might not regroup when the sum of digits exceeds 9. (ME) • Learners may not identify key or relevant information from the problem needed to set up equations. (ME)

Courses
Mathematics 2

SECOND GRADE

Operations and Algebraic Thinking (OA) *Learners will analyze patterns and relationships to generate and interpret numerical expressions.*
2.AR.OA.4 Solve one- and two-step authentic word problems with subtraction within 100, including the use of unknowns.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
1.NO.NBT.4 1.NO.NBT.5 1.AR.OA.5	1.NO.NBT.1 1.AR.OA.6 2.NO.NBT.3 2.NO.NBT.4 2.NO.NBT.5 2.AR.OA.1 2.AR.OA.3	3.AR.OA.3 4.AR.OA.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Add and subtract within 20 automatically (vertically and horizontally). • Understand place value (ones, tens, hundreds). • Understand the concept of an unknown. • Solve a subtraction problem.	• subtraction • unknown • word problem	• Learners use objects, drawings, verbal explanations, and numerical representations to show or explain their thinking. (IN) • Build proficiency in solving word problems through the context of the problem rather than underlining or focusing on keywords or numbers. (IN) • Learners might not identify key or relevant information from the problem needed to set up equations. (ME) • When solving two-step problems, learners may not complete the second step of the problem. (ME) • When regrouping in subtraction, the learners may try to subtract the larger digit from the smaller digit rather than regroup. (ME) • When regrouping in subtraction, learners may not take a ten away from the tens when adding it to the one. (ME)

Courses
Mathematics 2

SECOND GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

2.AR.OA.5 Use repeated addition to find the total number of objects arranged in a rectangular array.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.AR.OA.7	1.NO.CC.5 2.NO.CC.4 3.NO.NBT.4	3.NO.NBT.4 3.AR.OA.1 3.AR.OA.4
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Develop an understanding of rectangular array. • Understand the concept of repeated addition. • Skip count by 2s and 5s up to 25 (5x5 range rectangular array).	• repeated addition • addition • rectangular array	• Skip counting builds verbal words for numerical patterns and for repeated addition. (IN) • When doing repeated addition, learners may try to count all objects rather than skip-counting or creating addition equations. (ME)

Courses
Mathematics 2

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

2.AR.OA.6 Identify a group of objects from 0 to 20 as even or odd by showing even numbers as a sum of two equal parts.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.AR.OA.7	1.AR.OA.2	3.AR.OA.6 4.AR.OA.6
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Skip count by twos. • Understand the concept of odd and even. • Compose combinations of 20.	• even • odd • sum	• Learners may not understand the academic vocabulary of odd versus even when describing groups of objects. (ME)

Courses
Mathematics 2

SECOND GRADE

Geometry and Measurement (GM)
Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties; represent and solve problems using a coordinate plane.

2.GM.G.1 Identify two-dimensional shapes (parallelograms and quadrilaterals).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.GM.G.1 1.GM.G.1 1.GM.G.3	2.GM.G.2 2.GM.G.3	3.GM.G.1
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify two-dimensional shapes (square, circle, triangle, rectangle, trapezoids, rhombuses, pentagons, hexagons, octagons). • Identify parallelograms and quadrilaterals.	• Two-dimensional shapes (squares, circles, triangles, rectangles, trapezoids, rhombuses, pentagons, hexagons, octagons, parallelograms, quadrilaterals) (B)	• Learners may not recognize the categories of parallelograms and quadrilaterals and their shared attributes. They may identify them as individual shapes. (ME)

Courses

Mathematics 2

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties; represent and solve problems using a coordinate plane.

2.GM.G.2 Identify two-dimensional shapes found within three-dimensional shapes.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.GM.G.1 1.GM.G.1 1.GM.G.2 1.GM.G.3	2.GM.G.1 2.GM.G.3	6.GM.GF.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Recognize simple shapes. • Identify two-dimensional shapes. • Identify three-dimensional shapes.	• two-dimensional shapes • three-dimensional shapes	• Learners might need to work with visuals to discover that three-dimensional shapes have varying two-dimensional shapes defining them. (IN) • Learners may have difficulty visualizing two-dimensional shapes in three-dimensional contexts. (ME)

Courses

Mathematics 2

SECOND GRADE

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties; represent and solve problems using a coordinate plane.

2.GM.G.3 Compose geometric shapes having specified geometric attributes, such as a given number of edges, angles, faces, vertices, and/or sides.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.GM.G.3 K.GM.G.4 1.GM.G.3 1.GM.G.4	2.GM.G.1 2.GM.G.2	3.GM.G1 3.GM.G.2 4.GM.G.1 4.GM.G.2 6.GM.GF.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify shapes (circle, square, triangle, rectangle, trapezoid, rhombus, pentagon, hexagon, octagon, parallelogram, quadrilateral). • Identify attributes of two-dimensional shapes, e.g., number of sides, number of angles, size of sides, parallel lines, etc. • Identify attributes of squares, circles, triangles, rectangles, trapezoids, rhombuses, pentagons, hexagons, and octagons. • Identify three-dimensional shapes/solids (cubes, spheres, cylinders, cones, triangular prisms, rectangular prisms). • Identify attributes of three-dimensional solids, e.g., number of faces, number of corners, number of edges, number of vertices. • Identify attributes of cubes, cylinders, spheres, cubes, triangular prisms, and rectangular prisms. • Distinguish attributes on a given shape. • Compose a geometric shape by combining two or more simple shapes.	• geometric shapes • compose (draw, build, create) • edges • angles • vertices • faces • sides • geometric attribute	• Composition includes age-appropriate drawing, building, and/or creating. (IN) • Learners might confuse different geometric attributes, e.g., edges, faces, vertices, angles. (ME)

Courses
Mathematics 2

SECOND GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

2.GM.M.1 Measure the length of an object using two different standard units of measurement. Describe how the two measurements relate to the size of the units chosen.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.GM.M.1	2.GM.M.2 4.GM.M.1	2.DPS.D.2 3.GM.M.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Measure length using non-standard units. • Identify standard units of measure (inches, feet, centimeters, meters). • Compare standard units of measurement. • Identify the appropriate unit to measure a given object. • Select the appropriate measurement tool to measure a given object. • Align the tool correctly during measurement. • Measure length using standard units of measurement.	• measure • length • standard unit of measure (inches, feet, centimeters, meters)	• Teachers should be mindful of the tools selected and check if learners are using them appropriately. (IN) • Learners may not understand the meaning of the marks on the measurement tool or how the tool is used to measure objects. (ME) • Learners tend to start measuring from the one rather than the beginning of the tool. (ME) • Learners may struggle with utilizing a second unit of measurement and relating it to the size of the unit chosen. (ME)

Courses
Mathematics 2

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

2.GM.M.2 Estimate and measure to determine how much longer one object is than another, expressing the difference with a standard unit of measurement.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.GM.M.1 1.GM.M.2	2.GM.M.1 4.GM.M.1	3.GM.M.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the concept of estimation. • Measure length using standard units of measurement.	• estimate • measure • standard unit of measurement (inches, feet, centimeters, meters)	• The skill of estimation precedes the measurement of objects as it lends to the conceptual understanding of measurement. (IN) • Learners tend to measure prior to estimation. (ME) • Learners may not understand the meaning of the marks on the measurement tool or how the tool is used to measure objects. (ME) • Learners tend to start measuring from the one rather than the beginning of the tool. (ME)

Courses
Mathematics 2

SECOND GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

2.GM.M.3 Tell and write time to the nearest five minutes (including quarter after and quarter to) with a.m. and p.m. using analog and digital clocks.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.GM.M.2 1.NO.CC.5 1.GM.M.3		3.GM.M.3 3.GM.M.4 4.GM.M.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify the difference between a digital and analog clock. • Identify the hour and minute hand on an analog clock and explain what they represent. • Tell and write time to hour and half hour. • Skip count by five. • Understand the concept of a.m. and p.m. • Demonstrate that quarter means four equal parts and show the relationship to the analog clock. • Tell and write the time to the quarter-hour. • Tell and write time to the nearest five minutes.	• minute • quarter hour (quarter to, quarter after) • a.m. • p.m. • time • clock • digital clock	• Learners have more understanding and exposure to digital clocks, so bringing connections from their previous knowledge to analog clocks will support their learning. (IN) • Learners tend to confuse the hour and minute hands. (ME) • Learners might misinterpret the position of the hour hand when it is close to the next hour or misinterpret the position of the minute hand when it is not on a multiple of five. (ME) • Learners tend to confuse terms quarter to and quarter after. (ME)

Courses
Mathematics 2

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

2.GM.M.4 Count collections of money (quarters, dimes, nickels, and pennies) relating to counting patterns by 1s, 5s, and 10s up to one dollar.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.NO.CC.5 1.GM.M.4 1.GM.M.5		3.GM.M.5 4.GM.M.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• One-to-one correspondence. • Count by ones, fives, and tens. • Identify coins (penny, nickel, dime, quarter). • Identify the value of coins (penny, nickel, dime, quarter). • Transition counting pattern based on the value of the coin.	• quarter • dime • nickel • penny • dollar • counting patterns	• Decimal notation is not required at this point. (IN) • Learners tend to lose their place when counting the coins. (ME) • Learners might get coin identity and value confused. (ME) • Learners may skip or double count a coin. (ME) • Learners might have difficulty switching counting between counting by tens (dimes), fives (nickels), and ones (pennies). (ME)

Courses
Mathematics 2

SECOND GRADE

Data, Probability, and Statistics (DPS)
Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.

Data (D)
Learners will represent and interpret data.

2.DPS.D.1 Formulate questions and collect, organize, and represent data with up to four categories using a single-unit scaled picture and bar graphs.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.DPS.D.1	2.DPS.D.2 2.DPS.D.3	3.DPS.D.1 4.DPS.D.1
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the difference between a question and a statement. • Form descriptive questions • Collect data (tally marks, counting, etc.). • Organize data (sort, categorize) • Identify the parts of a graph, (title, legend, scale, category). • Read bar and picture graphs. • Read scaled bar and picture graphs. • Make a graph using pre-collected data. • Analyze data by asking and answering descriptive questions.	• scaled graph (T) • picture graph • bar graph • data	• Learners misinterpret the scale on a bar or picture graph when reading data and answering questions. (ME) • Learners might have difficulty grouping similar items together into categories and counting items accurately. (ME) • Learners might have difficulty drawing bars or pictures proportional to the quantities represented. (ME) • Learners might have difficulty keeping items aligned and organized on the graph. (ME) • Learners may not label the categories or data values used on the graph. (ME)

Courses
Mathematics 2

Data (D)
Learners will represent and interpret data.

2.DPS.D.2 Generate data and create line plots marked in whole-number units.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.DPS.D.1 2.GM.M.1	2.DPS.D.1 2.DPS.D.3	3.DPS.D.2 4.DPS.D.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Collect, organize, and sort data. • Understand how to read and use a number line. • Read line plots. • Represent data on a line plot with whole-number units.	• data • line plot • whole-number unit (T)	• Measurement data could be used for the creation of line plots. (IN) • Learners may struggle with the iteration of a tick on a line plot if they are a discrete counter vs. an interval counter. (ME) • Learners may place data points incorrectly or space intervals unevenly. (ME)

Courses
Mathematics 2

SECOND GRADE

Data (D) *Learners will represent and interpret data.*		
2.DPS.D.3 Analyze data and interpret the results to solve one-step comparison problems using information from the graphs.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
1.DPS.D.2	2.DPS.D.1 2.DPS.D.2	3.DPS.D.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Add and subtract within 100. • Read bar and picture graphs. • Understand how to read a scaled bar or picture graph. • Understand how to read a line plot. • Make an observation using data from a graph. • Answer basic questions about the data shown.	• data • graphs • analyze • interpret (T) • comparison	• When learners are first analyzing and interpreting data, the comparison problems should be within the numerical proficiency range to allow learners to focus on the data comparison. (IN) • Learners might misinterpret the scale on the bar or picture graph when reading data and answering questions. (ME) • Learners might misinterpret the symbols used on a picture graph. (ME) • Learners may struggle with the addition or subtraction needed to solve the problem. (ME) • Learners might mix up the categories being compared. (ME)
Courses		
Mathematics 2		

THIRD GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

Counting and Cardinality (CC)

Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

3.NO.CC.1 Read and write numbers up to 10,000 using objects or visual representations, including standard, word, and expanded forms.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.NO.CC.1 2.NO.CC.2 2.NO.CC.3		4.NO.CC.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Count forward to and backward from 1000. • Understand place value. • Identify place values up to ten thousand. • Identify the value of a digit given the place value. • Understand how to read numbers in standard, word, and expanded forms. • Read and write numbers up to 1000 using standard, word, and expanded forms.	• ten thousand • standard form • word form • expanded form	• This standard involves a traditional understanding of place value, which supports the standard algorithm. (IN) • Spelling is not assessed. (IN) • Learners might errantly include the word “and” when naming numerals beyond 100. (ME) • Learners might skip or repeat numbers. (ME) • Learners may lose track of their place in the count. (ME)

Courses
Mathematics 3

THIRD GRADE

Counting and Cardinality (CC)
Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

3.NO.NBT.1 Compare two four-digit numbers using symbols >, <, and =. Justify comparisons based on the value of thousands, hundreds, tens, and ones.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.NO.NBT.1 2.NO.NBT.2	3.NO.NBT.2 3.NO.NBT.3	4.NO.NBT.2

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Count by ones, tens, hundreds, or thousands. • Understand and identify place value (thousands, hundreds, tens, ones). • State the value of each digit in a multi-digit number. • Understand the concept of greater than, less than, and equal to. • Identify and use the symbols >, <, =. • Compare two three-digit numbers using the symbols >, <, and =.	• place value (thousands, hundreds, tens, ones) • Symbols >, <, and = • digit • compare	• Teaching quantities using the vocabulary of greater than, less than, and equal to will lend to an understanding of thousands, hundreds, tens, and ones before moving to the symbolic level of comparing two four-digit numerals with symbols of <, >, or =. (IN) • Ordering numbers may be included for number comparisons. (IN) • Learners tend to confuse the < and > symbols. (ME) • Learners may compare digits individually without considering their place value. (e.g., Compare 285 and 542. Learners may think 285 is greater because 8 is larger than 4, ignoring the hundreds.) • Learners may transpose or reverse digits. (ME)

Courses
Mathematics 3

THIRD GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

3.NO.NBT.2 Apply place value understanding to round whole numbers to the nearest 10 or 100.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.NO.NBT.1	3.NO.NBT.1 3.NO.NBT.3 3.NO.NBT.4	4.NO.NBT.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Count forward and backward within 1000. • Skip count by tens within 1000. • Understand place value (thousands, hundreds, tens, ones). • Identify place value positions (hundreds, tens, ones). • Understand the concept of estimation and rounding. • Use a math tool to help identify and represent whole numbers' positions and how that relates to rounding (number line, hundreds chart). • Round numbers to the nearest ten. • Round numbers to the nearest hundred.	• place value • round (whole numbers to the nearest ten or hundred) • nearest	• Use tools to help identify and represent a number's position and how that relates to rounding rather than teaching the procedure of 5 and greater round-up. These tricks do not help when learners must apply rounding understanding to larger digit numerals, fractions, or decimals. (IN) • Learners may not round to the next ten or hundred when the digit to the right of the rounding place is a five or greater. (ME) • Learners may not understand which digit to look at when rounding. (ME)

Courses

Mathematics 3

THIRD GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

3.NO.NBT.3 Add and subtract within 1000 using place value strategies, algorithms, and/or the relationship between addition and subtraction.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.NO.NBT.3 2.NO.NBT.4 2.NO.NBT.5 2.AR.OA.1	3.NO.NBT.1 3.NO.NBT.2	3.AR.OA.3 4.NO.NBT.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand place value (thousands, hundreds, tens, ones). • Write numbers in expanded form. • Compose and decompose numbers using place value. • Add and subtract automatically within 20 (vertically and horizontally). • Add and subtract within 100. • Understand the concept of algorithm. • Use the standard algorithm to add and subtract numbers. • Understand the inverse relationship of addition and subtraction. • Add and subtract numbers without regrouping. • Add and subtract numbers with regrouping.	• addition • subtraction • place value • algorithm	• Addition and subtraction strategies and the relationship are foundational prior to the standard algorithm. Once the standard algorithm is taught, it is very difficult to build relationships and strategies within addition and subtraction. The order of teaching will be instrumental. (IN) • When adding, learners may not regroup the tens digit when the sum in a place value column exceeds 9. (ME) • When regrouping in subtraction, the learners may try to subtract the larger digit from the smaller digit rather than regroup. (ME) • When regrouping in subtraction, learners may not take a ten away from the tens when adding it to the one. (ME) • Learners might not line up the place values when adding and subtracting with the standard algorithm. (ME) • Since the standard algorithm is not expected until this standard, learners may struggle with subtraction if the minuend is a number in the hundreds (500-79). (ME)

Courses

Mathematics 3

THIRD GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

3.NO.NBT.4 Multiply one-digit whole numbers by multiples of 10 within 100.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.AR.OA.1 2.AR.OA.5	1.NO.CC.5 2.AR.OA.5 3.NO.NBT.2 3.AR.OA.1 3.AR.OA.2 3.AR.OA.4 3.AR.OA.6 4.NO.NBT.1	4.NO.NBT.5

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Use repeated addition to find the number of objects in a rectangular array. • Understand the concept of multiplication. • Multiply automatically to 5 (vertically and horizontally). • Skip count by tens. • Mentally add and subtract 10 to a given number.	• whole numbers • multiples	• Use of visual models will build an understanding of multiplication before moving to the use of numerals. (IN) • Building a strong understanding of the multiplication of 10s could lead to the multiplication of 5s. (IN) • Learners may need support in building an understanding of place value if their understanding is solely the procedure of adding a zero. (IN) • Learners may have difficulty multiplying multiples if they are not fluent with single-digit multiplication. (ME) • Learners may not understand multiplying by 10 shifts the digit one place to the left increasing the value tenfold. (ME) • Learners may incorrectly place or omit the zero in the product. (ME)

Courses
Mathematics 3

THIRD GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

3.NO.NF.1 Partition two-dimensional figures into equal areas and express the area of each part as a unit fraction of the whole. Describe using the language of sixths, eighths, a sixth of, and an eighth of.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.NO.NF.1 2.NO.NF.3	3.NO.NF.3 3.NO.NF.4 3.NO.NF.5	4.NO.NF.3 4.NO.NF.7

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify two-dimensional figures. • Understand the concept of equal shares. • Understand the concept of half, third, fourth, sixth, and eighth. • Understand the concept that dividing a shape into more equal shares results in smaller shares.	• unit fraction • partition • whole • sixth, eighth • sixth of, eighth of • _ out of 6, _ out of 8	• Fractional notation is not expected in first and second grade for 1/2, 1/4, and 1/3. The language of half, fourth, and third has been the focus within fractions of two-dimensional figures. (IN) • Learners may need support describing the shares with appropriate academic language. (IN) • Some learners may create equal parts but not use the entire whole. (ME) • Some learners may use the entire whole but not create equal parts. (ME)

Courses

Mathematics 3

THIRD GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

3.NO.NF.2 Represent and understand a fraction as a number on a number line.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
	2.NO.NF.2 3.NO.NF.3 3.NO.NF.4 3.NO.NF.5 3.GM.M.1 3.DPS.D.2	4.NO.NF.3 4.NO.NF.6

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify if a given shape is partitioned into equal shares. • Identify if a whole shape is used during partitioning. • Identify numerator and denominator. • Write standard fractions. • Order fractions with the same denominator from least to greatest. • Understand the concept of a half, third, fourth, sixth, and eighth. • Locate fractions on a number line. • Represent fractions on a number line with equal intervals (equidistant).	• fraction • whole number • number line • numerator • denominator	• Learners will need practice with number lines having varied intervals to see the connection between whole number intervals and fractional intervals. (IN) • Learners will have a better understanding of fractions on a number line if they have a strong foundation of part/whole fractions. (IN) • Learners may struggle with the representation of fractions when incremented on a number line that ends at one. (ME) • Learners might not scale fractions appropriately based on their denominators when placing them on the number line. (ME)

Courses
Mathematics 3

THIRD GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

3.NO.NF.3 Represent equivalent fractions using visual representations and number lines.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.NO.NF.2 2.NO.NF.3	3.NO.NF.1 3.NO.NF.2 3.NO.NF.4 3.NO.NF.5 3.GM.M.1 3.DPS.D.2	4.NO.NF.1 4.NO.NF.3 4.NO.NF.4 4.NO.NF.5 5.NO.NF.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the concept of a half, third, fourth, sixth, and eighth. • Locate fractions on a number line. • Represent fractions on a number line. • Represent fractions using visual models. • Understand the concept of equivalent fractions.	• equivalent fractions • fractions • visual model • number line	• Learners are not generating equivalent fractions until standard 4.NO.NF.3, but they are recognizing equivalent fractions using visual representations, such as fractions bars and number lines. (IN) • Learners might begin to recognize multiplicative patterns, but they do not need to be taught multiplication of fractions to represent equivalent fractions with visual representations and number lines. (IN) • Learners may assume the fraction with the larger denominator is the larger number. (ME) • Learners may confuse numerators and denominators. (ME) • Learners might have difficulty evenly partitioning number lines or fraction bars. (ME) • Learners may not place fractions on the number line accurately. (ME)

Courses
Mathematics 3

THIRD GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

3.NO.NF.4 Recognize whole numbers as fractions and express fractions that are equivalent to whole numbers.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
	3.NO.NF.1 3.NO.NF.2 3.NO.NF.3 3.NO.NF.5 3.GM.M.1 3.DPS.D.2	4.NO.NF.2 4.NO.NF.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the difference between a whole number and a fraction. • Understand the concept of a half, third, fourth, sixth, and eighth. • Locate fractions on a number line. • Represent fractions on a number line. • Represent fractions using visual models. • Identify equivalent fractions with a visual model and number line.	• whole number • fraction • equivalent fractions • unit fraction • proper fraction	• This standard goes beyond one as learners express fractions equivalent to whole numbers. (IN) • Learners might be able to write the fraction and their equivalency but may be unable to read it accurately, e.g., 4/4 and 4/1. (ME) • Learners may not conceptually understand the difference between 4/1 and ¼. (ME) • Learners might have difficulty visualizing and understanding what a fraction such as 4/1 or 8/1 represents in relation to a whole number. (ME)

Courses

Mathematics 3

THIRD GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

3.NO.NF.5 Compare fractions of the same whole having the same numerators or denominators, using symbols >, <, and = by reasoning about their size (fractions should be limited to denominators of 2, 3, 4, 6, and 8 and should not exceed the whole).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.NO.NF.3	3.NO.NF.1 3.NO.NF.2 3.NO.NF.3 3.NO.NF.4 3.GM.M.1 3.DPS.D.2	4.NO.NF.5

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the difference between a whole number and a fraction. • Identify what each part (numerator, denominator) of a fraction represents. • Understand the concept of a half, third, fourth, sixth, and eighth. • Locate fractions on a number line. • Represent fractions on a number line. • Represent fractions using visual models. • Compare fractions of the same whole using a visual model. • Identify equivalent fractions with a visual model and number line. • Identify numerators and denominators. • Understand the concept of greater than, less than, and equal to. • Identify and use the symbols >, <, =.	• fractions • numerator • denominator) • symbols >, <, and = • compare • whole	• Fractions must be compared within the same whole represented before learners begin comparing fractions of varied sizes of wholes. (IN) • Ordering numbers may be included for number and fraction comparisons. (IN) • Learners tend to mix up the < and > symbols. (ME) • Learners may assume the fraction with the larger denominator is the larger number. (ME) • Learners might confuse the roles of numerators and denominators and assume the smaller numerator is always smaller, regardless of the denominator. (e.g. Think 1/2 is smaller than 3/8 because 1 is smaller than 3). (ME)

Courses
Mathematics 3

THIRD GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

Operations and Algebraic Thinking (OA)

Learners will analyze patterns and relationships to generate and interpret numerical expressions.

3.AR.OA.1 Using mental strategies, multiply and divide basic facts within 100. Automatically multiply and divide up to 5 x 5 and 10s facts.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.NO.CC.4 2.AR.OA.5	2.AR.OA.1 3.NO.NBT.4 3.AR.OA.2 3.AR.OA.4 3.AR.OA.5	4.NO.NBT.5 4.NO.NBT.6 4.AR.OA.1 4.AR.OA.4 5.AR.OA.1 5.AR.OA.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Skip count by 2s, 3s, 4s, 5s, and 10s. • Understand the concepts of multiplication and division. • Use appropriate terms to represent digits in a multiplication or division equation (factors, product, quotient, divisor, dividend). • Recognize the formats of division problems and symbols used to indicate multiplication (x, (), •). • Identify and use multiplication and division strategies. • Add automatically to 20 (vertically and horizontally). • Use repeated addition to find the number of objects in a rectangular array. • Mentally add and subtract 10 to a given number.	• multiply • divide • mental strategies • automatically • factor • product • quotient • divisor • dividend	• Automaticity is built through strategy practice and exposure but is practiced through rehearsal once learners own their understanding. (IN) • As defined in the 2023 standards, automatic is to multiply without conscious thought or attention. (IN) • Learners might be fluent in their automaticity of multiplication but not division. (IN) • Learners might mix up operations (multiplication and addition, subtraction and division). (ME) • Learners may not understand the relationship between multiplication and division. (ME)

Courses

Mathematics 3

THIRD GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

3.AR.OA.2 Apply the properties of operations to solve multiplication and division equations and justify thinking.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
	2.AR.OA.2 3.NO.NBT.4 3.AR.OA.1 3.AR.OA.6	4.NO.NF.4 4.AR.OA.2 5.AR.OA.2

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the concepts of multiplication and division. • Multiply automatically to 5x5 and tens facts. • Identify the +, -, x, ÷ and = symbols. • Distinguish between the +, -, x, ÷ and = symbols. • Understand the concept of an equation. • Understand how the +, - , x, ÷ and = symbols are used to write equations. • Understand that the = sign shows a balance between either side of it rather than indicating the performance of an action. • Develop an understanding of the commutative property of addition and multiplication. • Develop an understanding of the inverse relationship between addition and subtraction and between multiplication and division. • Develop an understanding of the identity property of 0 (addition) and identity property of 1 (multiplication). • Develop an understanding of the property of 0 (multiplication). • Develop an understanding of associative property. • Develop an understanding of the distributive property.	• properties of operations (associative, commutative, distributive) (T-name specific B-skills within properties of operations) • multiplication • division • equations	• This standard develops an understanding of the properties and operations to solve multiplication and division equations. (IN) • Learners are not expected to name the properties of operations in third grade, but they will apply the properties within multiplication and division equations to justify their thinking. (IN) • Learners might not perform the operations in parentheses first when using the associative and distributive properties. (ME) • When using the distributive property, learners might distribute only one factor. (ME) • Learners might confuse the signs when distributing multiplication over subtraction. (ME)

Courses
Mathematics 3

THIRD GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

3.AR.OA.3 Solve two-step authentic word problems using addition and subtraction within 1000, including equations with a letter as an unknown.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.NO.NBT.3 2.NO.NBT.4 2.AR.OA.3 2.AR.OA.4 3.NO.NBT.3	3.GM.M.4 3.GM.M.5 3.GM.M.6 3.DPS.D.3	4.NO.NBT.4 4.NO.NF.6 4.AR.OA.3 4.GM.M.4 4.DPS.D.3 5.NO.NF.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Add and subtract within 20 automatically (vertically and horizontally). • Understand place value (ones, tens, hundreds). • Add and subtract numbers without regrouping within 500. • Add and subtract numbers by regrouping within 500. • Understand the concept of the unknown. • Identify the operation(s) necessary to solve a problem. • Determine the unknown (variable) in a problem. • Solve one-step problems. • Solve and evaluate the answer to a word problem.	• addition • subtraction • equation • unknown (letter as unknown) • word problem • variable	• Learners use objects, drawings, verbal explanations, and numerical representations to show or explain their thinking. (IN) • Build proficiency with solving word problems through the context of the problem rather than underlining or focusing on keywords or numbers. (IN) • When regrouping in subtraction, the learners may try to subtract the larger digit from the smaller digit rather than regroup. (ME) • When regrouping in subtraction, learners might not take the ten or hundred away when adding it to the ones or tens. (ME) • Learners may not perform the second step of the problem. (ME) • Learners might ignore key information stated in the problem or consider information that does not apply to the problem. (ME) • Learners may not label their answers. (ME)

Courses
Mathematics 3

THIRD GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

3.AR.OA.4 Use strategies and visual models to solve authentic word problems with multiplication within 100, including unknowns, using grouping models and equations.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.AR.OA.5	3.NO.NBT.4 3.AR.OA.1 3.AR.OA.5 3.GM.M.8 4.NO.NBT.5	4.AR.OA.3 4.AR.OA.5 4.GM.M.5

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the concept of multiplication and the symbols used to represent multiplication (x, (), •). • Understand the concept of multiplication and recognize the models used to represent multiplication problems. • Multiply automatically to 5x5 and tens facts (vertically and horizontally). • Skip count (twos, fives, tens). • Use repeated addition and rectangular arrays - multiplication strategies. • Understand the concept of the unknown. • Understand and use grouping models. • Recognize keywords in a word problem to accurately solve it (factor, product, equal groups, times, groups of). • Understand how to write equations and solve them. • Write a multiplication equation using the correct operation symbols and unknowns.	• visual models • multiplication • unknowns • grouping models • equations	• The standards for the area are included because the area model is often used as a visual model to support multiplication within 100. (IN) • Build proficiency with solving word problems through the context of the problem rather than underlining or focusing on keywords or numbers. (IN) • When solving two-step problems, learners may not complete the second step of the problem. (ME) • Learners might ignore key information stated in the problem or consider information that does not apply to the problem. (ME) • Learners may not label their answers. (ME) • Learners may not understand terms used for multiplication in the problem (e.g., product, times, groups of). (ME) • Learners might transpose digits or might include incorrect numbers when writing equations. (ME)

Courses
Mathematics 3

THIRD GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

3.AR.OA.5 Use strategies and visual models to solve authentic word problems with division within 100, including unknowns, using grouping models and equations.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
	3.AR.OA.1 3.AR.OA.4 4.NO.NBT.6	4.AR.OA.3 4.GM.M.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the concept of division and recognize the models used to represent division problems. • Understand the concept of division and the formats used to write division problems. • Multiply and divide automatically by 5x5 and tens. • Recall and apply division strategies. • Understand the concept of unknowns. • Understand and use grouping models. • Understand how to write equations and solve them. • Write a division equation using the correct operation symbol (÷, /) and unknowns.	• visual models • division • unknowns • grouping models • equations	• Build proficiency in solving word problems through the context of the problem rather than underlining or focusing on keywords or numbers. (IN) • Learners might ignore key information stated in the problem or consider information that does not apply to the problem. (ME) • When solving two-step problems, learners might not complete the second step of the problem. (ME) • Learners may not label their answers. (ME) • Learners might transpose digits or may include incorrect numbers when writing equations. (ME)

Courses

Mathematics 3

THIRD GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

3.AR.OA.6 Identify arithmetic patterns and explain them using the properties of operations.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.AR.OA.7 1.NO.CC.5 2.NO.CC.4 2.AR.OA.2 2.AR.OA.6	3.NO.NBT.4 3.AR.OA.2 4.AR.OA.2 5.AR.OA.5	4.AR.OA.6 5.NO.NBT.7

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand and identify patterns. • Skip count. • Identify and apply the properties of operations for addition/ subtraction and multiplication/ division.	• numerical patterns • properties of operations (associative, commutative, distributive) (T-name specific B-skills within properties of operations)	• The goal of this standard is to recognize patterns across the scope of mathematics. Patterning is found vertically throughout the grades to build upon knowledge. (IN) • Learners might have difficulty generating and/or repeating the pattern as the pattern rule gets more complex. (ME) • Learners may not recognize the increment or might miss an alternating increment pattern using irregular increments. (ME)

Courses
Mathematics 3

THIRD GRADE

Geometry and Measurement (GM)
Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties; represent and solve problems using a coordinate plane.

3.GM.G.1 In two-dimensional shapes, identify lines, angles (right, acute, obtuse), and perpendicular and parallel lines.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.GM.G.1 1.GM.G.3 2.GM.G.1 2.GM.G.3	3.GM.G.2 3.GM.M.7	4.GM.G.1 4.GM.G.2 4.GM.M.6 5.GM.G.1
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify two-dimensional shapes (circle, square, rectangle, trapezoid, triangle, right triangle, quadrilateral, parallelogram, rhombus, hexagon, pentagon, octagon). • Identify attributes of two-dimensional shapes. • Identify lines and angles. • Identify types of angles (right, acute, obtuse) and their attributes. • Identify and distinguish between parallel and perpendicular lines.	• lines • angles (right, acute, obtuse) • parallel lines • perpendicular lines • two-dimensional	• Learners may get acute and obtuse angles mixed up. (ME) • Learners might confuse parallel and perpendicular lines. (ME)

Courses
Mathematics 3

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties; represent and solve problems using a coordinate plane.

3.GM.G.2 Sort quadrilaterals into categories based on attributes.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
K.GM.G.3 1.GM.G.3 2.GM.G.3	3.GM.G.1 4.GM.G.2	5.GM.G.1
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify quadrilaterals. • Identify the attributes of quadrilaterals (sides, angles, parallel sides, right angles, congruent sides). • Identify attributes of specific quadrilaterals. • Sort and classify quadrilaterals based on attributes (trapezoid, parallelogram, rectangle, rhombus, square).	• quadrilaterals (trapezoid, parallelogram, rectangle, rhombus, square) • attributes	• Standards 3.GM.G.1 and 3.GM.G.2 can be taught in tandem as they are parallel standards. (IN) • Learners might confuse defining and non-defining attributes when sorting quadrilaterals. (ME)

Courses
Mathematics 3

THIRD GRADE

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties; represent and solve problems using a coordinate plane.

3.GM.G.3 Identify lines of symmetry in quadrilaterals.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
		4.GM.G.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Divide an object into two equal shares (halves). • Identify quadrilaterals (trapezoid, parallelogram, rectangle, square, rhombus). • Understand the concept of symmetry. • Understand the concept of lines of symmetry.	• symmetry • lines of symmetry • quadrilaterals (trapezoid, parallelogram, rectangle, rhombus, square)	• This standard connects to learning that is happening in middle and secondary grades. (IN) • Learners tend to believe a figure only has one line of symmetry when, in fact, it may have more. (ME)

Courses
Mathematics 3

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

3.GM.M.1 Measure lengths using rulers marked with halves and fourths of an inch.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.GM.M.1 2.GM.M.1 2.GM.M.2	3.NO.NF.2 3.NO.NF.3 3.NO.NF.4 3.NO.NF.5 3.DPS.D.2	4.GM.M.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify fractions on a number line from 0 to 1. • Identify where to start measuring an object on a ruler. • Measure length using non-standard units. • Identify standard units of measure (inches). • Measure length using a ruler to the nearest whole number unit (inch). • Identify the fourth and half marks on a ruler. • Measure length to the nearest half-inch. • Measure length to the nearest fourth (quarter) inch.	• ruler • inch • half inch • fourth inch • measure • length	• Teachers should be mindful of the tools selected and check if learners are using them appropriately. (IN) • Learners may not understand the meaning of the marks on the measurement tool or how the tool is used to measure objects. (ME) • Learners may start measuring from the one rather than the beginning of the tool. (ME) • Learners tend to have difficulty finding the markings for half and quarter inch on the ruler and get the lines on the ruler confused. (ME) • Learners may not look for the zero on the ruler instead of just starting to measure at the end of the ruler. (ME)

Courses
Mathematics 3

THIRD GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

3.GM.M.2 Measure and estimate liquid volumes and masses of objects using standard units. Solve one-step authentic word problems involving masses or volume given in the same units.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
		4.GM.M.1 4.GM.M.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify the parts of measurement tools and understand how to read them for measurement. • Understand the concept of measuring mass (weight). • Identify standard units used to measure mass (lb., oz., g, kg). • Understand the concept of measuring liquid volume. • Understand standard units used to measure liquid volume (cup, quart, gallon, liter, milliliter). • Add and subtract to solve for unknown. • Multiply to solve for unknown.	• volume • mass • units	• Learners may not understand the relationship between standard units used to measure mass and volume. (ME) • Learners may not label their answers. (ME) • Learners might ignore key information stated in the problem or consider information that does not apply to the problem. (ME) • Learners might measure before they estimate rather than estimate first. (ME) • Learners may not understand the difference between mass (amount of matter in an object) and volume (amount of space and object takes up). (ME) • Learners might have difficulty reading measurement tools (balance, graduated cylinder) accurately or might misinterpret the scales used on the tools. (ME)

Courses
Mathematics 3

THIRD GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

3.GM.M.3 Tell and write time to the nearest minute and measure time intervals in minutes.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.GM.M.3 2.GM.M.3	3.GM.M.4	4.GM.M.1 4.GM.M.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Tell and write time to hour and half hour. • Skip count by five. • Understand the concept of a.m. and p.m. • Tell and write the time to the quarter-hour. • Tell and write time to the nearest five minutes. • Tell and write time to the nearest minute. • Understand the concept of time intervals to the nearest minute.	• minute • hour • half-hour • quarter-hour	• Learners have more understanding and exposure to digital clocks, so bringing connections from their previous knowledge to analog clocks will support their learning. (IN) • This is the last time in which learners will be focusing on telling time. Moving forward, time is incorporated into word problems. (IN) • Learners tend to have difficulty identifying time to the nearest minute on an analog clock as they move from counting by fives to counting by ones. (ME) • Learners might misinterpret the position of the hour hand when it is close to the next hour or misinterpret the position of the minute hand when it is not on a multiple of five. (ME) • Learners might mix up the hour and minute hand on an analog clock. (ME)

Courses

Mathematics 3

THIRD GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

3.GM.M.4 Solve elapsed time authentic word problems on the hour and the half-hour, using a variety of strategies.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.GM.M.3 2.GM.M.3	3.AR.OA.3 3.GM.M.3	4.GM.M.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Read an analog clock. • Tell and write time to the nearest hour and half hour. • Skip count by hours. • Skip count by half hours. • Understand the concept of time intervals. • Understand the concept of elapsed time. • Use addition and subtraction to solve problems.	• elapsed time • hour • half-hour • word problem	• Build proficiency with solving word problems through the context of the problem rather than underlining or focusing on keywords or numbers. (IN) • Discrete counters may need additional support with the understanding of elapsed time. (IN) • Learners might have difficulty envisioning the amount of time that has elapsed, either prior or future. (ME) • Learners may struggle with elapsed time since it is based on increments of 60 minutes, which contrasts with their typical focus on numbers within 100 when connecting and solving problems. (ME)

Courses

Mathematics 3

THIRD GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

3.GM.M.5 Solve authentic word problems involving dollar bills, quarters, dimes, nickels, and pennies using the $ and ¢ symbols appropriately.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.GM.M.5 2.GM.M.4	3.AR.OA.3	4.GM.M.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify coins (penny, nickel, dime, quarter, dollar bill). • Identify the value of coins (penny, nickel, dime, quarter dollar bill). • Identify and use the symbols $ and ¢. • Skip count by fives and tens. • Use addition, subtraction, and multiplication to solve problems.	• dollar bill • quarter • dime • nickel • penny • $ and ¢ symbols	• Learners may confuse the values of different coins. (ME) • Learners may not understand how the values of different coins and dollar bills relate. (ME) • Learners may not include the symbols when they record their answers or write the problem out. (ME) • Learners might have difficulty adding and subtracting amounts of money, especially when regrouping is involved. (ME) • Learners might have difficulty counting and determining correct change when given a purchase amount and payment. (ME) • Learners might have difficulty determining which operation is needed to solve the problem. (ME)

Courses
Mathematics 3

THIRD GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

3.GM.M.6 Solve problems involving the perimeters of rectangles given the side lengths or when given the perimeter and unknown side length(s).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
	3.AR.OA.3 3.GM.M.7 3.GM.M.8	4.AR.OA.3 4.GM.M.3 4.GM.M.5 5.GM.M.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify rectangles. • Identify the sides of rectangles. • Add automatically to 20 (vertically and horizontally). • Understand place value. • Understand the concept of perimeter. • Understand the concept of the unknown.	• perimeter • rectangle • side length • unknown	• Use rectangles with the same perimeter and different areas or with the same area and different perimeters. (IN) • Learners tend to mix up the concepts of perimeter and area. (ME) • Learners may struggle to add the lengths of the sides correctly when dealing with larger numbers. (ME)

Courses
Mathematics 3

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

3.GM.M.7 Recognize area as an attribute of plane figures and understand concepts of area measurement.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
	3.GM.G.1 3.GM.M.6 3.GM.M.8	4.GM.M.5 5.GM.M.2 5.GM.M.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify a rectangle. • Identify attributes (sides, angles) of a rectangle. • Understand the concept of area. • Understand how to use unit squares to find an area. • Use the addition of unit squares to find an area. • Multiply the side lengths to find an area.	• area • rectangle • side length • unit square • whole number	• A plane figure, which can be covered without gaps or overlaps by *n* unit squares, is said to have an area of *n* square units. (IN) • Learners tend to mix up the concepts of perimeter and area. (ME)

Courses
Mathematics 3

THIRD GRADE

Measurement (M) *Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.*
3.GM.M.8 Find the area of a rectangle with whole-number side lengths by modeling with unit squares; show that area can be additive and is the same as would be found by multiplying the side lengths.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
	3.AR.OA.4 3.GM.M.6 3.GM.M.7	4.GM.M.3 4.GM.M.5 5.GM.M.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify a rectangle. • Identify attributes (sides, angles) of a rectangle. • Understand the concept of area. • Understand how to use unit squares to find an area. • Use the addition of unit squares to find an area. • Multiply the side lengths to find an area.	• area • rectangle • side length • unit square • whole number	• Learners tend to mix up the concepts of perimeter and area. (ME) • Learners may incorrectly count or misplace unit squares when covering a rectangle's area. (ME) • Learners may not completely cover the entire area of the rectangle with unit squares. (ME) • Learners may overlap unit squares resulting in double-counting the area of certain parts of the rectangle. (ME) • Learners might not align the unit squares with the sides of the rectangle. (ME)

Courses
Mathematics 3

THIRD GRADE

Data, Probability, and Statistics (DPS)

Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.

Data (D)

Learners will represent and interpret data.

3.DPS.D.1 Formulate questions to collect, organize, and represent data with more than four categories using scaled pictures and bar graphs.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.DPS.D.1 2.DPS.D.1		4.DPS.D.1
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the difference between a question and a statement. • Form questions. • Collect data (tally marks, counting, etc.). • Organize data (sort, categorize). • Read bar and picture graphs. • Read scaled bar and picture graphs.	• scaled • picture graph • bar graphs • collect • organize • represent • data	• This will be the first time learners are using scaled graphs. They may need additional practice with scales, creating scales, and scales that involve half of something. (IN) • When using tally marks to record information, learners forget to cross the five after they have four marks recorded. (ME) • Learners may have difficulty drawing bars or pictures proportional to the quantities represented or following the scale of the graph consistently. (ME) • Learners may not consider the scale when representing data on a scaled bar or picture graphs. (ME)

Courses
Mathematics 3

THIRD GRADE

Data (D) *Learners will represent and interpret data.*		
3.DPS.D.2 Generate data and create line plots marked in whole numbers, halves, and fourths of a unit.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
2.DPS.D.2	3.NO.NF.2 3.NO.NF.3 3.NO.NF.4 3.NO.NF.5 3.GM.M.1	4.NO.NF.3 4.NO.NF.5 4.GM.M.3 4.DPS.D.2 5.DPS.D.1
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Collect, organize, and sort data. • Understand how to read and use a number line. • Read line plots. • Measure length in whole-number units (inches, feet, yards, centimeters, meters). • Measure length to the nearest half or quarter (fourth) of an inch.	• data • line plots • whole numbers • halves • fourths • unit	• Measurement data could be used for the creation of line plots. (IN) • The standard utilizes unit fractions and proper fractions. (IN) • Learners may be more successful starting with line plots containing whole numbers, then with an added layer of halves, and then the full standard of mixing units. (IN) • Learners will need proficiency in creating equivalent fractions prior to being able to utilize the understanding in the creation of line plots. (IN) • Learners may not understand how fractions relate to units of measurement and may have difficulty applying them accurately in data representations. (ME) • Learners might misplace data points on line plots or group them incorrectly. (ME) • Learners might have difficulty marking and spacing data points on a line plot according to fractional units. (ME)

Courses
Mathematics 3

THIRD GRADE

Data (D) *Learners will represent and interpret data.*		
3.DPS.D.3 Analyze data and make simple statements to solve one- and two-step problems using information from the graphs.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
1.DPS.D.2 2.DPS.D.3	3.AR.OA.3	4.AR.OA.3 4.DPS.D.3 5.DPS.D.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Read bar and picture graphs. • Understand how to read a scaled bar or picture graph. • Understand how to read a line plot.	• data • graphs	• Learners may not read and interpret the scales and labels used on bar or picture graphs correctly (ME) • Learners might misinterpret the intervals and units used on graphs and line plots. (ME) • Learners might not recognize which data points or categories are relevant to solving the problem. (ME) • Learners might make errors calculating totals, differences, or averages from graphed data. (ME)
Courses		
Mathematics 3		

FOURTH GRADE

Number and Operations (NO)
Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

Counting and Cardinality (CC)
Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

4.NO.CC.1 Read numbers to the millions place, including word, standard, and expanded form. Write numbers to the millions place, including standard and expanded forms.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.NO.CC.1		5.NO.CC.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Count forward and backward to 1000. • Understand place value. • Understand how to read numbers in standard, word, and expanded form.	• millions • place value • standard form • expanded form • word form	• Spelling is not assessed. (IN) • Learners might errantly include the word “and” when naming numerals beyond 100. (ME) • Learners tend to misidentify the value of each digit or forget to include zero placeholders when writing a number in expanded form. (ME)

Courses
Mathematics 4

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

4.NO.NBT.1 Understand that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.NO.NBT.1	3.NO.NBT.4 4.NO.NBT.2 4.NO.NBT.3 4.NO.NBT.4 4.NO.NBT.5 4.NO.NBT.6	5.NO.NBT.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand place value (millions, hundred thousands, ten thousands, thousands, hundreds, tens, ones).	• whole number • place value • ones • tens • hundreds • thousands • ten thousands • hundred thousands	• Tricks and procedures of adding zeros will not lend to learners' place value understanding within multi-digit whole numbers. (IN) • Learners may struggle to understand that the value of a digit is determined by its position in the number, not just its numerical value. (ME)

Courses
Mathematics 4

FOURTH GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

4.NO.NBT.2 Compare two numbers to the millions place and decimals to the hundredths place, using symbols >, <, and =. Justify comparisons based on the value of the digits.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.NO.NBT.1	4.NO.NBT.1 4.NO.NBT.3 4.NO.NF.1	5.NO.CC.1 5.NO.NBT.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand place value (millions, hundred thousand, ten thousand, thousands, hundreds, tens, ones). • Understand the place value of decimals (tenths, hundredths). • Understand the concept of greater than, less than, and equal to. • Identify and use the symbols >, <, =.	• millions • place value • decimals • hundredths • symbols >, <, and =	• Comparing visual models will build an understanding that will lend to the justification of place value comparison prior to moving to the symbolic notation of decimal comparison. (IN) • Visual models that were previously used to model 100 are often used to represent one, with the smaller pieces representing decimals. Learners may need time to adjust to the visual's changing meaning. (IN) • Ordering numbers may be included for number comparisons. (IN) • Learners tend to mix up the < and > symbols. (ME) • When working decimals, learners may not realize that the larger numbers are actually smaller in size. (ME) • Learners may transpose or reverse digits. (ME)

Courses
Mathematics 4

FOURTH GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

4.NO.NBT.3 Apply place value understanding to round multi-digit whole numbers to any place.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.NO.NBT.1 3.NO.NBT.2	4.NO.NBT.1 4.NO.NBT.2 4.NO.NBT.4 4.AR.OA.3	5.NO.NBT.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand and identify place value (thousands, hundreds, tens, ones). • Understand the concept of estimation and rounding. • Use a math tool to help identify and represent whole numbers' positions and how that relates to rounding (number line, hundreds chart). • Locate the place value to which the number is to be rounded. • Round numbers to the nearest ten. • Round numbers to the nearest hundred. • Round numbers to the nearest thousand. • Find the range for the given rounded place value.	• place value • round • whole numbers • multi-digit	• When the digit to the right of the rounding place is five or greater, the learners may not round up. (ME) • When rounding within a three-digit or four-digit number, learners may have difficulty identifying which digit to focus on if rounding to the nearest ten or hundred. (ME)

Courses
Mathematics 4

FOURTH GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

4.NO.NBT.4 Add and subtract multi-digit whole numbers to the one million place using strategies, including the algorithm.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.NO.NBT.2 2.NO.NBT.3 2.NO.NBT.4 3.NO.NBT.3 3.AR.OA.3	4.NO.NBT.1 4.NO.NBT.3 4.GM.M.4	5.NO.NBT.5 5.NO.NBT.6 5.NO.NF.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand and apply place value (ones, tens, hundreds, thousands, ten thousand, hundred thousand, millions). • Write numbers in expanded form up to 10,000. • Compose and decompose numbers using place value. • Understand the value of the digits in the algorithm. • Add and subtract automatically to 20. • Add and subtract multi-digit whole numbers without regrouping within 1000. • Add and subtract multi-digit whole numbers with regrouping within 1,000. • Use the standard algorithm.	• add • subtract • whole number • millions • algorithm • place value	• When subtracting with regrouping, the learners tend to try to subtract the smaller digit from the larger digit. (ME) • Learners may not regroup the tens digit when the sum in a place value column exceeds 9. (ME) • Learners might misalign digits when adding or subtracting multi-digit numbers. (ME) • Learners may not attend to the symbols and mix up the addition and subtraction operations. (ME)

Courses
Mathematics 4

FOURTH GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

4.NO.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number and multiply two two-digit numbers. Show and justify the calculation using equations, rectangular arrays, and models.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.NO.NBT.4 3.AR.OA.1	3.AR.OA.4 4.NO.NBT.1 4.NO.NBT.6 4.AR.OA.1 4.AR.OA.2 4.AR.OA.3 4.AR.OA.5 4.GM.M.5 4.DPS.D.3	5.NO.NBT.4 5.NO.NBT.5 5.NO.NBT.6 5.NO.NBT.7

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Add automatically to 20. • Multiply automatically to 10x10 (vertically and horizontally). • Use rectangular arrays, equations, and visual models to solve problems. • Recognize the symbols used to indicate multiplication (x, (),•). • Understand and apply place value (ones, tens, hundreds, thousands). • Add multi-digit numbers with and without regrouping. • Find the product when multiplying single-digit factors. • Multiply two digits by one digit. • Multiply three digits by one digit. • Identify an equation that matches a model of multiplication.	• multiply • whole number • equation • rectangular array	• This standard does not include the standard algorithm. (IN) • Learners may forget to multiply by all the digits. (ME) • Learners may not include the zero placeholder when writing larger numbers. (ME) • Learner may not regroup when the product in a place value column exceeds 9. (ME)

Courses

Mathematics 4

FOURTH GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

4.NO.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors using place value strategies. Show and justify the calculation using equations, rectangular arrays, and models.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.AR.OA.1	3.AR.OA.5 4.NO.NBT.1 4.NO.NBT.5 4.AR.OA.1 4.AR.OA.3	5.NO.NBT.6

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Add and subtract automatically through 20. • Multiply and divide whole numbers through 10x10. • Understand and apply place value (ones, tens, hundreds, thousands). • Use rectangular arrays, equations, and visual models to solve problems. • Understand parts of a division problem (divisor, dividend, quotient, remainder). • Understand formats of a division problem (÷, ⌐, fraction representation). • Add and subtract multi-digit numbers with and without renaming. • Find the product when multiplying single-digit factors. • Multiply multi-digit numbers. • Identify the inverse relationship between multiplication and division. • Use place value strategies to perform multi-digit division. • Divide two digits by one digit without remainders. • Divide three digits by one digit without remainders. • Divide four digits by one digit without remainders. • Understand the concept of remainders. • Divide two digits by one digit with a remainder. • Divide three digits by one digit with a remainder. • Identify an equation that matches a model of division.	• whole number • quotient • remainder • dividend • divisor • equation • rectangular array	• This standard does not include the standard algorithm. (IN) • Learners might forget to divide by all the digits. (ME) • Learners might forget the zero placeholder when writing larger numbers. (ME) • Learners might leave out the remainder or write it incorrectly. (ME) • Learners may not complete all the steps in the long division problem. (ME)

Courses
Mathematics 4

FOURTH GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

4.NO.NF.1 Express equivalent fractions with a denominator of 10 and a denominator of 100 to generate a decimal notation.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.NO.NF.3	4.NO.NBT.2 4.NO.NF.3 4.NO.NF.5 4.GM.M.2 4.DPS.D.2	5.NO.NBT.1 5.NO.NBT.2 5.NO.NBT.3 5.NO.NBT.5 5.NO.NF.1
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify numerators and denominators in fractions. • Understand the concept of equivalent fractions. • Identify and use place value to tenths and hundredths. • Read and write decimal numbers. • Understand the relationship between fractions and decimal numbers.	• equivalent fractions • denominator • decimal notation	• Standard 4.NO.NF.1 relates to standard 4.GM.M.2 through metric conversions. (IN) • Learners might misplace the decimal when representing fractions as decimals. (ME) • Learners may have difficulty connecting the fraction with the decimal equivalent. (ME) • Learners may not understand how equivalent fractions relate to each other. (ME)

Courses
Mathematics 4

FOURTH GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

4.NO.NF.2 Explain and demonstrate how a mixed number is equivalent to a fraction greater than one and how a fraction greater than one is equal to a mixed number using visual fraction models and reasoning strategies (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.NO.NF.4	4.NO.NF.4 4.NO.NF.6 4.GM.M.3	5.NO.NF.2

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the concept of mixed numbers. • Identify mixed numbers. • Place fractions on a number line. • Understand how to use a visual fraction model. • Understand concepts and differences between proper and improper fractions. • Identify proper and improper fractions.	• mixed number • equivalent • visual fraction model • proper fractions • improper fractions • denominator	• Standard 4.NO.NF.1 relates to standard 4.GM.M.3 through fractions within length. (IN) • Learners may not understand that a mixed number and an improper fraction can represent the same quantity. (ME) • Learners might not divide correctly or may not include the remainder in the fraction part of the mixed number when converting improper fractions to mixed numbers. (ME) • Learners may not add the numerator when converting mixed numbers to improper fractions. (ME) • Learners may not understand that an improper fraction represents a value greater than one. (ME)

Courses
Mathematics 4

FOURTH GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

4.NO.NF.3 Generate equivalent fractions using numerical representations, visual representations, and number lines (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.NO.NF.1 3.NO.NF.2 3.NO.NF.3 3.DPS.D.2	4.NO.NF.1 4.NO.NF.5 4.GM.M.2 4.GM.M.3 4.DPS.D.2	4.NO.NF.4 5.NO.NF.1 5.NO.NF.3 5.DPS.D.1

Prerequisite Skills	Key Vocabulary	Common Misconceptions (Teacher and Learner) and Instructional Notes (IN)
• Understand the concept of equivalent fractions. • Represent fractions on number lines and using visual models. • Understand concepts and differences between proper and improper fractions. • Identify proper and improper fractions.	• equivalent fractions • numerical representation • visual representation • number line • proper fractions • improper fractions • denominator	• Learners are developing an understanding of equivalent fractions through visual representations and number lines to generate equivalent fractions using a numerical representation. (IN) • When placing fractions on a number line, learners may assume that a fraction with a larger denominator is larger than the one with a small denominator. (ME) • Learners may not understand that a mixed number and an improper fraction can represent the same quantity. (ME)

Courses
Mathematics 4

FOURTH GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

4.NO.NF.4 Demonstrate how equivalent fractions are generated by multiplying a fraction equivalent to 1 or the properties of multiplication (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.NO.NF.3 3.NO.NF.4 3.AR.OA.2 4.NO.NF.3	4.NO.NF.2 4.NO.NF.5 4.AR.OA.1 4.AR.OA.5	5.NO.NF.2 5.NO.NF.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the concept of equivalent fractions. • Represent fractions on number lines and using visual models. • Understand concepts and differences between proper and improper fractions. • Identify proper and improper fractions. • Apply multiplication facts within 100. • Apply the properties of multiplication (associative, property of 0, identification property of multiplication, distributive). • Identify the inverse relationship between multiplication and division up to 100. • Identify the numerator and denominator. • Identify fractions that are equivalent to 1.	• equivalent fractions • fraction equivalent • properties of multiplication • proper fractions • improper fractions • denominator • numerator	• Learners are developing an understanding of fractions through visual representations and number lines in standard 4.NO.NF.3 to notate equivalent fractions using a numerical representation and properties of multiplication. (IN) • When multiplying a fraction by a whole number to find an equivalent fraction, learners might not multiply both the numerator and denominator by the whole number. (ME) • Learners may not understand that a mixed number and an improper fraction can represent the same quantity. (ME)

Courses
Mathematics 4

FOURTH GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

4.NO.NF.5 Compare and order fractions having unlike numerators or denominators. Record comparisons using the symbols >, <, and =. Justify using a visual fraction model (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.NO.NF.3 3.NO.NF.5 3.DPS.D.2	4.NO.NF.1 4.NO.NF.3 4.NO.NF.4	5.NO.NBT.1 5.NO.NF.1
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand and apply symbols >, <, = to compare whole numbers. • Identify numerators and denominators. • Understand the concept of like and unlike denominators. • Identify if two fractions have like denominators. • Compare and order fractions with like denominators. • Represent fractions with like denominators on number lines and using visual models. • Identify equivalent fractions. • Create equivalent fractions. • Represent fractions with unlike denominators on number lines and using visual models. • Identify proper and improper fractions. • Understand and apply symbols >, <, = to compare fractions.	• numerator • denominator • order • like numerators and denominators • unlike numerators and denominators • symbols >, <, and = • proper fraction • improper fraction	• Fourth graders need to build a strong understanding of comparing and ordering fractions to support the progression in standards 5.NO.NF.1 and 5.NO.NBT.2. (IN) • Educators are encouraged to use part-whole comparison and number lines as visual fraction models. (IN) • Learners tend to confuse the < • and > symbols. (ME) • Learners tend to assume the numbers with larger denominators are larger rather than smaller than fractions with smaller denominators. (ME) • Learners may not understand that a mixed number and an improper fraction can represent the same quantity. (ME)

Courses
Mathematics 4

FOURTH GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

4.NO.NF.6 Solve authentic word problems by adding and subtracting fractions and mixed numbers with like denominators (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.NO.NF.1 3.NO.NF.2 3.AR.OA.3	4.NO.NF.2 4.GM.M.2 4.GM.M.3 4.DPS.D.2 4.DPS.D.3	5.NO.NF.3 5.GM.M.2 5.DPS.D.1 5.DPS.D.2

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Model using unit fractions. • Compose and decompose unit fractions. • Understand the meaning of and identify mixed numbers. • Understand the concept of adding and subtracting fractions with like denominators. • Identify proper and improper fractions. • Identify mixed numbers. • Convert between mixed numbers and improper fractions. • Understand the concept of equivalent fractions. • Determine if word problems require addition or subtraction. • Generate an equation and use fractions to solve a problem.	• add • subtract • fractions • mixed numbers • denominator • proper fraction • improper fraction • equivalent fractions • like denominators	• This is the first standard where learners add and subtract fractions and mixed numbers with like denominators. There would need to be instruction involving joining and separating parts with reference to the same whole and the use of visual fraction models and equations prior to building proficiency within word problems of addition and subtraction of fractions and mixed numbers. (IN) • Learners often have difficulty visualizing what is happening in a story problem. Pictures help them break the problem down into manageable parts. (ME) • Learners might struggle to identify common denominators when adding or subtracting fractions. (ME) • Learners might misinterpret the whole number part or the fractional part of mixed numbers. (ME) • Learners may add or subtract the denominators instead of just the numerators. (ME) • When working with mixed numbers, learners might forget to add or subtract the whole number parts. (ME)

Courses

Mathematics 4

FOURTH GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

4.NO.NF.7 Solve problems by multiplying fractions and whole numbers using visual fraction models (proper and improper fractions limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.NO.NF.1	4.AR.OA.1 4.AR.OA.5 4.GM.M.1 4.GM.M.2 4.GM.M.3 4.DPS.D.3	5.NO.NF.4 5.GM.M.2 5.DPS.D.2
Prerequisite Skills	**Key Vocabulary**	**Common Misconceptions (Teacher and Learner) and Instructional Notes (IN)**
• Apply multiplication facts within 100. • Understand how to use a visual fraction model. • Identify numerators and denominators. • Identify proper and improper fractions. • Understand and apply the concept of equivalent fractions.	• multiply • fractions • whole numbers • visual fraction models • proper fraction • improper fraction • denominator • equivalent fractions	• This is the first standard where learners are multiplying fractions and whole numbers. The use of visual models by teachers and students is imperative to build understanding and application of solving problems. (IN) • Learners need to use visual models to understand the distribution of multiplying fractions across wholes. (IN) • Learners often have difficulty visualizing what is happening in a story problem. Pictures help them to break the problem down into manageable parts. (ME) • When multiplying fractions, learners may not understand how the numerator and denominator interact during multiplication. (ME) • Learners may forget to simplify fractions or whole numbers before multiplying them. (ME) • Learners may not realize that multiplying a fraction by a whole number may result in a product that is smaller than the original fraction depending on the size of the whole number. (ME) • Learners might multiply the whole number by the denominator instead of the numerator. (ME) • Learners might multiply both the numerator and denominator by the whole number. (ME) • Learners may not understand that a whole number can be expressed by a fraction. (ME)

Courses

Mathematics 4

FOURTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

Operations and Algebraic Thinking (OA)

Learners will analyze patterns and relationships to generate and interpret numerical expressions.

4.AR.OA.1 Automatically multiply and divide through 10 x 10.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.AR.OA.1	4.NO.NBT.5 4.NO.NBT.6 4.NO.NF.4 4.NO.NF.7 4.AR.OA.3 4.AR.OA.4 4.GM.M.4 4.GM.M.5 4.DPS.D.3	5.AR.OA.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the concepts of multiplication and division. • Understand and apply multiplication and division terms (factors, product, quotient, divisor, dividend). • Recognize the formats of division problems and symbols used to indicate multiplication (x, (), •). • Add automatically to 20 (vertically and horizontally). • Use repeated addition to find the number of objects in a rectangular array. • Multiply automatically to 5x5 (vertically and horizontally) and tens. • Skip count by twos, fives, and tens. • Apply strategies to find multiplication and division facts of 6, 7, 8, and 9. • Mentally add and subtract 10 to a given number.	• multiply • divide • automatically	• Automaticity is built through strategy practice and exposure but is practiced through rehearsal once students own their understanding. (IN) • As defined in the 2023 standards, automaticity is multiplying without conscious thought or attention. (IN) • Learners who are not fluent may use pictures, groups, repeated addition, and/or counting to solve. (IN) • Learners may be fluent in their automaticity of multiplication but not division. (IN) • Learners might mix up operations (multiplication and addition, subtraction and division). (ME) • Learners may not understand the relationship between multiplication and division. (ME)

Courses

Mathematics 4

FOURTH GRADE

Operations and Algebraic Thinking (OA) *Learners will analyze patterns and relationships to generate and interpret numerical expressions.*
4.AR.OA.2 Identify and apply the properties of operations for addition, subtraction, multiplication, and division and justify thinking.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
2.AR.OA.2 3.AR.OA.2	3.AR.OA.6 4.NO.NBT.5	5.NO.NBT.5 5.AR.OA.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify the properties of operations (associative, commutative, additive property, multiplicative property, distributive property, identity). • Understand how and when to use properties of operations. • Add, subtract, multiply, and divide whole numbers.	• properties of operations (associative, commutative, additive property, multiplicative property, distributive property) • addition • subtraction • multiplication • division	• Learners have been working with the properties of operations in second and third grade, but this is the first time they are expected to work with the vocabulary of the property of operation names. (IN) • Learners may not perform the operations in parentheses first when using the associative and distributive properties. (ME) • When using the distributive property, learners may distribute only one factor. (ME) • Learners might confuse the signs when distributing multiplication over subtraction. (ME)

Courses
Mathematics 4

FOURTH GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

4.AR.OA.3 Solve multi-step authentic word problems using the four operations, including problems with interpreted remainders. Represent problems using equations, including a symbol as an unknown.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.AR.OA.4 3.AR.OA.3 3.AR.OA.4 3.AR.OA.5 3.GM.M.6 3.DPS.D.3	4.NO.NBT.3 4.NO.NBT.5 4.NO.NBT.6 4.AR.OA.1 4.GM.M.2 4.GM.M.3 4.GM.M.4 4.GM.M.5 4.DPS.D.3	5.NO.NF.3 5.NO.NF.4 5.GM.M.1 5.DPS.D.2

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Add and subtract whole numbers with and without regrouping. • Understand the concepts of multiplication, division, addition, and subtraction. • Understand the symbols used in equations for addition, subtraction, multiplication, and division. • Identify the operations necessary to solve a problem. • Use letters or symbols to represent unknowns in equations. • Generate an equation. • Multiply and divide whole numbers (division with and without remainders). • Apply the operation and evaluate the answer. • Interpret the remainder.	• add (B) • subtract (B) • multiply (B) • divide (B) • remainders • equation • symbol • unknown • operations	• Use drawings and equations with a symbol for an unknown number or variable to represent the problem. (IN) • Learners may not perform the operations in parentheses first when using the associative and distributive properties. (ME) • Learners may have difficulty understanding the meaning of the remainder, or they do not consider it important, so they leave it off the answer. (ME) • When solving two-step problems, learners may not complete the second step of the problem. (ME) • Learners might ignore key information stated in the problem or consider information that does not apply to the problem. (ME) • Learners may not label their answers. (ME)

Courses
Mathematics 4

FOURTH GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

4.AR.OA.4 Find factor pairs and multiples within the range of 1-36 while classifying numbers as prime or composite.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.AR.OA.1	4.AR.OA.1	5.AR.OA.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the concept of factors. • Identify factors. • Divide whole numbers to 36. • Understand the concept of multiples. • Identify multiples. • Understand the concept of a prime and composite number. • Identify prime and composite numbers.	• factor pairs • multiples • prime number • composite number	• This is the first exposure students will have with factor pairs and prime and composite numbers. (IN) • Learners need to be proficient in multiplication facts to master this standard. (IN) • Learners might confuse prime and composite numbers. (ME) • Learners may confuse factors and multiples. (ME) • Learners might miss less obvious factor pairs resulting in incomplete factor lists. (ME) • Learners might mistakenly assume any even number is composite. (ME) • Learners may incorrectly classify 1 as prime or composite when it is neither. (ME)

Courses
Mathematics 4

FOURTH GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

4.AR.OA.5 Interpret multiplication equations as a comparison. Represent multiplicative comparisons as multiplication equations.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.AR.OA.4	4.NO.NBT.5 4.NO.NF.4 4.NO.NF.7 4.GM.M.1	5.NO.NF.2 5.NO.NF.4 5.AR.OA.3 5.GM.M.1
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Compare two sets of items. • Multiply through 10x10. • Understand the symbols used in a multiplication equation. • Understand the use of letters as unknowns. • Write multiplication equations including letters as unknowns. • Interpret multiplicative comparisons. • Understand that the = shows balance between either side of it, rather than indicating the performance of an action.	• multiplication • equation • multiplicative comparison	• Learners should be able to distinguish between additive comparisons and multiplicative comparisons. (IN) • Learners may not interpret the phrase "__times as many" as meaning __ x __ . (ME) • Learners might confuse multiplicative comparisons ("__ times as many") with additive comparisons ("__more than"). (ME)

Courses
Mathematics 4

FOURTH GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

4.AR.OA.6 Generate a number or shape pattern that follows a given rule while identifying apparent features of the pattern that were not explicit in the rule itself.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.AR.OA.7 2.AR.OA.6 3.AR.OA.6		5.AR.OA.5
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand and identify patterns. • Repeat, extend, or create patterns. • Identify the rule the pattern follows. • Identify pattern features not explained by the rule.	• number pattern • shape pattern • rule	• When making shape patterns that include a transformation, such as a rotation, learners have difficulty seeing the pattern. (ME) • Learners might not recognize the pattern when it involves a combination of operations. (ME) • Learners might not apply the rule consistently across multiple steps leading to errors as the pattern progresses. (ME) • When generating longer patterns, learners may lose track of their place in the sequence. (ME) • Learners may struggle when dealing with more complex shape patterns involving changes in size, orientation, and/or color. (ME)

Courses
Mathematics 4

FOURTH GRADE

Geometry and Measurement (GM)

Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

Geometry (G)

Learners will compose and classify figures and shapes based on attributes and properties; represent and solve problems using a coordinate plane.

4.GM.G.1 Identify, label, and draw points, lines, line segments, rays, and angles (right, acute, obtuse).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.GM.G.3 3.GM.G.1	4.GM.G.2 4.GM.M.6 4.GM.M.7	5.GM.G.1 7.GM.GF.1 7.GM.GF.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify and label points. • Identify and label lines. • Identify and label line segments. • Identify and label rays. • Identify and label angles (right, acute, obtuse). • Draw points, lines, line segments, rays, and angles.	• point • line • line segment • ray • angle (acute, obtuse, right) • vertex • degree symbol for angle measures • angle symbol (∠)	• Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint. (IN) • Points, lines, line segments, rays, and angles can be identified within two-dimensional shapes. (IN) • Symbols are used to label and describe points, lines, line segments, rays, and angles. (IN) • When labeling rays, learners may not start with the endpoint. (ME) • Learners might confuse acute and obtuse angles. (ME)

Courses

Mathematics 4

FOURTH GRADE

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties; represent and solve problems using a coordinate plane.

4.GM.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of specified size.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.GM.G.3 3.GM.G.1	3.GM.G.2 4.GM.G.1 4.GM.M.6 4.GM.M.7	5.GM.G.1 7.GM.GF.2

Prerequisite Skills	Key Vocabulary	Common Misconceptions (Teacher and Learner) and Instructional Notes (IN)
• Identify two-dimensional figures. • Identify parallel and perpendicular lines. • Identify angles of different sizes (right, acute, obtuse, scalene). • Identify the attributes of two-dimensional figures (sides, angles, parallel sides, right angles, congruent sides). • Sort and classify two-dimensional figures based on attributes (parallel or perpendicular lines, angle size).	• classification • quadrilateral • trapezoid • parallelogram • rectangle • rhombus • square • attribute • parallel lines • perpendicular lines • angles • degree symbol for angle measures • angle symbol (∠)	• Points, lines, line segments, rays, and angles will be labeled with symbols so learners can name specific items when classifying them within two-dimensional figures. (IN) • Properties of two-dimensional shapes would include identifying the number of sides and angles found in polygons and triangles. (IN) • Learners might confuse parallel and perpendicular lines. (ME) • Learners might confuse acute and obtuse angles. (ME)

Courses
Mathematics 4

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties; represent and solve problems using a coordinate plane.

4.GM.G.3 Draw lines of symmetry in two-dimensional figures.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.GM.G.3		8.GM.GF.2

Prerequisite Skills	Key Vocabulary	Common Misconceptions (Teacher and Learner) and Instructional Notes (IN)
• Use attributes to describe two-dimensional figures. • Divide an object into two equal shares (halves). • Identify quadrilaterals (trapezoid, parallelogram, rectangle, square, rhombus). • Understand the concept of symmetry. • Understand the concept of line of symmetry. • Recognize two-dimensional figures with symmetry. • Identify the line(s) of symmetry in two-dimensional figures. • Draw the lines of symmetry in two-dimensional figures.	• lines • line of symmetry • symmetry • symmetrical • two dimensional	• Learners may not realize that shapes may have more than one line of symmetry. (ME)

Courses
Mathematics 4

FOURTH GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

4.GM.M.1 Know the relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min., sec. Record measurement equivalents in a two-column table.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.GM.M.2 3.GM.M.3	2.GM.M.1 2.GM.M.2 4.NO.NF.7 4.AR.OA.5 4.GM.M.2 4.GM.M.3	5.GM.M.1 7.GM.GF.2

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify standard units of measurement used to measure length, mass, weight, capacity, time, and temperature (customary and metric). • Understand the size of different standard units of measurement. • Understand the equivalency of standard units of measurement within one system (customary, metric).	• measurement units (km, m, cm; kg, g; lb., oz.; l, ml; hr., min., sec) • two-column table • equivalent	• Standard 4.GM.M.1 is needed to generate simple conversions from a larger unit to a smaller unit in standard 4.GM.M.2. (IN) • Learners might confuse the equivalency of standard units of customary measurement, especially when working with liquid measures. (ME) • Learners may not understand the basic relationships between different units within a system. (e.g., how many inches in a foot). They may not understand how the sizes of units relate to each other (e.g., a kilogram is much larger than a meter). (ME)

Courses
Mathematics 4

FOURTH GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

4.GM.M.2 Generate simple conversions from a larger unit to a smaller unit to solve authentic problems within a single system of measurement, both customary and metric systems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
	4.NO.NF.1 4.NO.NF.3 4.NO.NF.6 4.NO.NF.7 4.AR.OA.3 4.GM.M.1 4.GM.M.3	5.NO.NF.2 5.GM.M.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify and use the units used to measure length, mass, weight, capacity, time, and temperature (customary and metric). • Understand the size and equivalency of standard units of measurement (4 cups =1 quart, 16 oz. =1 lb., 1kg=1000 g), 1m=100 cm). • Add, subtract, multiply, and divide whole numbers, benchmark fractions, and decimals.	• conversions • customary system of measurement • metric system of measurement • measurement units	• There are no conversions across customary and metric systems in fourth grade. (IN) • Learners might confuse the equivalency of standard units of customary measurement, especially when working with liquid measures. (ME) • Learners may not understand the basic relationships between different units within a system. (e.g., how many inches in a foot). They may not understand how the sizes of units relate to each other (e.g., a kilogram is much larger than a meter). (ME)

Courses
Mathematics 4

FOURTH GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

4.GM.M.3 Identify and use the appropriate tools, operations, and units of measurement, both customary and metric, to solve problems involving time, length, weight, mass, and capacity.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.GM.M.1 3.GM.M.2 3.GM.M.3 3.GM.M.4 3.GM.M.6 3.GM.M.8 3.DPS.D.2	4.NO.NF.2 4.NO.NF.3 4.NO.NF.6 4.NO.NF.7 4.AR.OA.3 4.GM.M.1 4.GM.M.2	5.NO.NF.3 5.NO.NF.4 5.GM.M.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Differentiate between customary and metric systems. • Know the relative sizes of measurement units within one system of unit. • Identify and use the tools used to measure length, mass, weight, capacity, time, and temperature. • Identify and use the units used to measure length, mass, weight, capacity, time, and temperature (customary and metric). • Record measurement equivalents in a two-column table within each system. • Determine an appropriate tool and unit of measurement to solve a problem. • Add, subtract, multiply, and divide whole numbers, benchmark fractions, and decimals.	• units of measure • customary measurement units • metric measurement units • time • length • weight • mass • capacity	• There are no conversions across customary and metric systems in fourth grade. (IN) • Learners may not be able to differentiate between the tools of measurement to select one appropriate for the problem. (ME) • Learners might struggle with recognizing, reading, and using measurement tools. (ME) • Learners may not label units when recording their answers. (ME)

Courses

Mathematics 4

FOURTH GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

4.GM.M.4 Solve authentic word problems involving dollar bills, quarters, dimes, nickels, and pennies using $ and ¢ symbols and decimal notation appropriately.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
1.GM.M.5 2.GM.M.4 3.AR.OA.3 3.AR.OA.5 3.GM.M.5	4.NO.NBT.4 4.NO.NBT.5 4.AR.OA.1 4.AR.OA.3	5.NO.NBT.5

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify coins and dollar bills (penny, nickel, dime, quarter, dollar bill). • Identify the value of money (penny, nickel, dime, quarter dollar bill). • Identify and use the symbols $ and ¢. • Skip count by fives and tens. • Use addition, subtraction, and multiplication to solve problems. • Identify decimals to the hundredths. • Understand the use and positioning of the decimal point when adding, subtracting, and multiplying. • Determine the operation needed to solve a problem. • Notate money appropriately using $ and ¢.	• dollar bills • quarter • dime • nickel • penny • $ and ¢ symbols • decimal notation	• There are instances where money notation has been written with the $ coming at the end of the total, and it would be appropriate to teach how to use the $ and ¢ accurately when notating a total of money formally rather than informally, e.g., $4.06 vs. 4.06$. (IN) • Formal notation of the $ and ¢ is needed within this standard. (IN) • Learners may not use the $ and ¢ symbols when recording their answers for problems. (ME) • Learners may not read problems carefully to identify the operation needed to solve the problem. (ME) • Learners might have difficulty counting and determining correct change when given a purchase amount and payment. (ME) • Learners may have difficulty determining which operation is needed to solve the problem. (ME) • Learners might not move the decimal when multiplying. (ME) • Learners might not use the decimal when recording their answers. (ME)

Courses
Mathematics 4

FOURTH GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

4.GM.M.5 Apply the area and perimeter formulas for rectangles, including connected rectangular figures, in problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.AR.OA.4 3.GM.M.6 3.GM.M.7 3.GM.M.8	4.NO.NBT.5 4.AR.OA.1 4.AR.OA.3	5.GM.M.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the concept of the perimeter of plane figures. • Understand how to measure perimeter. • Understand the concept of the area of plane figures. • Distinguish between area and perimeter. • Determine when to use area or perimeter. • Understand how to measure the area of a plane figure using square units. • Understand the formula for measuring perimeter. • Understand the formula for measuring area. • Understand how to solve problems using the formulas for perimeter and area to solve problems. Find and label the perimeter of a rectangle. • Find and label the area of a rectangle. • Understand what a connected rectangle is. • Understand how to use the formulas to find the perimeter of a connected rectangle. • Understand how to use the formulas to find the area of a connected rectangle.	• area (area formula) • perimeter (perimeter formula) • rectangle (rectangular) • connected rectangular figures	• Learners tend to mix up perimeter and area. (ME) • Learners may not label units or are unable to differentiate between area and perimeter labels. (ME) • Learners may lack understanding of units squared when labeling area measurement. (ME)

Courses
Mathematics 4

FOURTH GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

4.GM.M.6 Measure angles in whole-number degrees using a protractor. Using a protractor and ruler, draw angles of a specified measure.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3. GM.G.1	4.GM.G.1 4.GM.G.2 4.GM.M.7	7.GM.GF.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify angles. • Use a ruler or straight edge to draw angles. • Understand how to use a protractor to measure angles. • Understand angles are measured in degrees. • Use protractors and rulers to draw angles. • Use protractors and rulers to draw angles of a specific measure.	• angles • vertex • whole-number degrees • protractor • ruler • degree symbol for angle measures • angle symbol (∠)	• Understand concepts of angle measurement-An angle that turns through n one-degree angles is said to have an angle measure of n degrees. (IN) • This skill is necessary for successive skills and standards. Learners must be hands-on during the measurement of angles. (IN) • Learners may struggle with obtuse and acute angles when determining which number to look at on the protractor. Learners need to understand the ”hole” in the protractor must be placed on the vertex of the angle to determine the angle measurement. (ME)

Courses
Mathematics 4

FOURTH GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

4.GM.M.7 Recognize angle measures as additive and solve addition and subtraction problems to find unknown angles on a diagram.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
	4.GM.G.1 4.GM.G.2 4.GM.M.6	7.GM.GF.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify angles. • Understand how to measure angles using a protractor. Measure angles in whole-number degrees. • Understand angle measures are additive. • Add and subtract to solve problems. • Understand the concept of an unknown angle.	• angle • angle measure • unknown angles • addition • subtraction • diagram	• When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. (IN) • Proficiency in standard 4.GM.M.6 is needed to solve addition and subtraction problems with angle measurement. (IN) • Learners may not understand that they can use their known measurements to solve for unknowns and think they need to measure all angles with the protractor. (ME)

Courses
Mathematics 4

FOURTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*
Data (D) *Learners will represent and interpret data.*
4.DPS.D.1 Formulate questions to collect, organize, and represent data to reason with math and across disciplines.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
2.DPS.D.1 3.DPS.D.1	4.DPS.D.2 4.DPS.D.3	5.DPS.D.1 6.DPS.D.1
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the difference between a question and a statement. • Form questions. • Collect data (tally marks, counting, etc.). • Organize data (sort, categorize). • Read bar and picture graphs. • Read scaled bar and picture graphs.	• collect • organize • represent • data • reason	• This standard continues the learning from standard 3.DPS.D.1. (IN) • Learners may not form specific, clear or focused questions that are not too broad or vague. (ME) • Learners might have difficulty deciding the most appropriate method to use to collect data (survey, observation, experiment). (ME) • Learners might have difficulty deciding how to categorize and organize data. (ME) • Learners may have difficulty selecting the most appropriate way to represent data (e.g., bar graph, line plot, picture graph, table, pie graph). (ME)

Courses
Mathematics 4

FOURTH GRADE

Data (D)
Learners will represent and interpret data.

4.DPS.D.2 Generate data and create line plots to display a data set of fractions of a unit (½, ¼, ⅛). Solve problems involving addition and subtraction of fractions by using information presented in line plots.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.DPS.D.2 3.DPS.D.2	4.NO.NF.1 4.NO.NF.3 4.NO.NF.6 4.DPS.D.1 4.DPS.D.3	5.NO.NF.3 5.NO.NF.4 5.DPS.D.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Collect, organize, and sort data. • Understand how to read and use a line plot. • Measure length in whole-number units (inches, feet, yards, centimeters, meters). • Measure length to the nearest half or quarter (fourth) of an inch. • Interpret data in a given line plot. • Determine the question asked to generate a given data set. • Construct a line plot including unit fractions. • Solve problems using addition or subtraction.	• data set • line plots • unit fractions • addition • subtraction	• Measurement data could be used for the creation of line plots. (IN) • This standard is not isolated to unit fractions. (IN) • Learners might confuse the fractional measurement lines on the ruler and may not ensure they start measuring from the 0 mark. (ME) • Learners may have difficulty selecting the appropriate scales and intervals for graphs. (ME) • Learners may not label the categories or data values used on the graph. (ME)

Courses

Mathematics 4

FOURTH GRADE

Data (D)
Learners will represent and interpret data.

4.DPS.D.3 Utilize graphs and diagrams to represent and solve authentic word problems using the four operations involving whole numbers, benchmark fractions, and decimals.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.AR.OA.3 3.DPS.D.3	4.NO.NBT.5 4.NO.NF.6 4.NO.NF.7 4.AR.OA.1 4.AR.OA.3 4.DPS.D.1 4.DPS.D.2	5.DPS.D.2

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Collect, organize, and sort data. • Read bar and picture graphs. • Understand how to read a scaled bar or picture graph. • Understand how to read a line plot. • Use data to create graphs and line plots. • Analyze graphs and diagrams using whole numbers, benchmark fractions, and/or decimals. • Determine which operation should be used to solve problems. • Add, subtract, multiply, and divide whole numbers, benchmark fractions, and decimals to solve problems.	• graphs • whole numbers • benchmark fractions • decimals • diagram • operations	• Data includes distances, intervals of time, liquid volumes, masses of objects, and money. (IN) • Learners might not consider the scale of the graph when interpreting information. (ME) • Learners may have difficulty identifying the parts of the graph or diagram that pertain to the question asked in the problem. (ME)

Courses

Mathematics 4

FIFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

Counting and Cardinality (CC)

Learners will understand the relationship between numerical symbols, names, quantities, and counting sequences.

5.NO.CC.1 Read and write decimals to the thousandths including standard, word, and expanded forms.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
4.NO.CC.1 4.NO.NBT.2	5.NO.NBT.1 5.NO.NBT.3	6.NO.NS.1 6.NO.NS.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand place value (tenths, hundredths, thousandths). • Understand the concept of decimals. • Understand how to read numbers in standard, word, and expanded forms. • Distinguish between standard, word, and expanded forms. • Generate the other two number forms when given a form.	• decimal • thousandths • hundredths • tenths • standard form • word form • expanded form	• Spelling is not assessed. (IN) • Learners might errantly include the word “and” when naming whole numbers. The use of “and” should be reserved for reading numerals with a decimal. (ME) • Learners may not include zero placeholders when writing numbers in expanded form. (ME)

Courses

Mathematics 5

FIFTH GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

5.NO.NBT.1 Understand that in a multi-digit number, a digit in one place represents ten times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
4.NO.NBT.1 4.NO.NF.1 4.NO.NF.5	5.NO.CC.1 5.NO.NBT.2 5.NO.NBT.3 5.NO.NBT.4 5.NO.NBT.6 5.NO.NBT.7	8.NO.NS.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand place value (millions, hundred thousand, ten thousand, thousands, hundreds, tens, ones). • Understand the fraction 1/10.	• whole number • digit • place value • ones, tens, hundreds, thousands, ten thousand, hundred thousand, millions	• Tricks and procedures of adding zeros will not lend to students' place value understanding within multi-digit whole numbers. (IN) • Place value understanding of decimals goes through thousandths like standards 5.NO.CC.1 and 5.NO.NBT.2. (IN) • Learners may not realize that larger decimal numbers are actually smaller in size. (ME) • Learners may struggle to understand that the value of a digit is determined by its position in the number, not just its numerical value. (ME)

Courses

Mathematics 5

FIFTH GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

5.NO.NBT.2 Compare two decimals to the thousandths place using symbols >, <, and =. Justify comparisons based on the value of the digits.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
4.NO.NBT.2 4.NO.NF.1	5.NO.NBT.3	6.NO.NS.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand and identify the place value of decimals (tenths, hundredths, thousandths). • Understand the concept of greater than, less than, and equal to. • Identify and use the symbols >, <, = to compare numbers. • Understand the value of tenths, hundredths, and thousandths. • Understand the importance of decimal placement when comparing decimal numbers.	• decimal • symbols >, <, and = • thousandths • hundredths • tenths • digit	• Comparing visual models will build an understanding that will lend to the justification of place value comparison prior to moving to the symbolic notation of decimal comparison. (IN) • Visual models that were previously used to model 100 are often used to represent 1, with the smaller pieces representing decimals. Students may need time to adjust to the visual's changing meaning. (IN) • Ordering numbers may be included for number comparisons. (IN) • Learners may confuse the < and > symbols. (ME) • Learners may not realize that larger decimal numbers are actually smaller in size. (ME)

Courses
Mathematics 5

FIFTH GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

5.NO.NBT.3 Apply place value understanding to round decimals to any place.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
4.NO.NBT.3 4.NO.NF.1	5.NO.CC.1 5.NO.NBT.1 5.NO.NBT.2	7.GM.AV.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand place value positions (tenths, hundredths, thousandths). • Understand the concept of estimation and rounding. • Identify the place value to which a number is to be rounded. • Identify the range for the given rounded place value. • Round numbers to the nearest tenth. • Round numbers to the nearest hundredth. • Round numbers to the nearest thousandth.	• place value • rounding • decimal	• When the digit to the right of the rounding place is five or greater, the learners may not round up. (ME) • When rounding within multi-digit numbers, learners might have difficulty identifying which digit to focus on when rounding to a specific place. (ME)

Courses
Mathematics 5

FIFTH GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

5.NO.NBT.4 Multiply multi-digit whole numbers using strategies flexibly, including the algorithm.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
4.NO.NBT.5	5.NO.NBT.1 5.NO.NBT.5 5.NO.NBT.6 5.NO.NBT.7 5.AR.OA.1 5.GM.M.1 5.GM.M.2	6.NO.O.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Add automatically to 20. • Multiply automatically to 10x10 (vertically and horizontally). • Use rectangular arrays, equations, and visual models to solve problems. • Recognize the symbols used to indicate multiplication (x, (), •). • Understand and apply place value to hundred thousand. • Understand and apply the concept of standard US algorithm. • Add multi-digit numbers with and without regrouping. • Find the product when multiplying single-digit factors. • Multiply multi-digit numbers with and without regrouping. • Identify an equation that matches a model of multiplication. • Change a horizontal math problem to a vertical one with correct place value alignment.	• multiply • algorithm • whole numbers • multi-digit	• Mastery of the standard US multiplication algorithm is expected. (IN) • Learners are expected to build understanding beyond thousands. (IN) • Learners might miss steps or misalign digits when using the standard algorithm for solving multiplication. (ME) • Learners may incorrectly carry over digits to the next column or might not carry digits at all. (ME)

Courses
Mathematics 5

FIFTH GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

5.NO.NBT.5 Use concrete models, drawings, place value strategies, properties of operations, and/or relationships to add, subtract, and multiply decimals to hundredths.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
4.NO.NBT.4 4.NO.NBT.5 4.NO.NF.1 4.AR.OA.2 4.GM.M.4	5.NO.NBT.4 5.NO.NBT.7 5.NO.NF.3 5.NO.NF.4 5.AR.OA.1	6.NO.O.2 6.NO.O.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Add and subtract automatically to 20. • Multiply automatically to 10x10. • Understand and apply place value to hundredths. • Add and subtract with and without regrouping. • Multiply with and without regrouping. • Understand and apply the properties of operations (commutative, associative, distributive, identity, inverse operations, 0 property of multiplication). • Add, subtract, and multiply problems involving money. • Identify decimals to the hundredths. • Understand the use and positioning of the decimal point when adding, subtracting, and multiplying. • Change a horizontal math problem to a vertical one with correct place value alignment.	• place value strategies • properties of operations • add • subtract • multiply • decimals • hundredths	• The mastery of the standard algorithm with decimals is not expected within this standard. However, it may be included within the strategies a student chooses to use. (IN) • Concrete models and drawings precede place value strategies, properties of operations, and/or relationships between addition, subtraction, and multiplication. (IN) • Learners may not line up the place values when adding and subtracting. (ME) • Learners might mistakenly assume that when multiplying two decimals, the answer will be larger instead of smaller. (ME)

Courses

Mathematics 5

FIFTH GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

5.NO.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and two-digit divisors using place value strategies. Show and justify the calculation by using equations, rectangular arrays, and/or area models.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
4.NO.NBT.4 4.NO.NBT.5 4.NO.NBT.6	5.NO.NBT.1 5.NO.NBT.4 5.NO.NBT.7 5.AR.OA.1 5.GM.M.1	6.NO.O.1 6.NO.O.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Add and subtract automatically through 20. • Multiply and divide whole numbers through 10 x 10. • Understand and apply place value to thousands. • Use rectangular arrays, equations, and visual models to solve problems. • Understand parts of a division problem (divisor, dividend, quotient, remainder). • Understand formats of a division problem (÷, ⌐, fraction representation). • Understand the inverse relationship of multiplication and division. • Add and subtract multi-digit numbers with and without regrouping. • Multiply multi-digit numbers. • Select and perform the steps of a multi-digit division problem using place value strategies. • Divide multiple digits (up to four) by one digit without remainders. • Understand the concept of remainders. • Divide multiple digits (up to four) by one digit with a remainder. • Divide multiple digits (up to four) by two-digit divisors without remainders. • Divide multiple digits (up to four) by two-digit divisors, with a remainder. • Identify an equation that matches a model of division.	• quotient • remainder • dividend • divisor • equation • rectangular array • area model • place value	• The use of visual models shows and justifies calculations of division. The standard division algorithm is not included in this standard. (IN) • Visual models may include arrays, area models, and/or partial quotients (IN) • Learners may not include the remainder when recording the quotient. (ME) • Learners may be more successful with multi-digit dividends and one-digit divisors but struggle when moving to using two-digit divisors. (ME)

Courses

Mathematics 5

FIFTH GRADE

Base Ten (NBT)
Learners will understand the place value structure of the base-ten number system and represent, compare, and perform operations with multi-digit whole numbers and decimals.

5.NO.NBT.7 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.AR.OA.6 4.NO.NBT.5	5.NO.NBT.1 5.NO.NBT.4 5.NO.NBT.5 5.NO.NBT.6 5.AR.OA.1 5.GM.M.1	6.AR.EE.1 8.NO.NS.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Add and subtract automatically to 20. • Multiply automatically to 10x10. • Understand and apply place value. • Understand the concept of powers of 10. • Understand what an exponent is and what it represents. • Identify decimals. • Understand the use of the decimal point when multiplying by a power of 10. • Identify equivalencies between multiplying powers of 10 and multiplying by 10, 100, and 1000. • Identify equivalencies between dividing powers of 10 and dividing by 10, 100, and 1000.	• powers of 10 • multiply • decimal • decimal point • division • whole number exponents • patterns	• Learners may not use the decimal point. (ME) • Learners may not place the decimal point correctly when multiplying by a power of ten. (ME) • Learners might recognize the pattern but may lack the understanding of the change of value based on the decimal. (ME)

Courses
Mathematics 5

FIFTH GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

5.NO.NF.1 Generate equivalent forms of commonly used fractions and decimals (e.g., halves, fourths, fifths, tenths).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.NO.NF.3 4.NO.NF.1 4.NO.NF.3 4.NO.NF.5	5.NO.NF.3 5.NO.NF.4 5.GM.M.1	6.NO.NS.2 6.NO.O.2 6.AR.RP.1 6.AR.RP.4 7.NO.NS.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify numerators and denominators in fractions. • Understand the concept of equivalent fractions. • Understand the location of fractions and decimals on number lines. • Represent fractions and decimals on visual models. • Identify equivalent forms of commonly used fractions and decimals, e.g., ½ = .5.	• equivalent forms (fractions, decimals) • fractions • decimals	• Learners might mistakenly assume the denominator represents the same digit when converting to a decimal. (e.g., 1/4 = 0.4 instead of 0.25). (ME)

Courses
Mathematics 5

FIFTH GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

5.NO.NF.2 Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number and explain why multiplying a given number by a fraction less than one results in a product smaller than the given number.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
4.NO.NF.2 4.NO.NF.4 4.AR.OA.5 4.GM.M.2	5.NO.NF.4	6.NO.O.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/Errors (ME)
• Multiply automatically through 10x10. • Able to draw a visual model to represent the problem presented. • Represent fractions and whole numbers on number lines and in visual models. • Represent and multiply whole numbers by fractions. • Represent and multiply fractions by whole numbers. • Generalize results of fractional problems.	• multiply • fractions • proper fractions • improper fractions • compare	• Interpret multiplication as scaling (resizing) by comparing the size of a product to the size of one factor on the basis of the size of the other factor without performing the indicated multiplication. (IN) • The use of visual models should precede procedural instruction to build a deeper understanding of operations (IN) • Learners may be able to complete the procedure to solve but lack the understanding of the product. (ME) • Learners might anticipate the answer is the opposite of what it actually is (e.g., 2 x ¼ = ½, so the product is less than 2). (ME)

Courses
Mathematics 5

FIFTH GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

5.NO.NF.3 Solve authentic word problems by adding and subtracting fractions and mixed numbers with unlike denominators using visual fraction models and equations.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.AR.OA.3 4.NO.NBT.4 4.NO.NF.3 4.NO.NF.4 4.NO.NF.6 4.AR.OA.3 4.GM.M.3 4.DPS.D.2	5.NO.NBT.5 5.NO.NF.1 5.DPS.D.1 5.DPS.D.2	6.NO.O.2

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Add and subtract automatically to 20. • Understand the meaning of and identify mixed numbers. • Represent fractions using visual models. • Convert fractions with unlike denominators into fractions with common denominators using equivalent fractions. • Convert a mixed number into an improper fraction. • Convert an improper fraction into a mixed number. • Interpret word problems and determine the operation needed to solve them. • Represent problems involving fractions using equations. • Add and subtract fractions with like denominators. • Identify proper and improper fractions. • Understand the concept of equivalent fractions. • Add and subtract fractions with unlike denominators.	• add • subtract • fractions • mixed numbers • denominator • unlike denominator • equations • visual fraction model	• In a progression, learners may practice adding and subtracting fractions and mixed numbers with unlike denominators prior to using the understanding in word problems. (IN) • Once learners are working beyond unit fractions and are working on proper fractions, the numerator may be alike or unlike, but the focus is on the denominator. (IN) • Learners might get confused when reading a story problem. Drawing a picture helps them visualize what is happening in the problem. (IN) • Learners who lack proficiency with word problems may look for key words to help them comprehend the word problem and task (identifying the operation required). (IN) • Learners might have difficulty converting between mixed numbers and improper fractions. (ME) • Learners may not label their answer. (ME)

Courses
Mathematics 5

FIFTH GRADE

Fractions (NF)
Learners will understand fractions and equivalency to represent, compare, and perform operations of fractions and decimals.

5.NO.NF.4 Solve authentic word problems by multiplying fractions and mixed numbers using visual fraction models and equations.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
4.NO.NF.7 4.AR.OA.3 4.AR.OA.5 4.GM.M.3 4.DPS.D.2	5.NO.NBT.5 5.NO.NF.1 5.NO.NF.2 5.AR.OA.1 5.DPS.D.2	6.NO.O.3 6.GM.AV.2

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Multiply automatically through 10x10. • Convert a mixed number into an improper fraction. • Convert an improper fraction into a mixed number. • Represent fractions and whole numbers using visual models. • Represent problems involving fractions using equations. • Multiply fractions. • Multiply whole numbers by fractions. • Identify proper and improper fractions. • Understand the concept of equivalent fractions. • Multiply fractions and mixed numbers. • Interpret word problems. • Interpret the solution of a word problem.	• multiply • fractions • mixed numbers • equations • visual fraction models	• Model with visuals how fractions are multiplied, rather than using the standard algorithm for multiplication with fractions. (IN) • This includes the multiplication of a fraction or a whole number by a fraction. (IN) • Interpret the product (a/b) x q as a part of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a x q ÷ b. (IN) • Learners might get confused when reading a story problem. Drawing a picture helps them visualize what is happening in the problem. (IN) • Learners who lack proficiency with word problems may look for keywords to help them comprehend the word problem and task. (IN) • Learners may have difficulty converting between mixed numbers and improper fractions before multiplying. (ME) • Learners may not label their answers. (ME)

Courses
Mathematics 5

FIFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

Operations and Algebraic Thinking (OA)

Learners will analyze patterns and relationships to generate and interpret numerical expressions.

5.AR.OA.1 Automatically multiply and divide through 12 x 12.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.AR.OA.1 4.AR.OA.1	5.NO.NBT.4 5.NO.NBT.5 5.NO.NBT.6 5.NO.NBT.7 5.NO.NF.4 5.GM.M.2	6.NO.O.1 6.NO.O.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the concepts of multiplication and division, including the use of the terms factors, product, quotient, divisor, and dividend. • Recognize the formats of division problems and symbols used to indicate multiplication (x, (), •). • Add automatically to 20 (vertically and horizontally). • Use repeated addition to find the number of objects in a rectangular array. • Multiply automatically to 10 x 10 (vertically and horizontally). • Skip count. • Mentally add and subtract 10 to a given number. • Apply strategies to find multiplication and division facts of 11 and 12.	• multiply • divide • product • factor • quotient • dividend • divisor	• Automaticity is built through strategy practice and exposure but is practiced through rehearsal once students own their understanding. (IN) • As defined by the 2023 standards, automaticity is multiplying without conscious thought or attention. (IN) • Learners might be fluent in their automaticity of multiplication but not division. (IN) • Learners may not understand the inverse relationship of multiplication and division. (ME)

Courses
Mathematics 5

FIFTH GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

5.AR.OA.2 Analyze problems using the order of operations to solve and evaluate expressions while justifying thinking.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.AR.OA.2 4.AR.OA.2	5.AR.OA.3	6.AR.EE.1 6.AR.EE.2 6.AR.EE.3 6.AR.EE.4
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify and write numerical expressions. • Understand and apply the order of operations. • Add and subtract multi-digit numbers with and without regrouping. • Multiply and divide numbers with and without remainders. • Apply the properties of operations. • Perform operations in an appropriate order. • Identify the function of parentheses, brackets, and braces within the order of operations.	• simple numerical expressions • order of operations • evaluate • parentheses • brackets (T) • braces (T)	• Knowing the order of operations to solve and evaluate expressions exceeds the acronym of PEMDAS because it creates misconceptions as learners progress in their math learning beyond fifth grade. (IN) • When using the PEMDAS acronym, learners may think multiplication always comes before division and that addition always comes before subtraction. (ME) • Learners might skip necessary steps leading to errors. (ME) • Learners may ignore the parentheses, performing the operations inside out of order or neglecting them entirely. (ME) • When using the distributive property, learners might distribute only one factor. (ME) • Learners might confuse the signs when distributing multiplication over subtraction. (ME)

Courses
Mathematics 5

FIFTH GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

5.AR.OA.3 Write simple expressions that record calculations with numbers. Interpret numerical expressions without evaluating them.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
4.AR.OA.5	5.AR.OA.2	6.AR.EE.1 6.AR.EE.2 6.GM.GF.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify and write numerical expressions. • Understand and apply the order of operations. • Add and subtract multi-digit numbers with and without regrouping. • Multiply and divide numbers with and without remainders. • Apply the properties of operations.	• interpret (B) • expressions (B) • evaluating (B)	• Learners may initially need to solve expressions to interpret them. They may need additional time and rehearsal to interpret simple expressions without evaluating them. (ME) • Learners may ignore the order of operations. (ME) • Learners may use incorrect or inconsistent variables. (ME)

Courses
Mathematics 5

FIFTH GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

5.AR.OA.4 Find factor pairs and multiples within the range of 1-100 while classifying numbers as prime or composite.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.AR.OA.1 4.AR.OA.4		6.NO.O.4
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the concept of factors. • Identify factors. • Divide whole numbers to 100. • Understand the concept of multiples. • Identify multiples. • Distinguish between factors and multiples. • Understand the concept of a prime and composite number. • Identify prime and composite numbers. • Identify factor pairs for numbers up to 100. • Identify multiples for numbers up to 100.	• factor pairs • multiples • prime numbers • composite numbers	• In standard 4.AR.OA.4, learners have worked with factor pairs and multiples in the range of 1-36 and describe them as prime or composite. (IN) • Learners who rely on skip counting or repeated addition for multiplication and division will have difficulty finding factor pairs and multiples. (IN) • Learners may not have enough practice with factor pairs and multiples. If fluency with division is lacking, it may be difficult to solve problems without writing out all combinations. (IN) • Learners might miss less obvious factor pairs resulting in incomplete factor lists. (ME) • Learners might mistakenly assume any even number is composite. (ME) • Learners may incorrectly classify 1 as prime or composite when it is neither. (ME)

Courses
Mathematics 5

FIFTH GRADE

Operations and Algebraic Thinking (OA)
Learners will analyze patterns and relationships to generate and interpret numerical expressions.

5.AR.OA.5 Generate two numerical patterns using two given rules and form ordered pairs consisting of corresponding terms from the two patterns. (Graphing on a coordinate plane).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
4.AR.OA.6	3.AR.OA.6 5.GM.G.2 5.GM.G.3	6.GM.GF.1 8.AR.F.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand and identify patterns. • Repeat, extend, or create patterns. • Identify the rule the pattern follows. • Identify pattern features not explained by the rule. • Generate a pattern following a given rule. • Generate two numerical patterns following two given rules. • Understand the concept of an ordered pair. • Understand the concept of the coordinate plane. • Identify the x and y axis on the coordinate plane. • Identify the x and y coordinates in an ordered pair. • Use the x and y coordinates in an ordered pair to graph points on the coordinate plane.	• numerical patterns • rules • ordered pairs • coordinate plane • corresponding terms	• Example: Given the rule "add 3" and the starting number of 0 and given the rule "add 6" and the starting number of 0, generate terms in the resulting sequences. In this case, observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so. (IN) • The coordinate plane and rules are fundamental prerequisites for middle school math. (IN) • Learners may inconsistently apply the rules or miss steps when generating successive terms in the pattern. (ME) • Learners may not recognize patterns in the sequences of numbers generated by the rules. They may struggle to predict or extend the pattern based on the given rules. (ME) • Learners might have difficulty managing patterns with multiple operations or rules. (ME) • When creating and graphing the ordered pairs, learners may mix up the x and y coordinates. (ME)

Courses
Mathematics 5

FIFTH GRADE

Geometry and Measurement (GM)

Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

Geometry (G)

Learners will compose and classify figures and shapes based on attributes and properties; represent and solve problems using a coordinate plane.

5.GM.G.1 Classify two-dimensional figures in a hierarchy based on properties.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.GM.G.1 3.GM.G.2 3.GM.G.3 4.GM.G.1 4.GM.G.2		7.GM.GF.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify attributes of two-dimensional figures (number of sides, number of angles, number of parallel sides). • Sort quadrilaterals into categories based on attributes. • Identify and label points, lines, line segments, rays, and angles (acute, right, obtuse), parallel lines, and perpendicular lines. • Identify angles as acute, obtuse, or right. • Identify two-dimensional figures by their specific name (rhombus, rectangle, parallelogram, isosceles triangle). • Classify quadrilaterals into categories based on attributes. • Classify two-dimensional figures based on attributes.	• Two-dimensional figures • hierarchy • properties of two-dimensional figures • classification of quadrilaterals • quadrilateral • trapezoid • parallelogram • rectangle • rhombus • square	• See Appendix D in ND Math Standards for hierarchy visual. (IN) • Properties of two-dimensional shapes and the names of the specific shapes would include identifying the number of sides, angles, and lines of symmetry found in polygons and triangles (equilateral, isosceles, scalene). (IN) • Learners may struggle with keeping track of multiple attributes to classify figures. (ME) • Learners might have difficulty recognizing more than one line of symmetry in two-dimensional shapes. (ME)

Courses
Mathematics 5

FIFTH GRADE

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties and represent and solve problems using a coordinate plane.

5.GM.G.2 Identify the x-coordinate and y-coordinate to graph and name points in the first quadrant of the coordinate plane.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
	5.AR.OA.5	5.GM.G.3 6.GM.GF.1 6.GM.GF.2

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the concepts of ordered pairs, coordinates, quadrants, and the coordinate plane. • Identify the x-axis, y-axis, and origin. • Identify the x and y coordinate in an ordered pair and differentiate between them. • Identify the x and y axis on the coordinate plane. • Graph a given ordered pair. • Identify the quadrants of the coordinate plane. • Graph points from ordered pairs in the first quadrant of the coordinate plane.	• x-coordinate • y-coordinate • quadrant • coordinate plane • ordered pair	• In a progression, learners may begin by learning about the origin in the direction of the axis and how the coordinates correspond. (IN) • This is the first exposure with x and y coordinates, ordered pairs, and coordinate planes. Learners will need direct instruction to learn to order. (IN) • Using alternating shaded rows can help learners track information on the graph. (IN) • Learners might mix up the x and y coordinates when writing ordered pairs. (ME)

Courses
Mathematics 5

FIFTH GRADE

Geometry (G)
Learners will compose and classify figures and shapes based on attributes and properties; represent and solve problems using a coordinate plane.

5.GM.G.3 Form ordered pairs and graph points in the first quadrant on the coordinate plane to solve authentic word problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.GM.G.2	5.AR.OA.5	6.GM.GF.1 6.GM.GF.2
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the concepts of ordered pairs, coordinates, quadrants, and the coordinate plane. • Identify the x and y coordinate in an ordered pair. • Identify the x and y axis on the coordinate plane. • Identify the quadrants of the coordinate plane. • Identify the quadrants of the coordinate plane. • Complete a function table with information from a given problem. • Graph points from ordered pairs in the first quadrant of the coordinate plane.	• x-coordinate • y-coordinate • quadrant • coordinate plane • ordered pair	• Learners need to have adequate proficiency within standard 5.GM.G.2 to focus attention and skill on graphing within the first quadrant to solve authentic word problems. (IN) • Learners might mix up the x and y coordinates when graphing an ordered pair. (ME) • Learners may not realize that (0,0) is the origin. (ME) • Learners might misinterpret the scale on the coordinate plane, leading to incorrectly placed points. (ME)

Courses
Mathematics 5

FIFTH GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

5.GM.M.1 Generate conversions among different-sized standard measurement units within a given measurement system, both customary and metric. Use these conversions to solve multi-step, authentic word problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
4.AR.OA.3 4.AR.OA.5 4.GM.M.1 4.GM.M.2 4.GM.M.3	5.NO.NBT.4 5.NO.NBT.5 5.NO.NBT.6 5.NO.NBT.7 5.NO.NF.1	6.AR.RP.5

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify measurement tools and their use. • Identify and use the units used to measure length, mass, weight, capacity, time, and temperature (customary and metric). • Understand the size and equivalency of standard units of measurement (4 cups =1 quart, 16 oz. =1 lb., 1kg=1000 g), 1m=100cm). • Determine an appropriate tool and unit of measure to solve a specific problem. • Differentiate between customary and metric systems. • Know the relative sizes of customary and metric measurement units. • Record equivalents in a two-column table within each system. • Add, subtract, multiply, and divide whole numbers, benchmark fractions, and decimals. • Use mathematical operations within units of measurement in one system. • Determine the operation to complete the conversion. • Identify the steps needed to solve a problem.	• conversions • standard measurement units • customary measurement system • metric measurement system	• In fourth grade, learners are converting within one system of measurement and that will lead to conversion across measurement systems in fifth grade. (IN) • Learners might have difficulty using a ruler correctly, including reading the fractional parts and making sure to start measuring at the 0 on the ruler instead of automatically assuming 0 is the end of the ruler. (ME) • Learners may not understand the size and equivalency of the standard units of measurement used. (ME)

Courses
Mathematics 5

FIFTH GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

5.GM.M.2 Find the area and perimeter of a rectangle, including connected rectangular figures, with fractional side lengths.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.GM.M.6 3.GM.M.7 3.GM.M.8 4.NO.NF.6 4.NO.NF.7 4.GM.M.5	5.NO.NBT.4 5.AR.OA.1	6.GM.AV.1 6.GM.GF.3

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the concept of the perimeter of plane figures. • Understand how to measure perimeter. • Understand the concept of the area of plane figures. • Distinguish between area and perimeter and determine when to use area or perimeter. • Understand how to measure the area of a plane figure using square units. • Understand the formula for measuring perimeter. • Understand the formula for measuring area. • Demonstrate the connection between the formula and model for area and perimeter. • Understand how to solve problems using the formulas for perimeter and area to solve problems. • Understand what a connected rectangle is. • Understand how to use the formulas to find the perimeter of a connected rectangle. • Understand how to use the formulas to find the area of a connected rectangle. • Multiply and add fractions. • Measure side lengths of a rectangle in whole and fractions of standard measurement units. • Find and label the area of rectangles with whole and fractional side lengths. • Apply the formulas to find the perimeter and area of rectangular figures with fractional side lengths.	• area • perimeter • rectangle (rectangular figures) • connected rectangular figures • fractional side lengths	• Learners need to have adequate proficiency with finding the area and perimeter of rectangles, including connected rectangular figures with whole numbers, prior to working with fractional side lengths. (IN) • Learners need to have adequate proficiency with adding and multiplying fractions with like and unlike denominators prior to applying their understanding to the area and perimeter of rectangles and connected rectangular figures. (IN) • Learners might have difficulty figuring out a missing side length when finding the perimeter of a connected rectangle. (ME) • Learners might confuse the concepts of perimeter and area. (ME) • Learners may omit sides or double count shared sides when calculating the perimeter of connected rectangles. (ME) • Learners may struggle to identify the length and width of connected rectangles. (ME)

Courses
Mathematics 5

FIFTH GRADE

Measurement (M)
Learners will represent and calculate measurement data, including time, money, and geometric measurement, and convert like measurement units within a given system.

5.GM.M.3 Recognize volume as an attribute of rectangular prisms and measure volume by counting unit cubes.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.GM.M.7		6.GM.AV.2

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify the attributes of rectangular prisms (edges, vertices). • Understand the concept of the volume of three-dimensional figures. • Understand volume is measured in cubic units. • Measure the volume of a rectangular prism by counting unit cubes.	• volume • rectangular prism • unit cubes	• In a progression, learners may begin by recognizing that volume is additive when measuring volume by counting unit cubes. (IN) • The volume formula is used in standard 6.GM.AV.2. Learners are developing the concept of volume to lead to sixth-grade learning. (IN) • Length, width, and height are attributes of rectangular prisms that learners need to understand to solve problems by counting unit cubes when not all cubes are visible. (IN) • Learners may struggle with counting unit cubes when the cube is hidden. They will need to visualize the three-dimensional shape to count all the cubes. (ME)

Courses
Mathematics 5

FIFTH GRADE

Data, Probability, and Statistics (DPS)

Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.

Data (D)

Learners will represent and interpret data.

5.DPS.D.1 Generate data and create line plots to display a data set of fractions of a unit (½, ¼, ⅛). Use grade-level operations for fractions to solve problems involving information presented in line plots.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.DPS.D.2 4.NO.NF.3 4.NO.NF.6 4.DPS.D.1 4.DPS.D.2	5.NO.NF.3 5.DPS.D.2	6.DPS.D.1 6.DPS.D.3 6.DPS.D.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Collect, organize, and sort data. • Understand how to read and use a number line. • Read line plots. • Measure length in whole-number units (inches, feet, yards, centimeters, meters). • Measure length using unit fractions (½, ¼, ⅛). • Add, subtract, and multiply with fractions and whole numbers. • Add, subtract, and multiply with fractions to solve problems involving line plots. • Identify the steps to solve a problem.	• data tables • line plots • whole numbers • fractions • decimals • graphs	• Learners need to be proficient with generating data and creating line plots, as well as solving problems using the data presented in line plots. (IN) • For scaffolded practice, see preceding standards 2.DPS.D.2, 3.DPS.D.2, and 4.DPS.D.2. (IN) • Learners will need to be able to be proficient in ordering fractions with like and unlike denominators to create a line plot. (IN) • When using information to solve problems, learners might misread plotted points or misinterpret the frequency of data points. (ME)

Courses

Mathematics 5

FIFTH GRADE

Data (D)
Learners will represent and interpret data.

5.DPS.D.2 Utilize graphs and diagrams to represent, analyze, and solve authentic word problems using information presented in one or more tables or line plots including whole numbers, fractions, and decimals.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
3.DPS.D.3 4.NO.NF.6 4.NO.NF.7 4.AR.OA.3 4.DPS.D.3	5.NO.NF.3 5.NO.NF.4 5.DPS.D.1	6.DPS.D.2 6.DPS.D.3 6.DPS.D.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Collect, organize, and sort data. • Read and interpret data tables. • Record data in data tables. • Read bar and picture graphs. • Understand how to read a scaled bar or picture graph. • Understand how to read a line plot. • Determine which visual representation to use. • Use data to create graphs, diagrams, tables, and line plots. • Interpret information shown on graphs, diagrams, tables, or line plots. • Add, subtract, multiply, and divide whole numbers, benchmark fractions, and decimals to solve problems. • Identify the steps to solve a problem. • Determine which operation to use to solve problems.	• graph • data tables • line plots • whole-numbers • fractions • decimals • diagram	• The assessment question or stem should give learners the information they need from a data source that they are familiar with (tables or line plots within K-4 DPS standards) to allow them to represent with any graph or diagram of their choosing. (IN) • When analyzing data, learners may quickly jump to conclusions. They need to think about what questions they can ask and what other variables might be involved with the data. (ME) • Learners might not use or accurately interpret the scale of the graph when analyzing data. (This standard continues applying skills from standard 3.DPS.D.1). (ME) • Learners may have difficulty connecting or comparing different sets of data represented on graphs or tables. (ME)

Courses
Mathematics 5

SIXTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

Number Systems (NS)

Learners will expand their knowledge of the number system to create connections and solve problems within and across concepts.

6.NO.NS.1 Explain and show the relationship between non-zero rational numbers and their opposites using horizontal and vertical number lines, including authentic problems.
Use rational numbers to represent quantities in authentic contexts and explain the meaning of 0 in certain situations.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.NO.CC.1	6.DPS.D.2	6.NO.NS.2 6.AR.EE.5 7.NO.NS.1 7.NO.O.1 7.AR.EE.2 7.AR.EE.3

Prerequisite Skills	Key Vocabulary, Notations, and Representations	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Distinguish between whole and rational numbers. • Represent rational numbers on a horizontal number line. • Represent rational numbers on a vertical number line. • Determine the distance from zero on a horizontal and vertical number line. • Identify the opposites of rational numbers. • Recognize when numbers are positive or negative in real-world situations. • Draw and label number lines.	• rational numbers • horizontal number line • vertical number line • opposite • zero • distance • absolute value • positive numbers • negative numbers • zero pair (2 numbers that add to zero) <u>Notations and Representations</u> • Negative Symbol • Vertical Numbers Lines • Horizontal Number Lines	• Teachers should explain that numbers further to the right on the horizontal number line are always greater than numbers on the left rather than teaching separate rules for comparing positive and negative numbers. At times, this is taught as numbers closer to zero are greater. However, that is only true for negative numbers. (IN) • Teachers need to help learners understand that a negative symbol represents the opposite of a number. (IN) • Teachers need to teach both vertical and horizontal number lines in a variety of ways (open number line, building a number line on the floor, clothesline, comparing it to a thermometer, etc.) Highlight that opposites are the same distance from zero on opposite sides of zero. (IN) • Teachers should focus on the concept of absolute value, but formal notation is not required at this level. (IN) • Learners may not realize every number has an opposite. (ME) • Learners may not understand a negative negative means the “opposite of negative.” (ME)

Courses

Mathematics 6
General Math
Applied Math

SIXTH GRADE

Number Systems (NS)
Learners will expand their knowledge of the number system to create connections and solve problems within and across concepts.

6.NO.NS.2 Write, interpret, and explain statements of order for rational numbers on a number line and in authentic contexts.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.NO.CC.1 5.NO.NBT.2 5.NO.NF.1 6.NO.NS.1	6.NO.NS.1 6.AR.EE.6 6.DPS.D.2	7.NO.NS.1 7.NO.NS.2 8.NO.NS.2
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the concept of rational numbers. • Represent rational numbers on a number line. • Represent quantities in authentic contexts. • Represent quantities in authentic contexts. • Recognize when numbers are positive or negative in authentic situations. • Compare positive fractions. • Compare positive decimals to the thousandth.	• inequality • rational numbers • horizontal number line • vertical number line • opposite • zero <u>Notations and Representations</u> • Negative Symbol • Inequality Symbols • Vertical Number Lines • Horizontal Number Lines	• Teachers should use words, symbols, and number lines to compare numbers written in different ways (decimals, fractions, integers) and use zero as a reference point. (IN) • Learners may be confused by the comparison phrases (no less than, no more than, etc.). (ME)

Courses
Mathematics 6 General Math Applied Math

SIXTH GRADE

Operations (O)
Learners will expand their computational fluency to create connections and solve problems within and across concepts.

6.NO.O.1 Divide multi-digit whole numbers up to four-digit dividends and two-digit divisors using strategies or procedures.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.NO.NBT.6 5.AR.OA.1	6.NO.O.2 6.NO.O.3	7.NO.O.1 7.NO.O.2 7.NO.O.3
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Multiply and divide whole numbers through 12x12. • Understand and apply place value (ones, tens, hundreds, thousands). • use rectangular arrays, equations, and visual models to solve problems. • Understand parts of a division problem (divisor, dividend, quotient, remainder). • Multiply multi-digit numbers. • Understand the concept of remainders. • Divide multiple digits (up to four) by two-digit divisors with remainders. • Understand the relationship between multiplication and division.	• divisor • dividend • quotient • remainder • place value • divide/division • digit <u>Notations and Representations</u> • understand formats of a division problem (a/b; a ÷ b; using long division symbol) • understand formats of a multiplication problem (a x b; ab; a(b); a • b)	• Teachers need to connect to strategies taught in elementary school. (IN) • Learners should be able to reason using number relationships and logic to choose an efficient strategy to solve each problem. (IN) • This is when learners may formalize their understanding of division using a procedure such as the standard algorithm for the division of multi-digit numbers. (IN) • Learners might switch the dividend and the divisor. (ME) • Learners might assume the larger number is always the dividend. (ME) • Learners might assume there is no remainder. (ME) • Learners may ignore place value. (ME)

Courses
Mathematics 6 General Math

SIXTH GRADE

Operations (O)
Learners will expand their computational fluency to create connections and solve problems within and across concepts.

6.NO.O.2 Add and subtract fractions and decimals up to the hundredths place, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.NO.NBT.5 5.NO.NF.1 5.NO.NF.3	6.NO.O.1 6.NO.O.3	7.NO.O.2 7.NO.O.3
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Add and subtract whole numbers automatically to 20. • Understand the meaning of and identify mixed numbers. • Add and subtract fractions with like and unlike denominators using models. • Understand the concept of equivalent fractions. • Convert between mixed numbers, improper fractions, and decimals using visual models. • Represent fractions and decimals using visual models and number lines. • Understand and apply the place value of decimals to the thousandths. • Change a horizontal math problem to a vertical one with correct place value alignment.	• fractions • decimals • add/addition • subtract/subtraction • equivalent fractions • place value	• Teachers need to connect to strategies taught in elementary school. (IN) • Fractions should include mixed numbers and improper fractions. (IN) • Learners should be able to reason using number relationships and logic to choose an efficient strategy to solve each problem. (IN) • Learners may not understand subtraction is not commutative. (ME) • Learners may not use common denominators when performing the operations (addition and subtraction). (ME) • Learners may ignore place value or set up the problem incorrectly. (ME)

Courses
Mathematics 6

SIXTH GRADE

Operations (O)
Learners will expand their computational fluency to create connections and solve problems within and across concepts.

6.NO.O.3 Apply multiplication and division of fractions and decimals to solve and interpret problems using visual models, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.NO.NBT.4 5.NO.NBT.5 5.NO.NBT.6 5.NO.NF.2 5.NO.NF.4 5.AR.OA.1	6.NO.O.1 6.NO.O.2 6.AR.EE.5 6.GM.AV.2	7.NO.NS.2 7.NO.O.2 7.NO.O.3

Prerequisite Skills	Key Vocabulary, Notations, and Representations	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Multiply automatically through 12x12. • Understand the concept of multiplying fractions. • Understand the concept of multiplying decimals and the positioning of the decimal point when multiplying. • Represent fractions and decimals using visual models and equations. • Understand and apply the concept of simplifying fractions. • Understand the concept of multiplicative inverses.	• multiply/multiplication • divide/division • fractions • decimals • reciprocal fractions • numerator • denominator • place value • multiplicative inverse <u>Notations and Representations</u> • understand formats of a division problem (a/b; a ÷ b; using long division symbol) • understand formats of a multiplication problem (a x b; ab; a(b); a • b)	• Teachers need to connect to strategies taught in elementary school. (IN) • Fractions should include mixed numbers and improper fractions. (IN) • Learners should be able to reason using number relationships and logic to choose an efficient strategy to solve each problem. (IN) • Teachers should focus on leading learners to develop strong number sense rather than just teaching the procedure. The focus on procedure will occur in seventh grade. (IN) • Learners might find common denominators when not necessary. (ME) • Learners may try to use remainders. (ME) • Learners might not move the decimal point when dividing by a decimal. (ME) • Learners might make number sense errors (e.g., not understanding that dividing by 1/2 leads to a bigger number, not a smaller number). (ME)

Courses
Mathematics 6

SIXTH GRADE

Operations (O)
Learners will expand their computational fluency to create connections and solve problems within and across concepts.

6.NO.O.4 Determine the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.AR.OA.4		7.AR.EE.1
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Multiply and divide automatically through 12x12. • Find factor pairs and multiples for a given number.	• factor • multiple • greatest common factor (GCF) • least common multiple (LCM)	• This standard leads to algebraic topics, including factoring expressions and the distributive property with variables. The focus should not be on finding the least common denominators or simplifying fractions. (IN) • A good strategy for this standard may be to use factor trees to decompose numbers. (IN) • Learners might mix up factors and multiples. (ME) • Learners may not be confident in their multiplication facts and may struggle with this standard. (ME)

Courses
Mathematics 6 General Math

SIXTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

Ratios and Proportional Relationships (RP)

Learners will use ratios, rates, and proportions to model relationships and solve problems.

6.AR.RP.1 Describe the concept of a ratio relationship between two quantities using ratio language and visual models.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.NO.NF.1	6.AR.RP.4 6.AR.RP.5 6.GM.AV.2	6.AR.RP.2 6.AR.RP.3 7.AR.RP.2 6.DPS.D.1 6.DPS.D.2
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the concept of a fraction as a comparison of a part to a whole.	• ratio • part • whole <u>Notations and Representations</u> • Represent ratios three ways using: "to", ":", and a fraction bar • tape diagram • double number line • picture • equation • table	• Learners may place the numbers in the incorrect spot because they assume the smaller number always must come first. (ME)

Courses
Mathematics 6 Algebra I Semester I Applied Math Consumer Math

SIXTH GRADE

Ratios and Proportional Relationships (RP)
Learners will use ratios, rates, and proportions to model relationships and solve problems.

6.AR.RP.2 Describe and calculate a unit rate when given a ratio relationship between two quantities using rate language and visual models.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.AR.RP.1	6.AR.RP.4 6.AR.RP.5	6.AR.RP.3 7.AR.RP.1 7.AR.RP.2 7.AR.RP.3
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Divide fractions and decimals, • Understand the concept of a ratio.	• rate • unit rate • ratio • scale up/scale down <u>Notations and Representations</u> • Unit rate involves a comparison of 2 different quantities, so notation must have both labels (mph; $/hr.) • tape diagram • double number line • picture • equation • table	• This is not the first learners have seen ratios. Connect the concept to prior learning involving multiplication and scaling up. (IN) • Make the connection to equivalent fractions. A unit rate is just an equivalent fraction. (IN) • Learners might figure out hours per mile rather than miles per hour because they are not connecting the unit label with the math being done. (ME) • Learners may not understand that fractions represent division. (ME)

Courses
Mathematics 6 Applied Math

SIXTH GRADE

Ratios and Proportional Relationships (RP)
Learners will use ratios, rates, and proportions to model relationships and solve problems.

6.AR.RP.3 Make and use tables of equivalent ratios, tape diagrams, double number line diagrams, and equations to solve problems involving ratios, rates, and unit rates, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.AR.RP.1 6.AR.RP.2	6.AR.RP.4 6.AR.RP.5	7.AR.RP.1 7.AR.RP.2 7.AR.RP.3 7.AR.RP.4
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the difference between a ratio, rate, and unit rate. • Multiply and divide positive rational numbers.	• ratios • equivalent ratios • ratio tables • tape diagrams • double number line diagrams • rates • unit rates • scale up/scale down	• This is not the first time learners have seen ratios. Connect the concept to prior learning involving multiplication and scaling up. (IN) • Make the connection to equivalent ratios, unit rates, and equivalent fractions. (IN) • Learners might place the numbers in the incorrect spot because they may assume the smaller number always must come first. (ME) • Learners might figure out hours per mile rather than miles per hour because they are not connecting the unit label with the math being done. (ME) • Learners may incorrectly create a ratio that is not equivalent, e.g.,10/3 written as 20/3; forgetting to scale up/down both numerator and denominator. (ME)

Courses

Mathematics 6
Applied Math
Consumer Math

SIXTH GRADE

Ratios and Proportional Relationships (RP)
Learners will use ratios, rates, and proportions to model relationships and solve problems.

6.AR.RP.4 Calculate a percent of a quantity as a rate per 100. Solve problems using ratio reasoning involving finding the whole when given a part and the percent.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.NO.NF.1	6.AR.RP.1 6.AR.RP.2 6.AR.RP.3	7.AR.RP.4 7.DPS.P.1 7.DPS.P.2
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify part and whole. • Create equivalent forms of common fractions. • Understand the concept of a ratio.	• percent • whole • part • rate per 100 <u>Notations</u> • % • tape diagram • double number line • picture • equation • table	• Learners will be expected to find parts when given a whole. (IN) • This standard is an extension of standard 6.AR.RP.3. The focus of this standard is working on equivalent ratios. (IN) • Learners might mix up part and whole. (ME) • Learners might create fractions that are not equivalent (learners may not do the same math to the numerator and denominator). (ME)

Courses
Mathematics 6 Applied Math Consumer Math

Ratios and Proportional Relationships (RP)
Learners will use ratios, rates, and proportions to model relationships and solve problems.

6.AR.RP.5 Convert measurement units within and between measurement systems using ratio reasoning given conversion factors.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.GM.M.1	6.AR.RP.1 6.AR.RP.2 6.AR.RP.3	7.AR.RP.4 9-10.NO.3 9-10.NO.5
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify and use the units used to measure length, mass, weight, capacity, time, and temperature (customary and metric). • Convert units within the same measurement system. • Add, subtract, multiply, and divide positive rational numbers. • Understand the concepts of ratios, rate, and unit rate.	• measurement units • ratio • conversion factors • customary measurement system • metric measurement system <u>Representations</u> • tape diagram • double number line • picture • equation • table	• Learners have converted within the same measurement systems before but have not yet converted between different systems. (IN) • This standard is an extension of standard 6.AR.RP.3. The focus of this standard is working with equivalent ratios. (IN) • Learners may not understand the relationships between the different units used within the measuring system. (ME)

Courses
Mathematics 6 Applied Math

SIXTH GRADE

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

6.AR.EE.1 Read, write, and evaluate numerical expressions including expressions with whole number exponents and grouping symbols.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.NO.NBT.7 5.AR.OA.2 5.AR.OA.3	6.AR.EE.2 6.AR.EE.3 6.GM.AV.2 6.DPS.D.2	7.AR.EE.1 8.NO.O.1

Prerequisite Skills	Key Vocabulary, Notations, and Representations	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Apply the properties of operations. • Evaluate powers of 10. • Identify words such as sum, difference, product, and quotient and match them to the operation they correspond to.	• numerical expressions • whole number exponents • grouping symbols • order of operations (See Appendix A Table 4 of the standards document) • term • base • power • evaluate <u>Notations</u> • grouping symbols include parentheses, brackets, braces, and fraction lines within the order of operations • exponent notation	• Teachers shouldn't focus entirely on simply evaluating the expressions but should also focus on writing an expression from a given picture, situation, or word phrase. (IN) • This is the first learners have evaluated exponents and used grouping symbols other than parentheses. (IN) • Learners may not follow the order of operations. (ME) • Learners might multiply by the exponent instead of applying the exponent. (ME) • Learners may not understand the function of a fraction line as division. (ME) • Learners may not understand that addition and subtraction must be performed from left to right. (ME) • Learners may not understand that multiplication and division must be performed from left to right. (ME) • Learners may not understand the difference between evaluate and solve. (ME)

Courses
Mathematics 6 General Math

SIXTH GRADE

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

6.AR.EE.2 Read and evaluate algebraic expressions, including expressions with whole number exponents and grouping symbols.
Write algebraic expressions to represent simple and authentic situations.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.AR.OA.2 5.AR.OA.3	6.AR.EE.1 6.AR.EE.3	6.AR.EE.5 7.AR.EE.1
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Evaluate numerical expressions using order of operations. • Apply the properties of operations. • Understand what variables are and how they are used. • Substitute a value for a variable. • Understand the difference between numerical and algebraic expressions.	• algebraic expression • whole number exponents • grouping symbols • order of operations (See Appendix A Table 4 of the standards document) • variables • term • base • power • evaluate • coefficient • constant <u>Notations</u> • no longer using x to symbolize multiplication (4x means 4 times x) • begin using coefficients instead • grouping symbols include parentheses, brackets, braces, and fraction lines within the order of operations • exponent notation	• Teachers shouldn't focus entirely on simply evaluating the expressions but should also focus on writing an expression from a given picture, situation, or word phrase. (IN) • This is the first learners have evaluated exponents and used grouping symbols other than parentheses. (IN) • Learners may not follow the order of operations. (ME) • Learners might multiply by the exponent instead of applying the exponent. (ME) • Learners may not understand the function of a fraction line as division. (ME) • Learners may not understand that addition and subtraction must be performed from left to right. (ME) • Learners may not understand that multiplication and division must be performed from left to right. (ME) • Learners might substitute incorrectly. (ME)

Courses

Mathematics 6
Applied Math
Consumer Math

SIXTH GRADE

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

6.AR.EE.3 Identify when two expressions are equivalent. Apply the properties of operations to generate equivalent expressions.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.AR.OA.2	6.AR.EE.1 6.AR.EE.2	7.NO.NS.2 7.AR.EE.1
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Apply the properties of operations • Understand the concept of equivalent expressions	• equivalent expressions • properties of operations (See Appendix A Table 1 of the standards document) • numerical expressions • algebraic expressions • Distributive Property • like terms • factor	• Both numeric and algebraic expressions are included in this standard. (IN) • Visual models such as rectangular arrays and/or algebra tiles could be used to show equivalency. (IN) • Learners might not apply the distributive property correctly. (ME) • Learners might incorrectly combine like terms. (ME)

Courses
Mathematics 6

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

6.AR.EE.4 Describe the concept of a solution to an equation and an inequality.
Determine whether a given number is a solution to an equation or an inequality.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.AR.OA.2		6.AR.EE.5 6.AR.EE.6 7.AR.EE.2 7.AR.EE.3
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish between expressions, equations, and inequalities. • Understand the concept of equality vs. inequality.	• solution to equation • solution to inequality • variable • expression • equation • equality • inequality • substitution <u>Notation</u> • <, >, ≥, ≤, =, ≠	• Teachers can have learners graph single solutions on a number line to lay the foundation for graphing inequalities on a number line and highlight the difference between a solution and a solution set. (IN) • Learners might use incorrect substitution. (ME) • Learners might read the inequality symbol incorrectly. (ME) • Learners may not understand the "or equal to" part of an inequality symbol. (ME)

Courses
Mathematics 6

SIXTH GRADE

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

6.AR.EE.5 Write and solve equations of the form x + p = q and px = q for cases in which p and q are non-negative whole numbers or decimals, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.NO.NS.1 6.AR.EE.2 6.AR.EE.4	6.NO.O.3 6.AR.EE.6	7.AR.EE.2 7.AR.EE.3
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Translate words into mathematical symbols. • Add, subtract, multiply, and divide multi-digit numbers and decimals. • Identify the operation that undoes another operation (inverse operations). • Determine whether a given number is a solution to an equation.	• equation • variable • inverse operations • properties of equality (see Appendix A, Table 2 of the standards document) • zero pair (2 numbers that add together to equal 0; opposites) • term	• Teachers shouldn't use only whole numbers within simple mental math problems rather than have learners think through the properties of equality. (IN) • Create visuals to match the equations to help learners conceptualize what they are doing when they solve equations. (IN) • Learners might not follow the properties of equality (not doing the same math for both sides of the equation). (ME) • Learners may not understand the concept of a zero pair. (ME) • Learners might use the incorrect inverse operation (trying to undo division with subtraction). (ME) • Learners might not translate the words into the correct mathematical symbols. (ME) • Learners may not put an equal sign into the equation. (ME)

Courses

Mathematics 6
Applied Math
General Math

SIXTH GRADE

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

6.AR.EE.6 Write a statement of inequality of the form x > c or the form x < c to represent a constraint or condition.
Recognize that inequalities of the form x > c or the form x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.AR.EE.4	6.NO.NS.2 6.AR.EE.5	7.AR.EE.3
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Graph values on a number line. • Understand the symbols <, >, ≥, ≤. • Determine whether a given number is a solution to an inequality.	• inequalities • constraint or condition • number line diagram • variable • infinitely many solutions • at most, at least, no more than <u>Notations</u> • <, > , ≥, ≤ • Open vs. closed circle • Shading a number line	• Teachers need to make sure all four inequality symbols are included even though the standard only explicitly states two of them. (IN) • Learners might be confused by the comparison phrases (no less than, no more than, at least, at most, etc.). (ME) • Learners may not identify correctly an open/closed circle on the number line. (ME) • Learners might shade the same direction as the inequality symbol points. (ME) • Learners might confuse the direction of the inequality symbol. (ME)

Courses
Mathematics 6 Applied Math Consumer Math General Math

SIXTH GRADE

Geometry and Measurement (GM)

Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

Area and Volume (AV)

Learners will use visualization and spatial reasoning to solve problems involving the area, surface area, and volume of geometric figures.

6.GM.AV.1 Derive the relationship of the areas of triangles using the area of rectangles.
Calculate the areas of triangles and quadrilaterals by composing and/or decomposing them into rectangles and triangles, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.GM.M.2	6.GM.GF.3	7.GM.AV.1 7.GM.AV.2 7.GM.AV.3
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify triangles and the different types of quadrilaterals. • Identify side lengths and angles (acute, obtuse, right). • Understand the concept of area • Multiply fractions, decimals, and whole numbers. • Decompose quadrilaterals into rectangles and/or triangles. • Compose quadrilaterals using rectangles and triangles.	• area • compose/decompose • base • height <u>Notation</u> • Square units	• The focus of this standard is to develop a fluent way of finding the area of a triangle based on the prior learning of composing and decomposing parallelograms in elementary school. (IN) • Learners need to be exposed to all three types of triangles (acute, obtuse, right). Draw attention to the fact that the height could be found inside, outside, or on the triangle. (IN) • Learners may incorrectly compose or decompose shapes. (ME) • Learners may calculate the triangle area without using the height. (ME) • Learners may not understand the concept of area and the need for square units (inches vs. square inches). (ME)

Courses

Mathematics 6
General Math

SIXTH GRADE

Area and Volume (AV)
Learners will use visualization and spatial reasoning to solve problems involving the area, surface area, and volume of geometric figures.

6.GM.AV.2 Describe the concept of volume of a right rectangular prism.
Apply given formulas to calculate the volume of right rectangular prisms, including fractional edge lengths, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.NO.NF.4 5.GM.M.3	6.NO.O.3 6.AR.RP.1 6.AR.EE.1 6.GM.GF.3	7.GM.AV.3
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Multiply multi-digit whole numbers, fractions, and decimals. • Substitute values into an expression. • Evaluate expressions. • Differentiate between two- and three-dimensional shapes. • Identify the attributes of rectangular prisms (edge length, vertices, faces). • Understand the concept of the volume of three-dimensional figures. • Differentiate between square units and cubic units. • Understand volume is measured in cubic units. • Measure the volume of a rectangular prism by counting unit cubes. • Identify the base and the height. • Find the area of the base.	• volume • right rectangular prism • edge length • base • height • face • edge • vertex <u>Notation</u> • Cubic Units	• In fifth grade, there is a similar standard (5.GM.M.3) that uses only whole numbers. This standard expands it to fractional edge lengths. (IN) • This is the first learners see the formula for the volume of a prism. (IN) • Learners may not distinguish between the height of a base and the height of the prism. (ME) • Learners may not understand the height can move (height isn't always vertical). (ME) • Learners may use the incorrect units (square units or units rather than cubic units). (ME) • Learners might think of volume as length x width x height rather than the concept of the area of the base x the height. Understanding the latter will help their understanding of volume in future learning. (ME)

Courses

Mathematics 6
General Math

SIXTH GRADE

Geometric Figures (GF)
Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics of figures, perform transformations, and construct logical arguments.

6.GM.GF.1 Identify and position ordered pairs of rational numbers in all four quadrants of a coordinate plane.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.AR.OA.5 5.GM.G.2 5.GM.G.3	6.NO.NS.1 6.GM.GF.2	7.AR.RP.2 7.AR.RP.3
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify the x and y coordinate in an ordered pair. • Identify the parts of a coordinate plane. • Graph points from ordered pairs in the first quadrant of the coordinate plane. • Plot rational numbers on a number line.	• coordinates • x-axis and y-axis (plural: axes) • coordinate plane • opposites • origin • ordered pair/coordinates • quadrant <u>Notations</u> • (x,y) • Quadrant I, II, III, and IV	• This is the first learners work in all four quadrants of the coordinate plane. (IN) • This standard should build off the idea of opposites from standard 6.NO.NS.1. (IN) • Learners might misnumber the quadrants. (ME) • Learners might mix up the x and y coordinates. (ME) • Learners may not understand horizontal movement comes before vertical. (ME) • Learners may not understand how to plot points on a number line. (ME)

Courses
Mathematics 6 General Math

Geometric Figures (GF)
Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics of figures, perform transformations, and construct logical arguments.

6.GM.GF.2 Draw polygons in the coordinate plane given for the vertices.
Determine the length of a side joining points with the same first or second coordinate, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.AR.OA.3 5.GM.G.2 5.GM.G.3	6.GM.GF.1	7.AR.RP.3 8.GM.GF.1 8.GM.GF.2 8.GM.GF.6
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify polygons and classify them based on their attributes (vertices, sides (length, parallel, perpendicular). • Identify and graph coordinate points from ordered pairs in all four quadrants of the coordinate plane.	• polygon • vertices • side length • point • ordered pair/coordinates <u>Notation</u> • (x, y)	• The focus is not on integer operations. The side of the polygons should not be diagonal. (IN) • Learners might plot points incorrectly. (ME) • Learners may not know the vocabulary associated with polygons. (ME)

Courses
Mathematics 6 Applied Math

SIXTH GRADE

Geometric Figures (GF)
Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics of figures, perform transformations, and construct logical arguments.

6.GM.GF.3 Represent three-dimensional figures using nets made up of rectangles and triangles (right prisms and pyramids whose bases are triangles and rectangles).
Calculate the surface area of prisms with rectangular and triangular bases using nets, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
2.GM.G.2 2.GM.G.3 5.GM.M.2	6.GM.AV.1 6.GM.AV.2	7.GM.AV.2
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify three-dimensional figures. • Calculate the area of rectangles and triangles. • Identify two-dimensional shapes found within three-dimensional shapes. • Compose geometric shapes having specified geometric attributes, such as a given number of edges, angles, faces, vertices, and/or sides.	• three-dimensional figure • rectangle • triangle • right prism (triangular or rectangular) • cube • pyramid (triangular, rectangular, or square) • base • surface area • net • edge • face • vertex <u>Notation</u> • Square units	• Teachers shouldn't expose learners to surface area formulas. The focus of this standard is on the area of nets. (IN) • Learners may not realize there is more than one way to draw a net. (ME) • Learners might confuse three-dimensional vs. two-dimensional figures. (ME) • Learners might mislabel the area. (ME) • Learners might misunderstand the height of the solid vs. the height of the base. (ME) • Learners may confuse volume and surface area (what's in the box vs. the wrapping paper). (ME)

Courses
Mathematics 6 Applied Math General Math

SIXTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*
Data Analysis (D) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, and making predictions.*
6.DPS.D.1 Write a statistical question that can be answered using measures of center or variability of a data set.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
4.DPS.D.1 5.DPS.D.1	6.DPS.D.2 6.DPS.D.3 6.DPS.D.4	7.DPS.D.1 7.DPS.D.2
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Formulate questions to collect, organize, and represent data to reason with math and across disciplines.	• statistical questions • measure of center (mean, median) • measure of variability (range, mean absolute deviation) • statistics • data	• Learners might write a question that cannot be answered using a measure of center or variability, e.g., asking learners within the same grade level what grade they are in. (ME)

Courses
Mathematics 6 Applied Math

SIXTH GRADE

Data Analysis (D)
Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, and making predictions.

6.DPS.D.2 Calculate measures of center (median and mean) and variability (range and mean absolute deviation) to answer a statistical question. Identify mode(s) if they exist.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.DPS.D.2	6.NO.NS.1 6.NO.NS.2 6.AR.EE.1 6.DPS.D.1 6.DPS.D.3 6.DPS.D.4	7.DPS.D.2
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Apply order of operations with and/or without technology. • Find absolute value. • Order rational numbers.	• measure of center (mean, median) • measure of variability (range, mean absolute deviation) • mode • data set	• Learners may use calculators incorrectly. (ME) • Learners might confuse opposites and absolute value. (ME) • Learners may not order numbers before finding the median. (ME) • Learners may not understand there can be one mode, no mode, or more than one mode. (ME) • Learners may not understand how to find the average of two middle values when finding the median. (ME) • Learners may not answer the statistical question.(ME)

Courses

Mathematics 6
Applied Math

SIXTH GRADE

Data Analysis (D)
Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, and making predictions.

6.DPS.D.3 Identify outliers by observation and describe their effect on measures of center and variability. Justify which measures would be appropriate to answer a statistical question.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.DPS.D.1 5.DPS.D.2	6.DPS.D.1 6.DPS.D.2 6.DPS.D.4	7.DPS.D.1 7.DPS.D.2 8.DPS.D.1
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Calculate measures of center and variability. • Write a statistical question. • Know how to read a data display.	• outliers • data set • measure of center (mean, median) • measure of variability (range, mean absolute deviation) • skew • spread • symmetry • tail	• Learners should be using graphs to help identify outliers and appropriate measures rather than a formula. (IN) • Learners might not understand the purpose of a measure of center. (ME) • Learners might not understand what makes an outlier an outlier. (ME) • Learners might not understand the outlier affects the mean but not the median. (ME)

Courses
Mathematics 6 Applied Math

Data Analysis (D)
Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences, conclusions, and making predictions.

6.DPS.D.4 Display numerical data in plots on a number line, including dot plots and histograms. Describe any overall patterns in data, such as gaps, clusters, and skews

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.DPS.D.1 5.DPS.D.2	6.NO.NS.2 6.DPS.D.1 6.DPS.D.2 6.DPS.D.4	7.DPS.D.1 7.DPS.D.2 8.DPS.D.1 8.DPS.D.4 9-10.DPS.1*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Collect, organize, and sort data. • Understand how to read a line plot. • Understand what data displays are and how they are used.	• dot plot/line plot • number line • histogram • gaps • cluster • skews • numerical data	• Teachers should include data sets that are uniform, skewed, symmetric, and normal (bell-shaped). (IN) • Learners should be connecting the shape of the graph to the measures of center and outliers. (IN) • Learners may not understand the difference between a bar graph and a histogram. (ME)

Courses
Mathematics 6 Applied Math

SEVENTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

Number Systems (NS)

Learners will expand their knowledge of the number system to create connections and solve problems within and across concepts.

7.NO.NS.1 Describe the absolute value of a number as its distance from zero on a number line.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.NO.NS.1 6.NO.NS.2	7.NO.O.1 7.NO.O.3 7.AR.EE.2 7.AR.EE.3	8.NO.NS.1 8.AR.EE.6

Prerequisite Skills	Key Vocabulary, Notations, and Representations	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Understand the concept of positive and negative numbers (opposites) and their placement on the number line. • Find the distance from zero on a horizontal and vertical number line. • Plot integers and their opposites on a horizontal number and vertical number line.	• absolute value • negative numbers • positive numbers • distance • opposite • zero • zero pair <u>Notations and Representations</u> • absolute value bars | | • horizontal number line • vertical number line	• Formal absolute value notation is introduced in this standard. (IN) • Teachers need to teach both vertical and horizontal number lines in a variety of ways (open number line, building a number line on the floor, clothesline, comparing it to a thermometer, etc.), highlighting that opposites are the same distance from zero on opposite sides of zero. (IN) • Learners may not realize distance cannot be negative. (ME) • Learners might not understand absolute value is not the same as opposite. (ME) • Learners may not understand absolute value as an operation (you don't need the bars after you've taken the absolute value). (ME) • Learners might not understand a negative negative means the opposite of negative (e.g., -(-3) = 3). (ME) • Learners may not realize that -|5| is not the same as |-5|. (ME)

Courses
Mathematics 7 Consumer Math

SEVENTH GRADE

Number Systems (NS)
Learners will expand their knowledge of the number system to create connections and solve problems within and across concepts.

7.NO.NS.2 Recognize common fractions and decimal equivalencies up to a denominator of 10. Convert a rational number to a decimal using technology.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.NO.NF.1 6.NO.NS.2 6.NO.O.3 6.AR.EE.3	7.NO.O.1 7.NO.O.2 7.NO.O.3	7.AR.RP.4 7.AR.EE.1 7.AR.EE.2 7.AR.EE.3 8.NO.NS.1
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the concept of rational numbers. • Understand the concept of equivalent fractions and decimals. • Use a calculator to divide.	• fraction • decimal • rational number • equivalent • repeating decimal • terminating decimal • numerator • denominator • convert <u>Notation</u> • line over numbers to indicate repeating digits (vinculum)	• Common fractions would include halves, thirds, fourths/quarters, fifths, eighths, and tenths. (IN) • Learners may use common fractions to expand their knowledge to sixteenths for measurement purposes. (IN) • Learners may not understand a fraction as division. (ME) • Learners may not understand repeating decimals are different than terminating decimals (thinking 0.3 is the same as 0.333…). (ME)

Courses
Mathematics 7 General Math

SEVENTH GRADE

Operations (O)
Learners will expand their computational fluency to create connections and solve problems within and across concepts.

7.NO.O.1 Add, subtract, multiply, and divide integers using visual models and properties of operations in multi-step problems, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.NO.NS.1 6.NO.O.1	7.NO.NS.1 7.NO.NS.2 7.NO.O.2 7.NO.O.3	7.AR.EE.1 7.AR.EE.2 7.AR.EE.3 8.NO.O.1 8.NO.O.2
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand what integers are. • Identify the opposite of a given integer. • Understand and apply the properties of operations and the order of operations.	• integers • properties of operations (See Appendix A Table 1) • zero pair • opposite • positive/negative <u>Representations</u> • visual models may include: • horizontal and vertical lines • algebra tiles • 2 color counters	• The development of the conceptual understanding of subtraction as adding the opposite is the focus of this standard. (IN) • This is where learners most often show shortcomings in number sense. (IN) • Learners may not understand how negative, subtraction, and adding opposites are related. (ME) • Learners may not realize subtraction doesn't always lead to smaller numbers. (ME)

Courses
Mathematics 7

SEVENTH GRADE

Operations (O)
Learners will expand their computational fluency to create connections and solve problems within and across concepts.

7.NO.O.2 Add, subtract, multiply, and divide non-negative fractions in multi-step problems, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.NO.O.1 6.NO.O.2 6.NO.O.3	7.NO.NS.2 7.NO.O.1 7.NO.O.3 7.AR.EE.1	8.NO.O.2
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Add and subtract fractions with like and unlike denominators. • Convert between mixed numbers and improper fractions. • Multiply and divide fractions using models. • Identify the reciprocal for a given fraction. • Understand and apply the properties of operations and the order of operations.	• fractions • reciprocal fractions • order of operations • numerator • denominator • multiplicative inverse • least common denominator <u>Notations and Representations</u> • understand formats of a division problem (a/b; a ÷ b; using long division symbol) • understand formats of a multiplication problem (a x b; ab; a(b); a • b)	• Teachers should expand on conceptual understanding from sixth grade to build fluency in fractional computations. (IN) • Learners might assume a common denominator is needed to multiply and divide fractions. (ME) • Learners may not find the common denominator when adding and subtracting fractions. (ME) • Learners might get confused about which fraction needs to be reciprocated when dividing. (ME) • Learners may assume dividing by a fraction should always lead to a smaller number. (ME)

Courses
Mathematics 7

SEVENTH GRADE

Operations (O)
Learners will expand their computational fluency to create connections and solve problems within and across concepts.

7.NO.O.3 Add, subtract, multiply, and divide non-negative decimals to the hundredth place in multi-step problems using strategies or procedures, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.NO.O.1 6.NO.O.2 6.NO.O.3	7.NO.NS.1 7.NO.NS.2 7.NO.O.1 7.NO.O.2 7.AR.EE.1	8.NO.O.2

Prerequisite Skills	Key Vocabulary, Notations, and Representations	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Add, subtract, multiply, and divide decimals (may use visual models). • Understand the positioning of the decimal when adding, subtracting, multiplying, and dividing. • Understand and apply the properties of operations and the order of operations.	• decimals • order of operations • place value <u>Notations and Representations</u> • understand formats of a division problem (a/b; a ÷ b; using long division symbol) • understand formats of a multiplication problem (a x b; ab; a(b); a • b)	• Learners should be able to reason using number relationships and logic to choose an efficient strategy to solve each problem. (IN) • Place value might be difficult for learners. (ME) • Learners might ignore place value while doing the operations. (ME) • Learners may not move the decimal point when dividing by a decimal. (ME) • Learners may not realize dividing doesn't always lead to a smaller quotient. (ME) • Learners might not realize multiplying doesn't always lead to a larger quotient. (ME)

Courses
Mathematics 7

SEVENTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

Ratios and Proportional Relationships (RP)

Learners will use ratios, rates, and proportions to model relationships and solve problems.

7.AR.RP.1 Calculate unit rates associated with ratios of rational numbers, including ratios of lengths, areas, and other quantities measured in like or different units.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.AR.RP.2 6.AR.RP.3	7.AR.RP.2 7.AR.RP.3 7.AR.RP.4	8.AR.EE.3 8.AR.EE.4 8.AR.F.4 9-10.NO.3 9-10.NO.4* 9-10.GM.35*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Convert between fractions, decimals, and percentages. • Understand the difference between a ratio, rate, and unit rate. • Divide fractions by fractions. • Divide decimals.	• unit rates • ratios • rate • unit <u>Representations</u> • unit rate can be represented as a fraction, decimal, and/or percent	• Make the connection to equivalent fractions. A unit rate is just an equivalent fraction. (IN) • Provide a variety of instances for learners to calculate unit rate (ratios of lengths, area, etc.). (IN) • Learners may not understand the difference between ratio, rate, and unit rate. (ME) • Learners might figure out hours per mile rather than miles per hour because they are not connecting the unit label with the math being done. (ME) • Learners may not realize that sometimes the unit rate is not a whole number (getting a solution of $1.50 unit price or 2.3 km/hour). (ME)

Courses
Mathematics 7 Applied Math

SEVENTH GRADE

Ratios and Proportional Relationships (RP)
Learners will use ratios, rates, and proportions to model relationships and solve problems.

7.AR.RP.2 Analyze the relationship between the dependent and independent variables of a proportional relationship using graphs and tables.
Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, k) where k is the unit rate.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.AR.RP.1 6.AR.RP.2 6.AR.RP.3 6.GM.GF.1	7.AR.RP.1 7.AR.RP.3 7.AR.RP.4	8.AR.EE.3 8.AR.EE.4 8.AR.F.1 8.AR.F.2 8.AR.F.3 8.AR.F.4 8.AR.F.5 9-10.NO.3 9-10.DPS.10
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Divide fractions. • Identify and name ordered pairs on the coordinate plane. • Understand the concept of unit rate. • Calculate the unit rate of a proportional relationship.	• dependent variable • independent variable • proportional relationship unit rate (also known as the constant of proportionality, k) • origin • ordered pair	• A proportional relationship (also known as a direct variation) is a subset of linear functions; however, the terms linear and slope are not taught until eighth grade. (IN) • The focus of this standard is analyzing the connection between different representations of a proportional relationship. (IN) • Learners might mix up dependent and independent variables. (ME) • Learners might mix up coordinates x and y of an ordered pair. (ME) • Learners might misinterpret scale on axes. (ME)

Courses
Mathematics 7 Algebra I Semester I Applied Math Consumer Math

SEVENTH GRADE

Ratios and Proportional Relationships (RP)
Learners will use ratios, rates, and proportions to model relationships and solve problems.

7.AR.RP.3 Identify the constant of proportionality in tables, graphs, equations, diagrams, and descriptions of proportional relationships.
Represent proportional relationships by an equation of the form y = kx, where k is the constant of proportionality, and describe the meaning of each variable (y, k, x) in the context of the situation.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.AR.RP.2 6.AR.RP.3 6.GM.GF.1 6.GM.GF.2	7.AR.RP.1 7.AR.RP.2 7.AR.RP.4	8.AR.EE.3 8.AR.EE.4 8.AR.F.2 8.AR.F.3 8.AR.F.4 8.AR.F.5
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Subtract and divide fractions, decimals, and whole numbers. • Identify the independent and dependent variables. • Understand the concept of unit rate. • Read a graph. • Read a table.	• constant of proportionality (also known as unit rate, k) • proportional relationship • independent variable • dependent variable • origin <u>Representations</u> • diagram • graph • table of values • equation in the form y = kx	• A proportional relationship (also known as a direct variation) is a subset of linear functions; however, the terms slope and linear are not taught until 8th grade. (IN) • To provide a deeper understanding, learners should model proportional relationships by making graphs, tables, and equations. (IN) • Learners might mix up dependent and independent variables. (ME) • Learners might mix up coordinates x and y of an ordered pair. (ME) • Learners might misinterpret scale on axes. (ME)

Courses
Mathematics 7 Algebra I Semester I Applied Math Consumer Math

SEVENTH GRADE

Ratios and Proportional Relationships (RP)
Learners will use ratios, rates, and proportions to model relationships and solve problems.

7.AR.RP.4 Use proportional relationships to solve multi-step problems involving ratios, percents, and scale drawings of geometric figures, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.AR.RP.3 6.AR.RP.4 6.AR.RP.5 7.NO.NS.2	7.AR.RP.1 7.AR.RP.2 7.AR.RP.3 7.GM.AV.2 7.DPS.P.1 7.DPS.P.2	8.AR.EE.3 8.GM.GF.1 8.GM.GF.2 8.DPS.D.4 9-10.GM.26 (+) 9-10.GM.29 9-10.GM.36* 9-10.DPS.9
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Convert percents to fractions or decimals. • Find equivalent fractions. • Understand the concepts of ratios, percentages, and scale. • Create ratio tables. • Create tape diagrams. • Create double-number line diagrams. • Understand the concept of the constant of proportionality. • Apply the order of operations.	• proportional relationship • ratio • percent • scale drawing • proportion • scaling up • scaling down • scale factor	• Learners may use ratio tables, tape diagrams, double number lines, or other strategies to solve these types of problems. (IN) • Most errors within this standard tend to be computational. (ME) • Learners may look for shortcuts to do the math and rely too heavily on procedures rather than understanding the concepts. (ME)

Courses

Mathematics 7
Algebra I Semester I
Applied Math
Consumer Math

SEVENTH GRADE

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

7.AR.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions involving variables, integers, and/or nonnegative fractions and decimals with an emphasis on writing equivalent expressions.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.NO.O.4 6.AR.EE.1 6.AR.EE.2 6.AR.EE.3 7.NO.NS.2 7.NO.O.1	7.NO.O.2 7.NO.O.3 7.AR.EE.2 7.AR.EE.3	8.AR.EE.1 8.AR.EE.5 8.AR.EE.6 9-10.AR.11
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify and apply properties of operations. • Identify and write numeric or algebraic expressions. • Identify the function of parentheses, brackets, braces, and fraction lines within the order of operations.	• properties of operations (see Appendix A Table 1) • variables • integers • equivalent expressions • factor an expression (undo the Distributive Property) • constant • coefficient • terms • combine like terms • Distributive Property	• The focus of this standard is to generate equivalent expressions using properties of operations. (IN) • Both numeric and algebraic expressions are included in this standard. (IN) • Visual models, such as rectangular arrays and/or algebra tiles, could be used to show equivalency. (IN) • Learners might forget to multiply the second term when applying the distributive property. (ME) • Learners may combine terms that are not like terms. (ME) • Learners might not follow the order of operations. (ME)

Courses
Mathematics 7 General Math

SEVENTH GRADE

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

7.AR.EE.2 Write and solve equations of the form px + q = r and p(x + q) = r , including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.NO.NS.1 6.AR.EE.4 6.AR.EE.5 7.NO.NS.2 7.NO.O.1	7.NO.NS.1 7.AR.EE.1 7.AR.EE.3	8.AR.EE.2 8.AR.EE.5 8.AR.EE.6 8.AR.F.4 9-10.AR.3*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Apply properties of operations. • Apply the properties of equality. • Translate written or verbal descriptions into mathematical equations. • Solve one-step equations. • Understand the concept of isolating the variable. • Describe the concept of the solution to an equation. • Determine whether a given value is a solution to an equation.	• properties of equality (see Appendix A Table 2) • equation • isolate • variable • coefficient • inverse operations • zero pairs • Distributive Property • collect like terms (see glossary)	• Teachers shouldn't use only whole numbers within simple mental math problems rather than have learners think through the properties of equality. (IN) • Use visuals to match the equations to help learners conceptualize what they are doing when they write and solve equations. (IN) • Learners may need to understand that the answer may not be a whole number. (IN) • Learners might not apply the distributive property correctly. (ME) • Learners may combine terms that are not like terms. (ME) • Learners might not perform the same operation on both sides of the equal sign. (ME) • Learners might try to complete the problem in their minds and make mistakes rather than focusing on the process. (ME) • Learners may not understand that 0 can be a solution. (ME) • Learners may not understand that a variable can be on either side of an equal sign. (ME)

Courses
Mathematics 7

SEVENTH GRADE

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

7.AR.EE.3 Write and solve one- or two-step inequalities where coefficients and solutions are integers and/or non-negative fractions and decimals, including authentic problems.
Graph the solution set of the inequality and interpret it in the context of the problem.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.NO.NS.1 6.AR.EE.4 6.AR.EE.5 6.AR.EE.6 7.NO.NS.2 7.NO.O.1	7.NO.NS.1 7.AR.EE.1 7.AR.EE.2	8.AR.EE.7 8.AR.EE.8 8.AR.F.4 9-10.AR.3*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/Errors (ME)**
• Understand the meaning of the symbols < , >, ≥, and ≤. • Understand how to isolate the variable. • Graph the solution set on a number line.	• inequality • properties of inequality (see Appendix A Table 3) • coefficient • isolate • variable • Distributive Property • zero pair • solution set <u>Notations</u> • <, >, ≥, ≤ • open vs. closed circle • shading a number line	• Teachers shouldn't use only whole numbers within simple mental math problems rather than have learners think through the properties of equality. (IN) • Use visuals to match the equations to help learners conceptualize what they are doing when they write and solve inequalities. (IN) • This is the first learners are solving inequalities. (IN) • Learners may not apply the distributive property correctly. (ME) • Learners might combine terms that are not like terms. (ME) • Learners may not perform the same operation on both sides of the inequality symbol. (ME) • Learners might try to complete the problem in their minds and make mistakes rather than focusing on the process. (ME) • Learners may not understand that the answer may not be a whole number. (ME) • Learners may not understand that 0 can be a solution. (ME) • Learners may not remember when to reverse the direction of the inequality symbol. (ME) • Learners might assume the inequality symbol indicates the direction of shading. (ME) • Learners may not realize a variable can be on either side of the inequality symbol. (ME)

Courses
Mathematics 7

SEVENTH GRADE

Geometry and Measurement (GM)

Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

Area and Volume (AV)

Learners will use visualization and spatial reasoning to solve problems involving the area, surface area, and volume of geometric figures.

7.GM.AV.1 Describe the relationship between the circumference and diameter of a circle (pi).
Apply given formulas to calculate the area and circumference of a circle, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.NO.NBT.3 6.GM.AV.1	7.GM.AV.2	8.GM.AV.1 9-10.NO.5 9-10.GM.22 9-10.GM.25 9-10.GM.26 9-10.GM.31 9-10.GM.35*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Round to a given place value. • Understand the concept of area. • Use a formula. • Apply the order of operations with and without a scientific calculator. • Label problems with appropriate units.	• circumference • diameter • pi • radius • area <u>Notations</u> • symbol for pi (π) • square units	• Use of approximations for π (22/7, 3.14, or π button) will result in different answers. (IN) • The focus should be on developing the concept of π before calculating area and circumference. (IN) • Learners might use the incorrect units and do not understand circumference as a length. (ME) • Learners may not round correctly. (ME) • Learners may not understand π goes on forever. (ME) • Learners might confuse circumference and area. (ME) • Learners might mix up radius and diameter. (ME) • Learners may not understand the relationship between diameter and radius. (ME)

Courses

Mathematics 7
Applied Math

SEVENTH GRADE

Area and Volume (AV)
Learners will use visualization and spatial reasoning to solve problems involving the area, surface area, and volume of geometric figures.

7.GM.AV.2 Calculate areas of polygons by composing and/or decomposing them into rectangles and triangles, including authentic problems.
Solve problems involving the surface area of prisms and right pyramids using nets, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.GM.GF.3 6.GM.AV.1	7.AR.RP.4 7.GM.AV.1 7.GM.AV.3	9-10.GM.30* 9-10.GM.32 9-10.GM.35* 9-10.GM.36*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify three-dimensional figures. • Understand what net is and how it is used. • Calculate the area of rectangles and triangles. • Use nets made of rectangles and triangles to represent three-dimensional figures. • Understand the concept of surface area. • Understand the difference between area and surface area. • Understand how nets may be used to calculate surface area. • Use nets to calculate the surface area of prisms with rectangular and triangular bases.	• area • surface area • net • compose/decompose • triangle • rectangle • square units • height • base • cube • prism • pyramid	• The focus of this standard is to develop a fluent way of finding the area of a polygon based on prior learning concepts of composing and decomposing polygons into rectangles and triangles. (IN) • Teachers shouldn't expose learners to surface area formulas. The focus of the standard is on the area of nets. (IN) • Learners might confuse area, surface area, and volume. (ME) • Learners might mislabel nets. (ME) • Learners might mislabel answers with the wrong units. (ME) • Learners might misunderstand the height of a solid vs. the height of a base. (ME) • Learners may not understand there is more than one way to draw a net. (ME)

Courses

Mathematics 7
Applied Math

SEVENTH GRADE

Area and Volume (AV)
Learners will use visualization and spatial reasoning to solve problems involving the area, surface area, and volume of geometric figures.

7.GM.AV.3 Solve problems involving the volume of prisms and composite solids, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.GM.AV.1 6.GM.AV.2	7.GM.AV.2	8.GM.AV.1 9-10.GM.31 9-10.GM.33 9-10.GM.35* 9-10.GM.36*

Prerequisite Skills	Key Vocabulary, Notations, and Representations	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Find the area of triangles and rectangles. • Understand the concept of the volume of three-dimensional figures. • Understand volume is measured in cubic units and label correctly for volume. • Understand what a composite solid is and identify composite solids. • Decompose a composite solid. • Distinguish between area, surface area, and volume. • Identify the base and height of the solid.	• volume • composite solid • prism • base • height • face • edge • vertex • cubic units	• When finding the area of the bases of a prism or composite solid, learners should find the area by composing and/or decomposing them into rectangles and triangles. (IN) • Learners may be unable to distinguish between the height of a base and the height of the prism. (ME) • Learners may not understand the height can move (height isn't always vertical). (ME) • Learners may use the incorrect units (square units or units rather than cubic units). (ME) • Learners might perceive volume as length x width x height rather than the concept of the area of the base x the height. The latter will help their understanding of volume in future learning. (ME) • Learners may not understand that the base doesn't always appear at the bottom of the figure. (ME) • Learners might decompose the figure incorrectly. (ME)

Courses

Mathematics 7
Applied Math

SEVENTH GRADE

Geometric Figures (GF)
Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics of figures, perform transformations, and construct logical arguments.

7.GM.GF.1 Draw triangles from given conditions using appropriate tools.
Defend whether a unique triangle, multiple triangles, or no triangle can be constructed when given three measures of angles or sides.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
4.GM.M.6 4.GM.G.1 4.GM.G.2 5.GM.G.1		8.GM.GF.1 8.GM.GF.3 8.GM.GF.5 8.GM.GF.6 9-10.GM.12 9-10.GM.28
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Use manipulatives, protractors, rulers, and/or technology to construct shapes. • Identify types of angles (acute, obtuse, right).	• triangle • unique triangles • angle measure • side measure • triangle classifications: acute, right, obtuse, equiangular, scalene, isosceles, equilateral <u>Notations</u> • degree symbol for angle measures • angle symbol (∠) • box symbol to represent 90°	• "Draw triangles" means using rulers, protractors, compasses, and/or technology. Formal constructions are not expected at this level. (IN) • Learners may not understand that in a triangle, the angle measures must add up to 180. (ME) • Learners might make assumptions that any three lengths can make a triangle. (ME)

Courses
Mathematics 7

SEVENTH GRADE

Geometric Figures (GF)
Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics of figures, perform transformations, and construct logical arguments.

7.GM.GF.2 Describe the following angle-pair relationships: supplementary angles, complementary angles, vertical angles, and adjacent angles.
Solve for an unknown angle in a figure by applying facts about these angles.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
4.GM.G.1 4.GM.G.2 4.GM.M.1 4.GM.M.7		8.GM.GF.4 9-10.GM.9
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Explain what an angle is and know how to measure one. • Identify types of angles. • Name angles using proper notation.	• angle • supplementary angles • complementary angles • vertical angles • adjacent angles • intersecting lines or line segments • congruent angles <u>Notations</u> • degree symbol for angle measures • angle symbol (∠) • congruent symbol (≅) • box symbol to represent 90°	• Notation clarification: Angle measures are equal, while angles are congruent. (IN) • The focus of this standard is on finding unknown angle measures, not solving equations, so do not use algebraic expressions for angle measures. (IN) • Learners might confuse supplementary and complementary. (ME) • Learners may use incorrect notation. (ME) • Learners might name angles incorrectly (not knowing that the vertex needs to be the middle letter when naming an angle using three letters). (ME) • Learners might misuse = and ≅ symbols. (ME)

Courses
Mathematics 7 Applied Math General Math

SEVENTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*
Data Analysis (D) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences, conclusions, and making predictions.*
7.DPS.D.1 Identify the strengths and weaknesses of a population sample including bias in the process of the data collection.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
6.DPS.D.1 6.DPS.D.3 6.DPS.D.4		7.DPS.D.2
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand what a statistical question is. • Understand the data analysis process (collect, organize, display).	• sample • population • bias • random • representative	• Learners should study sampling methods, statistical questions, sampling size, population demographics, etc., and then analyze these factors to compare and identify strengths and/or weaknesses. (IN) • Learners may not understand the statistical meaning of bias. (ME) • Learners may not understand there are many types of samples, but not all samples are random (representative of the population). (ME)

Courses
Mathematics 7 Applied Math

SEVENTH GRADE

Data Analysis (D)
Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences, conclusions, and making predictions.

7.DPS.D.2 Analyze and draw inferences about a population using single and multiple random samples by using given measures of center and variability for the numerical data set.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.DPS.D.1 6.DPS.D.2 6.SPD.D.3 6.DPS.D.4 7.DPS.D.1		8.DPS.D.1 9-10.DPS.2*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Describe a population (when collecting data). • Describe a random sample. • Explain the use of a statistical question to collect single and multiple random samples from a population. • Explain and identify measures of center (mean, median) for a numerical data set. • Explain and identify measures of variability (range, mean absolute deviation) for a numerical data set.	• population • random sample • measures of center (mean, median) • measures of variability (range, mean absolute deviation)	• Drawing inferences could include making predictions and determining the validity of predictions made. (IN) • Learners may make assumptions before thorough analysis, e.g., one data set is perceived as stronger overall simply because of an extremely high outlier. (ME) • Learners might confuse mean and median. (ME) • Learners might not consider sample bias or size when making inferences. (ME)

Courses

Mathematics 7
Applied Math

SEVENTH GRADE

Probability (P)
Learners will understand and apply basic concepts of probability.

7.DPS.P.1 Develop a probability model to find probabilities of theoretical events and contrast probabilities from an experimental model.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.AR.RP.1 6.AR.RP.4	7.AR.RP.4 7.DPS.P.2	8.DPS.D.4 9-10.DPS.5* 9-10.DPS.6* 9-10.DPS.7* 9-10.DPS.8* 9-10.DPS.9*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Describe the concept of a ratio relationship between two quantities. • Write numbers as a fraction, decimal, and percentage.	• probability model • theoretical events • experimental model • favorable outcomes • possible outcomes <u>Representations</u> • organized lists • tree diagrams • area models • simulations <u>Notation</u> • probability notation (P)	• This is the first learners have been exposed to the concept of probability. The basic concepts of probability and likelihood will need to be developed before fully addressing this standard. (IN) • Learners might mix up theoretical vs. experimental. (ME) • Learners might be unable to find all possible outcomes. (ME) • Learners might assume all outcomes are equally likely. (ME) • Learners may understand how to make the representations but don't always understand how to interpret what it means. (ME)

Courses
Mathematics 7 Applied Math Consumer Math

SEVENTH GRADE

Probability (P)
Learners will understand and apply basic concepts of probability.

7.DPS.P.2 Develop a probability model to find theoretical probabilities of independent compound events

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.AR.RP.1 6.AR.RP.4	7.AR.RP.4 7.DPS.P.1	9-10.DPS.5* 9-10.DPS.6* 9-10.DPS.7* 9-10.DPS.8* 9-10.DPS.9* 9-10.DPS.10*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Describe the concept of a ratio relationship between two quantities. • Write numbers as a fraction, decimal, and percentage.	• probability model • theoretical probability • compound event • independent events • Fundamental Counting Principle <u>Representations</u> • organized lists • tree diagrams • area models • simulations <u>Notation</u> • probability notation (P)	• Learners are not expected to use formulas for probability at this level. They should focus on the representations (organized lists, tree diagrams, area models, simulations, etc.). (IN) • Learners may be unable to find all possible outcomes. (ME) • Learners may assume all outcomes are equally likely. (ME) • Learners may understand how to make the representations but don't always understand how to interpret what it means. (ME) • Learners might misuse the fundamental counting principle and assume they should be adding possible outcomes instead of multiplying. (ME)

Courses

Mathematics 7
Applied Math
Consumer Math

EIGHTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

Number Systems (NS)

Learners will expand their knowledge of the number system to create connections and solve problems within and across concepts.

8.NO.NS.1 Compare and classify real numbers within the real number system.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.NO.NS.1 7.NO.NS.2	8.NO.NS.2 8.NO.NS.3 8.NO.O.1 8.NO.O.2 8.AR.EE.2 8.GM.AV.1	9-10.NO.1 11-12.NO.6

Prerequisite Skills	Key Vocabulary, Notations, and Representations	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Classify natural, whole, and rational numbers. • Convert between fractions and decimals. • Evaluate roots. • Locate a rational number on a number line.	• real numbers • real number system • rational numbers • irrational numbers • integers • whole numbers • natural numbers (counting numbers) • terminating/non-terminating • roots • perfect roots/non-perfect roots • fractions • decimal expansion • radicand • radical (√) • index	• Learners should be familiar with rational numbers and their subsets. This is the first time they will learn about irrational numbers and the real number system. (IN) • Learners might assume all roots are irrational and always in the same category. (ME) • Learners might assume all negative numbers are integers. (ME) • Learners might assume numbers can only be classified as one category. (ME) • Learners may compare numbers without putting them into the same form, e.g., trying to compare a radical, fraction, and decimal without making them all decimals. (ME)

Courses

Mathematics 8
Prealgebra
Algebra I Semester I
General Math

EIGHTH GRADE

Number Systems (NS)
Learners will expand their knowledge of the number system to create connections and solve problems within and across concepts.

8.NO.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them on a number line diagram, and estimate the value of irrational expressions involving one operation.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.NO.NS.2	8.NO.NS.1 8.NO.O.1 8.AR.EE.2 8.GM.AV.1 8.GM.GF.6	9-10.NO.1 9-10.NO.2 9-10.NO.5* 9-10.AR.10
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Explain the difference between rational and irrational numbers. • Plot rational numbers on a number line. • Compare rational numbers. • Identify irrational numbers. • Add, subtract, multiply, and divide rational numbers. • Estimate the value of an expression.	• rational numbers • irrational numbers • number line • rational approximation • expression • radicand • radical (√) • index • pi (π)	• Teachers need to teach both vertical and horizontal number lines in a variety of ways (open number line, building a number line on the floor, clothesline, comparing it to a thermometer, etc.). (IN) • Learners should be able to both approximate an irrational number and use that approximation to evaluate an expression involving that irrational number and operation. (IN) • Learners may evaluate the value of the radicand instead of the radical. (ME) • Learners may not follow the order of operations (not knowing where square roots fall in the order - see Appendix A Table 4 in the standards document). (ME) • Learners might confuse the location of the negative symbol and how it affects the value of the expression, e.g., -√2 vs. √-2. (ME)

Courses

Mathematics 8
Prealgebra
General Math

EIGHTH GRADE

Number Systems (NS)
Learners will expand their knowledge of the number system to create connections and solve problems within and across concepts.

8.NO.NS.3 Use scientific notation to represent very large or very small quantities.
Interpret scientific notation generated by technology.
Compare and order numbers in both scientific and standard notation.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.NO.NBT.1 5.NO.NBT.7	8.NO.NS.1 8.AR.EE.1	9-10.NO.1 9-10.NO.4* 9-10.AR.1
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Use and apply exponent properties. • Explain patterns in the number of zeros of the product when multiplying the number by powers of 10. • Explain patterns in the placement of the decimal point when a number is multiplied or divided by a power of 10.	• scientific notation • standard notation • exponents • base <u>Notation</u> • Scientific notation looks like "m × 10^n," where m can be known as the coefficient.	• Learners might assume that the exponent indicates the number of zeros instead of referring to place value. (ME) • Learners might compare only the coefficients without considering the exponent when comparing the expressions. (ME) • Learners may not understand that a negative exponent produces a number less than one. (ME) • Learners may not understand that a positive exponent produces a number greater than 10. (ME) • Learners might misinterpret the technology (not all calculators show scientific notation the same way). (ME)

Courses

Mathematics 8
Prealgebra
General Math

EIGHTH GRADE

Operations (O)
Learners will expand their computational fluency to create connections and solve problems within and across concepts.

8.NO.O.1 Evaluate mentally the square roots of perfect squares up to 225 and cube roots of perfect cubes up to 1000.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.AR.EE.1 7.NO.O.1	8.NO.NS.1 8.NO.NS.2 8.AR.EE.2 8.GM.GF.6	9-10.NO.1 9-10.NO.2 9-10.AR.10
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify the symbol for numbers squared (^{2}) and numbers cubed (^{3}). • Know how to square a number. • Know how to find the area of a square and the volume of a cube.	• square roots • perfect squares • cube roots • perfect cubes • radical • radicand • index • base • exponent <u>Notation</u> • square root ($\sqrt{}$) and cube root ($\sqrt[3]{}$)	• This is the first learners are evaluating radicals, so connections should be made to the area of squares and volume of cubes. (IN) • Learners may double the number instead of raising it to the 2nd power. (ME) • Learners might multiply the base by 3 instead of raising it to the 3rd power. (ME)

Courses
Mathematics 8 Prealgebra

EIGHTH GRADE

Operations (O)
Learners will expand their computational fluency to create connections and solve problems within and across concepts.

8.NO.O.2 Add, subtract, multiply, and divide rational numbers using strategies or procedures.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.NO.O.1 7.NO.O.2 7.NO.O.3	8.NO.NS.1	9-10.NO.2 9-10.AR.6 9-10.AR.11
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Add and subtract, multiply, and divide multi-digit whole numbers, integers, fractions, and decimals. • Apply the properties of operations and the order of operations.	• rational numbers	• The focus of this standard is to develop the conceptual understanding of subtraction as adding the opposite. (IN) • This concept is where learners frequently show shortcomings in number sense. (IN) • This is the last standard on operations so fluency at this level is crucial for later math classes. Learners should be moving away from visual models and towards proficiency in procedures and strategies. (IN) • Learners may not understand how negative, subtraction, and adding the opposite are related. (ME) • Learners may not understand subtraction doesn't always lead to smaller numbers. (ME)

Courses

Mathematics 8
Prealgebra
Algebra I Semester I
Consumer Math
General Math

EIGHTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

Expressions and Equations (EE)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

8.AR.EE.1 Explain the relationship between repeated multiplication and the properties of integer exponents. Apply a single exponent property to generate equivalent numeric and algebraic expressions that include numerical coefficients.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.AR.EE.1	8.NO.NS.3 8.AR.EE.2 8.AR.EE.6	9-10.NO.1 9-10.NO.2 9-10.AR.1 9-10.AR.11

Prerequisite Skills	Key Vocabulary, Notations, and Representations	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify exponents and explain how they are used. • Use exponential notation. • Identify numerical coefficients and explain how they are used. • Understand the coefficient as a constant in the variable term. • Identify and write numeric expressions. • Identify and write algebraic expressions. • Differentiate between numeric and algebraic expressions. • Write the reciprocal of a number. • Evaluate a positive exponent with a numerical base. • Write the expanded form of an exponential expression.	• integers • exponents • equivalent expressions • coefficients • base • power • reciprocal	• Learners should already be familiar with positive exponents. This is their first exposure to zero and negative exponents so time should be spent developing the concept of zero and negative exponents. (IN) • Learners have never worked with exponent properties before so they should be given the opportunity to conceptually develop each property. Learners should be expected to write the expression in expanded form and derive the formal rules from the patterns they see. (IN) • Learners might assume negative exponents make the expression negative. (ME) • Learners might multiply the base by the exponent. (ME) • Learners may confuse the operation as the exponent property. (ME) • Learners might assume anything to the 0 power is 0. (ME) • Learners might misunderstand the meaning of the parentheses in the base, e.g., 2x^2 vs. (2x)^2. (ME) • Learners may not understand what is being raised to the exponent. (ME) • Learners may not realize that every expression has an exponent. (ME)

Courses

Mathematics 8
Prealgebra
Algebra I Semester I

EIGHTH GRADE

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

8.AR.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a non-negative rational number.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.AR.EE.2	8.NO.NS.1 8.NO.NS.2 8.NO.O.1 8.AR.EE.1 8.AR.EE.5 8.AR.EE.6 8.GM.AV.1 8.GM.GF.5 8.GM.GF.6	9-10.NO.1 9-10.NO.2 9-10.AR.3* 9-10.AR.10
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify perfect squares to 225. • Identify perfect cubes to 1000. • Identify the symbol for square root ($\sqrt{}$) and cube root ($\sqrt[3]{}$). • Understand that one cannot take the square root of a negative number because it will result in a non-real solution.	• square root • cube root • equation • base • radical • exponent • index • perfect square • perfect cube • radicand • inverse operations <u>Notation</u> • ±	• x^2 equations can result in 1 or 2 real solutions (sometimes learners forget the second solution). x^3 equations will result in 1 real solution. (IN) • The standard is the beginning of square roots and cube roots. This concept will be covered more in-depth in high school. (IN) • Make the connection between geometric squares and cubes. (IN) • For x^2 equations, p should not be negative because it will lead to imaginary numbers. (IN) • In certain contexts, the second solution to x^2 equations is not viable. (IN) • Instead of taking the square root, learners may divide by two. (ME) • Learners might mistakenly assume it is impossible to take the cube root of a negative number. (ME)

Courses
Mathematics 8 Prealgebra Consumer Math

EIGHTH GRADE

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

8.AR.EE.3 Explain the characteristics of a linear relationship, including identifying the slope and y-intercept in tables, graphs, equations, and descriptions.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.AR.RP.1 7.AR.RP.2 7.AR.RP.3 7.AR.RP.4	8.AR.EE.4 8.AR.F.2 8.AR.F.3 8.AR.F.4 8.AR.F.5 8.DPS.D.1	8.DPS.D.2 9-10.AR.F.7 9-10.AR.F.11* 9-10.GM.27 9-10.GM.28
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify the x and y axes and ordered pairs on the coordinate plane. • Identify proportional relationships. • Represent proportional relationships using graphs, tables, and equations. • Write equivalent fractions.	• linear relationship • slope (also known as constant rate of change, m) • y-intercept (also known as initial value, b) <u>Representations</u> • diagram/picture representation • verbal description • graph • table of values • equation in the form y = mx + b	• A proportional relationship (also known as direct variation) is a subset of linear relationships. Proportional relationships are taught in seventh grade. Connections should be made to those relationships already learned. (IN) • Teachers should spend a lot of time developing the concept of slope (patterns in linear relationships). Connections should be made to prior learning in sixth and seventh grade with ratios, unit rates, constants of proportionality, etc. (IN) • Learners might invert slope (horizontal over vertical). (ME) • Learners might use the x-intercept instead of the y-intercept. (ME) • Learners may divide a y-coordinate by an x-coordinate instead of calculating the change in x- and y-values (students assume the relationships are proportional). (ME) • Learners might choose the first row of values on a table instead of finding the y-intercept (when x = 0). (ME) • Learners might assume all relationships are linear. (ME) • Learners may overlook the distinction between negative vs. positive slope. (ME)

Courses

Mathematics 8
Prealgebra
Algebra I Semester I

EIGHTH GRADE

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

8.AR.EE.4 Represent linear relationships using tables, graphs, equations, and descriptions when given a relationship in one of these forms.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.AR.RP.1 7.AR.RP.2 7.AR.RP.3	8.AR.EE.3 8.AR.F.2 8.AR.F.3 8.AR.F.4 8.AR.F.5 8.DPS.D.2	8.AR.EE.8 9-10.AR.4 9-10.AR.7* 9-10.AR.F.3* 9-10.AR.F.5* 9-10.GM.27
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify the x and y axes and ordered pairs on the coordinate plane. • Represent proportional relationships using tables, graphs, equations, and descriptions.	• linear relationships • slope (also known as constant rate of change, m) • y-intercept (also known as initial value, b) <u>Representations</u> • diagram/picture representation • verbal description • graph • table of values • equation in the form y = mx + b	• A proportional relationship (also known as direct variation) is a subset of linear relationships. Proportional relationships are taught in seventh grade. Connections should be made to those relationships already learned. (IN) • Teachers should spend a lot of time developing the concept of slope (patterns in linear relationships). Connections should be made to prior learning in sixth and seventh grade with ratios, unit rates, constant of proportionality, etc. (IN) • Only use linear equations in slope-intercept form. (IN) • Learners may invert the slope (horizontal over vertical). (ME) • Learners may use x-intercept instead of y-intercept. (ME) • Learners might divide a y-coordinate by an x-coordinate instead of calculating the change in x- and y-values (students assume the relationships are proportional). (ME) • Learners might choose the first row of values on a table instead of finding the y-intercept (when x = 0). (ME) • Learners might assume all relationships are linear. (ME) • Learners may overlook the difference between the negative and positive slope. (ME) • Learners might mix up slope and y-intercept when writing the equation. (ME) • Learners might assume all lines must go through the origin. (ME) • Learners may not understand

		when to use arrows on the ends of the graphed lines. (ME)

Courses
Mathematics 8 Prealgebra Algebra I Semester I

EIGHTH GRADE

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

8.AR.EE.5 Solve linear equations with rational number coefficients and variables on both sides, including equations that require using the distributive property and/or combining and collecting like terms.
Interpret the number of solutions.
Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.AR.EE.1 7.AR.EE.2	8.AR.EE.2 8.AR.EE.6 8.AR.EE.7	9-10.AR.2 9-10.AR.5 9-10.AR.6 9-10.AR.7* 9-10.AR.9 9-10.AR.10
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Solve two-step equations using the properties of equality and inverse operations. • Write equivalent expressions by combining like terms, applying the order of operations, and using the distributive property. • Create zero pairs.	• coefficients • variables • distributive property • combine like terms • collect like terms • number of solutions(infinitely many solutions, one solution, no solution) • constants • terms • like terms • inverse operations • properties of equality (See Appendix A Table 2) • isolate • zero pairs • equivalent equations • multiplicative inverse <u>Representations</u> (could include) • algebra tiles or other manipulatives • hanger diagrams • clothesline/number line	• Continue the work of seventh grade utilizing visual models and/or manipulatives to solve more complete equations and build towards procedural fluency. (IN) • Learners may not apply the distributive property correctly. (ME) • Learners may combine terms that are not like terms. (ME) • Learners may not perform the same operation on both sides of the equal sign. (ME) • Learners may not understand the difference between combining like terms and collecting like terms. (ME) • Learners might try to complete the problem in their minds and make mistakes rather than focusing on the process. (ME) • Learners may not understand that the answer may not be a whole number. (ME) • Learners may not understand that 0 can be a solution. (ME) • Learners may not understand that a variable can be on either side of an equal sign. (ME) • Learners might assume a solution of 0 means no solution. (ME) • Learners may not understand the concept of what a solution is. (ME) • Learners may use the incorrect operation to try to undo an operation. (ME) • Some learners may eliminate all terms from one side of an equal sign because they don't understand the concept of

		isolating the variable. (ME) • Some learners might not understand the equal sign indicates that two expressions are equal in value. (ME)

Courses
Mathematics 8 Prealgebra Algebra I Semester I

EIGHTH GRADE

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

8.AR.EE.6 Read, write, and evaluate numerical and algebraic expressions, including expressions involving absolute value.
Solve and graph equations of the form |x|=r where r is a nonnegative rational number.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.NO.NS.1 7.AR.EE.1 7.AR.EE.2	8.AR.EE.2 8.AR.EE.5	9-10.AR.1 9-10.AR.3* 9-10.AR.9
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify and apply the properties of operations. • Identify the function of parentheses, brackets, braces, and fraction lines within the order of operations. • Describe the absolute value of a number as its distance from zero on a number line. • Plot points on a number line.	• numerical expressions • algebraic expressions • absolute value • properties of operations (see Appendix A Table 1) • variables • integers • nonnegative fractions • equivalent expressions • factor an expression (undo the Distributive Property) • constant • coefficient • terms • combine like terms • Distributive Property <u>Representations</u> • horizontal number line • vertical number line • absolute value bars | |	• Continue the work from seventh grade utilizing visual models and/or manipulatives to work with numerical and algebraic expressions. (IN) • Teachers should utilize both horizontal and vertical lines to help learners understand the number or solutions and the symmetry that exists between them. (IN) • Learners might assume absolute value bars do not mean opposite. (ME) • Learners may not understand that absolute value bars are a grouping symbol. (ME) • Learners may not understand absolute value as an operation (You don't need the bars after you've taken the absolute value.). (ME) • Learners may incorrectly number the number lines (negative numbers get listed backward). (ME) • Learners may not follow the order of operations. (ME)

Courses
Mathematics 8
Prealgebra
Algebra I Semester I

EIGHTH GRADE

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

8.AR.EE.7 Solve and graph inequalities in one variable with rational number coefficients and variables on both sides, including inequalities that require using the distributive property and/or combining like terms.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.AR.EE.3	8.AR.EE.5 8.AR.EE.8	9-10.AR.3* 9-10.AR.6 9-10.AR.8 9-10.AR.9
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the meaning of the symbols < , >, ≥, and ≤. • Graph inequalities on a number line. • Understand when the direction of inequality changes and when it does not. • Determine when a given number is included in the solution set. • Solve a multi-step equation. • Determine the number of solutions when solving an equation.	• inequality • coefficients • variables • distributive property • combine like terms • collect like terms (see Glossary) • isolate • properties of inequality (see Appendix A Table 3) • integers • direction of inequality • Distributive Property • zero pair • solution set <u>Notations</u> • <, >, ≥, ≤ • open vs. closed circle • shading a number line	• Continue the work from seventh grade utilizing visual models and/or manipulatives to solve more complex inequalities to solve more complex inequalities and build toward procedural fluency. (IN) • Learners might not apply the distributive property correctly. (ME) • Learners might combine terms that are not like terms. (ME) • Learners may not perform the same operation on both sides of the inequality symbol. (ME) • Learners may not understand the difference between combining like terms and collecting like terms. (ME) • Learners might try to complete the problem in their minds and make mistakes rather than focusing on the process. (ME) • Learners may not understand that the answer may not be a whole number. (ME) • Learners may not understand that 0 can be a solution. (ME) • Learners may struggle to understand when to reverse the direction of the inequality symbol. (ME) • Learners might assume the inequality symbol indicates the direction of shading. (ME) • Learners may not understand a variable can be on either side of the inequality symbol. (ME)

Courses
Mathematics 8 Prealgebra Algebra I Semester I

EIGHTH GRADE

Expressions and Equations (EE)
Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adapting approaches in novel situations.

8.AR.EE.8 Graph linear inequalities in two variables on a coordinate plane. Interpret the possible solutions in the context of authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.AR.EE.3 8.AR.EE.4	8.AR.EE.7	9-10.AR.3* 9-10.AR.8 9-10.AR.9
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Common Misconceptions**
• Understand the meaning of the symbols < , >, ≥, and ≤. • Graph linear equations in two variables. • Understand when an ordered pair is a solution to an inequality.	• linear inequalities • slope (also known as constant rate of change, m) • y-intercept (also known as initial value, b) • half-plane • boundary line • solution set • viable solution <u>Representations</u> • diagram/picture representation • verbal description • graph inequalities in slope-intercept form <u>Notations</u> • <, >, ≥, ≤ • Solid vs. dashed line • Shading a half plane	• Only use inequalities in slope-intercept form. (IN) • Learners may not understand when to use a dashed boundary line vs. a solid boundary line. (ME) • Learners might not identify the entire solution set. (ME) • Learners might incorrectly choose which half-plane contains the solution. (ME) • Learners may test a point that lies on the boundary line. (ME) • Learners may assume points that lie on a dashed line are included in the solution set. (ME)

Courses
Mathematics 8 Prealgebra Algebra I Semester I

EIGHTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

8.AR.F.1 Defend whether a relation is a function from various representations using appropriate function language.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
5.AR.OA.5 7.AR.RP.2	8.AR.F.2 8.AR.F.3 8.AR.F.4 8.AR.F.5	9-10.AR.F.1 9-10.AR.F.2* 9-10.GM.2
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify the x values (input, independent variable) and y values (output, dependent variables) within ordered pairs, in tables and on graphs. • Generate and extend numerical patterns. • Identify the rule a numerical pattern follows.	• function • relation • input • output • independent variable • dependent variable <u>Representations</u> • graph • table of values • mapping diagram • ordered pairs	• This standard introduces functions as a concept. Functional notation is not expected at this level. (IN) • Teachers need to spend time helping learners conceptually understand what a function is. (IN) • When learners cannot recognize the pattern, they might assume it is not a function. (ME) • Learners may look for a linear relationship to determine if it is a function. (ME) • Learners might rely on the vertical line test as a trick without understanding the concept of a function. (ME) • Learners might mix up inputs and outputs. (ME) • Learners might assume any repeated numbers indicate that the relation is not a function without understanding the concept of a function. (ME)

Courses
Mathematics 8 Prealgebra Algebra I Semester I

EIGHTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

8.AR.F.2 Compare and contrast properties of two linear functions, each represented in a different way (algebraically, graphically, numerically in tables, and/or by descriptions).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.AR.RP.2 7.AR.RP.3	8.AR.EE.3 8.AR.EE.4 8.AR.F.1 8.AR.F.3 8.AR.F.4 8.AR.F.5	9-10.AR.F.3* 9-10.AR.F.4* 9-10.AR.F.6* 9-10.AR.F.7* 9-10.AR.F.10* 9-10.AR.F.11*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Describe linear functions. • Determine the rate of change and initial value for a linear function from a table of values, graph, equation, and verbal description. • Represent a linear function in multiple ways (algebraically, graphically, numerically in tables, and/or by descriptions).	• linear functions • slope (also known as constant rate of change, m) • y-intercept (also known as initial value, b) • function • input • output • independent variable • dependent variable <u>Representations</u> • graph • table of values • algebraic • description	• Learners tend to make more errors in finding slope and y-intercepts rather than have conceptual misunderstandings about this standard. (IN) • Learners might confuse the slope and y-intercept. (ME) • Learners might compare properties incorrectly (e.g., compare the y-intercept of one function with the slope of another). (ME) • Learners might mix up which function is which when switching between different representations. (ME)

Courses
Mathematics 8 Prealgebra Algebra I Semester I Consumer Math

EIGHTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

8.AR.F.3 Compare and contrast linear and non-linear functions represented in different ways (algebraically, graphically, numerically in tables, and/or by descriptions).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.AR.RP.2 7.AR.RP.3	8.AR.EE.3 8.AR.EE.4 8.AR.F.1 8.AR.F.2 8.AR.F.4 8.AR.F.5	9-10.AR.F.4* 9-10.AR.F.6* 9-10.AR.F.7* 9-10.AR.F.8*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Represent a function in multiple ways (algebraically, graphically, numerically in tables, and/or by descriptions). • Describe functions that are linear or not linear.	• linear function • nonlinear function • rate of change • slope (also known as constant rate of change, m) • y-intercept (also known as initial value, b) • function • input • output • independent variable • dependent variable <u>Representations</u> • graph • table of values • algebraic • description	• Learners are less familiar with equations of non-linear functions. (IN) • Learners might assume that any pattern in the values means it is a linear pattern. (ME) • Learners tend to make more errors in finding rates of change and initial values rather than having conceptual misunderstandings for this standard. (ME) • Learners may not understand how to distinguish between a linear and non-linear equation. (ME)

Courses
Mathematics 8 Prealgebra Algebra I Semester I Consumer Math

EIGHTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

8.AR.F.4 Model a linear function between two quantities by creating a table, graph, and equation. Interpret the rate of change and initial value of a linear function in terms of the situation it models.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.AR.RP.1 7.AR.RP.2 7.AR.RP.3 7.AR.EE.2 7.AR.EE.3	8.AR.EE.3 8.AR.EE.4 8.AR.F.1 8.AR.F.2 8.AR.F.3 8.AR.F.5 8.DPS.D.2 8.DPS.D.3	9-10.AR.3* 9-10.AR.4* 9-10.AR.F.1 9-10.AR.F.2* 9-10.AR.F.3* 9-10.AR.F.4* 9-10.AR.F.6* 9-10.AR.F.7* 9-10.AR.F.8* 9-10.AR.F.10* 9-10.AR.F.11* 9-10.DPS.3*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Represent linear relationships using tables, graphs, equations, and descriptions. • Determine the rate of change and initial value for a linear function from a table of values, graph, equation, and verbal description.	• linear functions • slope (also known as constant rate of change, m) • y-intercept (also known as initial value, b) • function • input • output • independent variable • dependent variable <u>Representations</u> • graph • table of values • equation	• A proportional relationship (also known as direct variation) is a subset of linear relationships. (IN) • Learners may invert slope (horizontal over vertical). (ME) • Learners may use the x-intercept instead of the y-intercept. (ME) • Learners may divide a y-coordinate by an x-coordinate instead of calculating the change in x- and y-values (learners assume the relationships are proportional). (ME) • Learners might choose the first row of values on a table instead of finding the y-intercept (when x = 0). (ME) • Learners might assume all relationships are linear. (ME) • Learners may overlook the difference between negative and positive slope. (ME) • Learners might mix up slope and y-intercept when writing the equation. (ME) • Learners might assume all lines must go through the origin. (ME) • Learners may not understand when to use arrows on the ends of the graphed lines. (ME)

Courses

Mathematics 8	Applied Math
Prealgebra	Consumer Math
Algebra I Semester I	

EIGHTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

8.AR.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph including where the function is constant, increasing, or decreasing; linear or nonlinear; and discrete or continuous. Create a graph that exhibits the qualitative features of a function described.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.AR.RP.2 7.AR.RP.3	8.AR.EE.3 8.AR.EE.4 8.AR.F.1 8.AR.F.2 8.AR.F.3 8.AR.F.4	9-10.NO.3 9-10.AR.4 9-10.AR.F.1 9-10.AR.F.2* 9-10.AR.F.3* 9-10.AR.F.4* 9-10.AR.F.5* 9-10.AR.F.6* 9-10.AR.F.7* 9-10.AR.F.8* 9-10.AR.F.10* 9-10.AR.F.11* 9-10.DPS.3*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Create appropriate scales and axis labels based on a given situation. • Determine the slope from a graph.	• function • constant • increasing • decreasing • linear • nonlinear • discrete • continuous • input • output • independent variable • dependent variable	• Learners may confuse constant functions with a constant rate of change. (ME) • Learners might misinterpret the graph. (ME) • Learners may not understand what the variables represent. (ME)

Courses
Mathematics 8 Prealgebra Algebra I Semester I Consumer Math

EIGHTH GRADE

Geometry and Measurement (GM)

Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

Area and Volume (AV)

Learners will use visualization and spatial reasoning to solve problems involving the area, surface area, and volume of geometric figures.

8.GM.AV.1 Apply given formulas to solve problems involving the volume of cones, cylinders, and spheres, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.GM.AV.1 7.GM.AV.3	8.NO.NS.1 8.NO.NS.2 8.AR.EE.2 8.GM.GF.6	9-10.NO.5 9-10.AR.2 9-10.GM.31 9-10.GM.33 9-10.GM.34 9-10.GM.36*

Prerequisite Skills	Key Vocabulary, Notations, and Representations	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify cones, cylinders, and spheres. • Identify the radius and diameter of a circle. • Calculate the area of a circle. • Identify the base and height of a cone and cylinder. • Identify the radius of a cone, cylinder, and sphere. • Apply the order of operations. • Evaluate square and cube roots. • Identify the correct label for volume (cubic units). • Understand what pi is and recognize the symbol for pi (π). • Round numbers to a specified place value.	• volume • cone • cylinder • sphere • radius • diameter • height • base • area • pi (π) <u>Notation</u> • cubic units	• Learners may not distinguish between the height of a base and the height of a figure. (ME) • Learners may not understand the height can move (height isn't always vertical). (ME) • Learners might use the incorrect units (square units or units rather than cubic units). (ME) • Learners might perceive volume as length x width x height rather than the concept of the area of the base x the height. The latter will help their understanding of volume in future learning. (ME) • Learners may not understand that the base doesn't always appear at the bottom of the figure. (ME) • Learners might decompose the figure incorrectly. (ME) • Learners may confuse circumference and area. (ME) • Learners might mix up radius and diameter. (ME) • Learners may use the formula incorrectly. (ME)

Courses

Mathematics 8
Prealgebra
Applied Math

EIGHTH GRADE

Geometric Figures (GF)
Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics of figures, perform transformations, and construct logical arguments.

8.GM.GF.1 Perform single transformations to a figure on the coordinate plane and determine whether the figures are congruent or similar.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.GM.GF.2 7.AR.RP.4	8.GM.GF.2 8.GM.GF.4	8.GM.GF.3 9-10.GM.1 9-10.GM.2 9-10.GM.3 9-10.GM.4 9-10.GM.5 9-10.GM.6 9-10.GM.7 9-10.GM.8 9-10.GM.15 9-10.GM.16 9-10.GM.17 9-10.GM.34

Prerequisite Skills	Key Vocabulary, Notations, and Representations	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Describe the concepts of congruent and similar shapes. • Draw polygons in the coordinate plane given coordinates for the vertices. • Describe the orientation of a figure on a coordinate plane. • Use proportional relationships to solve multi-step problems involving geometric figures.	• transformation • dilation • rotation • reflection • translation • congruent (≅) • scale factor • center of dilation • line of reflection • center of rotation • rigid • pre-image • image • degree of rotation/angle of rotation • similar (~) • coordinate plane • origin • orientation • corresponding parts (angles and sides) <u>Notation</u> • using prime notation for the image (')	• Learners do not have prior knowledge of the concepts of congruence and similarity. (IN) • This is the first time learners work with transforming shapes. (IN) • Learners might mix up clockwise and counterclockwise. (ME) • Learners may visualize rotations or reflections incorrectly. (ME) • Learners might mix up congruent vs. similar. (ME)

Courses

Mathematics 8
Prealgebra
Applied Math

EIGHTH GRADE

Geometric Figures (GF)
Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics of figures, perform transformations, and construct logical arguments.

8.GM.GF.2 Describe the characteristics of transformations on the coordinate plane using transformation language.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
4.GM.G.3 6.GM.GF.2 7.AR.RP.4	8.GM.GF.1 8.GM.GF.4	8.GM.GF.3 9-10.AR.F.9* 9-10.GM.1 9-10.GM.2 9-10.GM.3 9-10.GM.4 9-10.GM.5 9-10.GM.6 9-10.GM.7 9-10.GM.14 9-10.GM.15 9-10.GM.34

Prerequisite Skills	Key Vocabulary, Notations, and Representations	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Describe the concepts of congruent and similar shapes. • Draw polygons in the coordinate plane given coordinates for the vertices. • Describe the orientation of a figure on a coordinate plane. • Use proportional relationships to solve multi-step problems involving geometric figures.	• transformation • dilation • rotation • reflection • translation • congruent (≅) • scale factor • center of dilation • line of reflection • center of rotation • rigid • pre-image • image • degree of rotation/angle of rotation • similar (~) • coordinate plane • origin • orientation • corresponding parts (angles and sides) <u>Notation</u> • using prime notation for the image (')	• Characteristics could include congruent or similar orientation of the figure, the orientation of vertices, side lengths, angle measures, and distance from the origin. Learners may recognize patterns in coordinates before and after transformations even though the coordinate notation is not expected at this level. (IN) • Learners might mix up clockwise and counterclockwise. (ME) • Learners may visualize rotations or reflections incorrectly. (ME) • Learners might mix up congruent vs. similar. (ME)

Courses

Mathematics 8
Prealgebra

EIGHTH GRADE

Geometric Figures (GF)
Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics of figures, perform transformations, and construct logical arguments.

8.GM.GF.3 Name the type of transformation needed to map a pre-image to its image.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.GM.GF.1 8.GM.GF.1 8.GM.GF.2	8.GM.GF.4	9-10.AR.F.9* 9-10.GM.1 9-10.GM.2 9-10.GM.3 9-10.GM.4 9-10.GM.5 9-10.GM.6 9-10.GM.14 9-10.GM.15 9-10.GM.34

Prerequisite Skills	Key Vocabulary, Notations, and Representations	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Describe the concepts of congruent and similar shapes. • Draw polygons in the coordinate plane given coordinates for the vertices. • Describe the orientation of a figure on a coordinate plane. • Use proportional relationships to solve multi-step problems involving geometric figures.	• image • pre-image • transformation • dilation • rotation • reflection • translation • congruent (≅) • scale factor • center of dilation • line of reflection • center of rotation • rigid • degree of rotation/angle of rotation • similar (~) • coordinate plane • origin • orientation • corresponding parts (angles and sides) <u>Notation</u> • using prime notation for the image (')	• Learners need to specify the type of transformation and details about that transformation, e.g., Translate how many units and in what direction. Reflect over which axis. Rotate in which direction and how many degrees. (IN) • Learners might mix up clockwise and counterclockwise. (ME) • Learners may visualize rotations or reflections incorrectly. (ME) • Learners might mix up congruent vs. similar. (ME)

Courses

Mathematics 8
Prealgebra

EIGHTH GRADE

Geometric Figures (GF)
Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics of figures, perform transformations, and construct logical arguments.

8.GM.GF.4 Describe the following angle-pair relationships: interior and exterior angles of triangles and angles formed when a transversal cuts parallel lines or intersecting lines.
Solve for an unknown angle in a figure by applying facts about these angles.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.GM.GF.2	8.GM.GF.1 8.GM.GF.2 8.GM.GF.3	9-10.GM.1 9-10.GM.9 9-10.GM.10 9-10.GM.11 9-10.GM.16 9-10.GM.17

Prerequisite Skills	Key Vocabulary, Notations, and Representations	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify intersecting lines. • Identify parallel lines. • Identify angle-pair relationships vertical, complementary, supplementary, adjacent). • Determine the sum of angles within a triangle.	• interior angles • exterior angles • alternate interior angles • alternate exterior angles • corresponding angles • vertical angles • same-side interior angles • same-side exterior angles • complementary • supplementary • adjacent angles • transversal • parallel lines • intersecting lines <u>Notations</u> • degree symbol for angle measures • angle symbol (∠) • parallel line symbol (||) • box symbol to represent 90°	• Learners could use the idea of transformations to formalize the relationships found in angle pairs. (IN) • The focus should not be on memorizing the vocabulary but on reasoning through the diagram to find the angle pair relationships that exist. (IN) • Learners might make assumptions based on a diagram (e.g., It looks like a right angle but doesn't have the notation on the diagram). (ME)

Courses
Mathematics 8
Prealgebra

EIGHTH GRADE

Geometric Figures (GF)
Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics of figures, perform transformations, and construct logical arguments.

8.GM.GF.5 Describe the relationship between the leg lengths and the hypotenuse length of a right triangle. Determine whether a triangle is a right triangle using this relationship.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.GM.GF.1	8.AR.EE.2 8.GM.GF.6	9-10.GM.9 9-10.GM.10 9-10.GM.20
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify right triangles. • Identify perfect squares to 225. • Apply order of operations, including exponents and square roots.	• leg • hypotenuse • right triangle • area • square root • Pythagorean Theorem • converse of the Pythagorean Theorem	• Learners should be given the opportunity to discover the relationship between the leg lengths and the hypotenuse lengths of a right triangle. (IN) • Learners may insert values into the incorrect part of the equation (mixing up leg and hypotenuse). (ME) • Learners might divide a value by 2 instead of taking the square root. (ME) • Learners may double the number instead of squaring it. (ME) • Learners might mislabel the answer. (ME)

Courses

Mathematics 8
Prealgebra
Applied Math

EIGHTH GRADE

Geometric Figures (GF)
Learners will use visualization, spatial reasoning, and geometric modeling to investigate the characteristics of figures, perform transformations, and construct logical arguments.

8.GM.GF.6 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in two and three dimensions on and off a coordinate plane, including authentic problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.GM.GF.2 7.GM.GF.1	8.NO.NS.2 8.NO.O.1 8.AR.EE.2 8.GM.GF.5 8.GM.AV.1	9-10.AR.3* 9-10.AR.10 9-10.GM.9 9-10.GM.10 9-10.GM.20 9-10.GM.28 9-10.GM.30*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify right triangles. • Interpret the meaning of the solution to problems. • Identify perfect squares to 225. • Apply order of operations, including exponents and square roots.	• Pythagorean Theorem • right triangle • area • square root • leg • hypotenuse • slant height of a cone • radius of a cone	• Learners may insert values into the wrong part of the equation (mixing up leg and hypotenuse). (ME) • Learners might divide a value by 2 instead of taking the square root. (ME) • Learners might double a number instead of squaring it. (ME) • Learners might mislabel the answer. (ME) • Learners may not understand the difference between height and slant height. (ME)

Courses
Mathematics 8 Prealgebra Applied Math

EIGHTH GRADE

Data, Probability, and Statistics (DPS)

Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.

Data Analysis (D)

Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences, conclusions, and making predictions.

8.DPS.D.1 Interpret scatter plots for bivariate measurement data to investigate patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.DPS.D.3 6.DPS.D.4 7.DPS.D.1 7.DPS.D.2	8.AR.EE.3 8.DPS.D.2 8.DPS.D.4	8.DPS.D.3 9-10.DPS.3* 9-10.DPS.4*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Interpret data on dot plots, line plots, and histograms. • Identify outliers, clusters, gaps, and skews. • Understand the concept of slope on a coordinate plane. • Compare and contrast linear and non-linear relationships. • Plot and name points on a coordinate plane.	• scatter plot • bivariate data • cluster • outlier • positive association • negative association • linear association • nonlinear association	• Teachers tend to teach this concept at the end of the linear relationship unit, but it can be taught before and throughout the learning involved in teaching linear relationships. (IN) • This is the first time learners see bivariate data as opposed to the univariate data they work with in sixth and seventh grade. (IN) • Learners might assume one or two dots make a cluster. (ME) • Learners might misunderstand the scale of a graph (thinking points are outliers when they're not). (ME) • Learners may not understand an association could look linear or non-linear (there are situations where it could be confusing). (ME) • Learners might mix up the axes. (ME) • Learners might assume that when both variables are decreasing, it is a negative association when it is actually a positive association. (ME) • Learners might assume there must be outliers and clusters in every scatter plot. (ME) • Learners might misidentify gaps. (ME) • Learners may not understand correlation does not mean causation. (ME)

Courses

Mathematics 8
Prealgebra

EIGHTH GRADE

Data Analysis (D)
Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences, conclusions, and making predictions.

8.DPS.D.2 Draw an informal trend line on a given scatter plot with a linear association and justify its fit by describing the closeness of the data points to the line.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.AR.EE.3	8.AR.EE.4 8.AR.F.4 8.DPS.D.1 8.DPS.D.4	8.DPS.D.3 9-10.AR.4* 9-10.AR.F.5* 9-10.AR.F.8* 9-10.DPS.3* 9-10.DPS.4*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Draw a straight line using appropriate tools. • Interpret data on scatter plots.	• trend line • scatter plot • linear association	• Learners need to understand their trend lines may vary (not all trend lines will be the same). (IN) • Learners might assume informal means not having to use a ruler. (ME) • Learners might assume the trend line always has to go through points. (ME) • Learners might assume the trend line always has to go through the origin. (ME) • Learners may try to draw a trend line in non-linear associations. (ME) • Learners may try to draw a trend line when there is no association in the data. (ME) • Learners might assume they can "connect the dots" with the data points instead of drawing a line. (ME)

Courses

Mathematics 8
Prealgebra

EIGHTH GRADE

Data Analysis (D)
Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences, conclusions, and making predictions.

8.DPS.D.3 Solve authentic problems in the context of bivariate measurement data by interpreting the slope and intercept(s) and making predictions using a linear model.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.DPS.D.1 8.DPS.D.2	8.AR.F.4 8.DPS.D.1 8.DPS.D.2 8.SPD.D.4	9-10.AR.2 9-10.AR.3* 9-10.AR.4* 9-10.AR.F.5* 9-10.AR.F.8* 9-10.AR.F.11* 9-10.DPS.3* 9-10.DPS.4*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/Errors (ME)**
• Identify the slope and y-intercept in graphs, equations, and descriptions. • Create a scatter plot using bivariate data. • Recognize the patterns of linear and nonlinear characteristics in scatter plots. • Draw a trend line on a given scatter plot. • Justify the fit of the trend line on a scatter plot.	• bivariate data • slope • y-intercept • slope intercept form • linear	• This standard is the application of all the linear relationships concepts developed in the seventh and eighth grade standards. (IN) • Learners might assume that they must calculate a slope using data points instead of points on the trend line. (ME) • Based on the situation, learners may assume they have to use data points to draw their trend line. (ME) • Learners might assume the trend line has to go through the origin. (ME) • Learners may not understand that the line extends beyond the graph. (ME) • Learners may not consider restrictions on the domain. (ME) • When making predictions, learners might mix up the x and y variables. (ME) • Learners may assume they are all supposed to get the same prediction/equation when their trend lines may be different. (ME)

Courses

Mathematics 8
Prealgebra

EIGHTH GRADE

Data Analysis (D)
Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences, conclusions, and making predictions.

8.DPS.D.4 Construct and interpret a two-way table summarizing bivariate categorical data collected from the same subjects.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.DPS.D.4 7.AR.RP.4 7.DPS.P.1	8.DPS.D.1 8.DPS.D.2 8.DPS.D.3	9-10.DPS.4* 9-10.DPS.10*
Prerequisite Skills	**Key Vocabulary, Notations, and Representations**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Find the percentage of a number. • Find the frequency of an event. • Construct a frequency table.	• two-way table • bivariate data • categorical data • relative frequency • joint frequency • marginal frequency	• Interpretations may include calculating joint and marginal frequencies. (IN) • Interpreting a two-way table can be connected to probability and/or percent concepts learned in seventh grade. (IN) • Learners may not understand marginal frequency as a total. (ME) • Learners might set up the two-way table incorrectly. (ME) • Learners might be unable to calculate relative frequencies. (ME) • Learners may be unable to interpret the relative frequencies in the context of a given problem. (ME)

Courses

Mathematics 8
Prealgebra
Applied Math

NINTH AND TENTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

9-10.NO.1 Explain how the definition of rational exponents follows from extending the properties of integer exponents; rewrite simple expressions involving radicals and rational exponents using the properties of exponents.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.NO.NS.1 8.NO.NS.2 8.NO.NS.3 8.NO.O.1 8.AR.EE.1 8.AR.EE.2	9-10.NO.2 9-10.AR.1 9-10.AR.6	11-12.NO.1 11-12.NO.2 11-12.NO.3 (+) 11-12.NO.11 (+) 11-12.NO.13 11-12.AR.2 11-12.AR.4* 11-12.AR.5 11-12.AR.12 11-12.AR.F.3b*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Compare and classify numbers within the real number system. • Understand and apply the properties of integer exponents. • Understand the concept of square and cube roots and interpret the symbols for each. • Understand the concept of radical expressions and how they are used. • Identify the radicand for square and cube roots written in radical form. • Understand the concept of rational exponents in expressions written in both fractional and radical forms. • Understand how to use properties of exponents to simplify rational expressions. • Perform operations on rational numbers using strategies or procedures.	• root index • radicand • radical • integer exponents • rational exponents	• With fractional exponents with a numerator other than 1, learners may confuse the index with the power. (ME) • Some learners may not understand that the exponent in a rational exponent denotes the root of the base. (ME) • Learners might multiply the base by the negative exponent. (ME) • Learners may confuse the Product Rule of Exponents with the Power Rule of Exponents. (ME)

Courses

Algebra I
Integrated Mathematics I
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester I
Algebra I Semester II

NINTH AND TENTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

9-10.NO.2 Perform basic operations on simple radical expressions to write a simplified equivalent expression.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.NO.NS.2 8.NO.O.1 8.NS.O.2 8.AR.EE.1 8.AR.EE.2	9-10.NO.1 9-10.AR.1 9-10.AR.F.6*	11-12.NO.1 11-12.NO.2 11-12.NO.3 11-12.NO.6 (+) 11-12.NO.11 11-12.AR.2 11-12.AR.4* 11-12.AR.5 11-12.AR.12 11-12.AR.F.3b*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Perform operations on rational numbers using strategies or procedures. • Understand and apply the properties of integer exponents. • Understand the concept of square and cube roots and interpret the symbols for each. • Understand the concept of radical expressions and how they are used. • Identify the radicand for square and cube roots written in radical form. • Understand the concept of rational exponents in expressions written in both fractional and radical forms. • Understand how to use properties of exponents to simplify rational expressions.	• radicals • rationalizing the denominator	• Learners may not realize that radical signs are grouping symbols which is important when it comes to the order of operations. (ME) • Learners might mistakenly distribute the exponent to the coefficient even when the coefficient is not included in parentheses raised to a power. (ME) • Learners may confuse the Product Rule of Exponents with the Power Rule of Exponents. (ME)

Courses

Algebra I
Integrated Mathematics I
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester II

NINTH AND TENTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

9-10.NO.3 Choose and interpret the scale and the units in graphs and data displays.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.AR.RP.5 7.AR.RP.1 7.AR.RP.2 8.AR.F.5 8.DPS.D.3	9-10.NO.4 9-10.NO.5 9-10.AR.4 9-10.GM.35* 9-10.GM.36* 9-10.DPS.1	11-12.NO.4* 11-12.NO.5* 11-12.AR.8* 11-12.AR.9* 11-12.AR.F.8* 11-12.AR.F.9* 11-12.AR.F.10* 11-12.AR.F.11* 11-12.DPS.2* 11-12.DPS.3* 11-12.DPS.4*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Describe and calculate a unit rate. • Convert measurement units within and between measurement systems using ratios. • Identify and interpret the units used in scaled graphs and data displays. • Use scientific notation to represent very large or very small quantities.	• unit analysis • scale	• Learners may neglect to use a consistent scale on the axes. (ME) • Learners may not consider the range of the data when selecting the scale leading to inaccurate representations of data distribution. (ME) • When interpreting scale, learners might misinterpret proportional relationships. (ME)

Courses

Algebra I
Integrated Mathematics I
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester I
Algebra I Semester II
Occupationally Applied Math
Applied Math

NINTH AND TENTH GRADE

Number and Operations (NO) *Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.*		
9-10.NO.4* Define appropriate quantities and units for the purpose of descriptive modeling.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.AR.RP.1 8.NO.NS.3	9-10.NO.3 9-10.NO.5 9-10.AR.F.4* 9-10.GM.35* 9-10.GM.36* 9-10.DPS.1	11-12.NO.4 11-12.NO.5 11-12.AR.8 11-12.AR.9* 11-12.AR.F.8* 11-12.AR.F.9* 11-12.AR.F.10* 11-12.AR.F.11* 11-12.DPS.2* 11-12.DPS.3* 11-12.DPS.4*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Describe and calculate a unit rate. • Convert measurement units within and between measurement systems using ratios. • Identify and interpret the units used in scaled graphs and data displays. • Use scientific notation to represent very large or very small quantities.	• modeling	• Learners may write conversion factors “upside down.” Reversing the numerator and denominator. (ME)
Courses		
Algebra I Geometry Integrated Mathematics I College Ready English and Math – College Learning Lab Math 12 Algebra I Semester I Algebra I Semester II Applied Geometry Occupationally Applied Math		

NINTH AND TENTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

9-10.NO.5 Choose a level of accuracy or precision appropriate to limitations on measurement when reporting quantities.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
6.AR.RP.5 7.GM.AV.1 8.NO.NS.2 8.GM.AV.1	9-10.NO.3 9-10.NO.4 9-10.GM.35* 9-10.GM.36* 9-10.DPS.1*	11-12.NO.4* 11-12.NO.5* 11-12.AR.8* 11-12.AR.9*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Describe and calculate a unit rate. • Convert measurement units within and between measurement systems using ratios. • Identify and interpret the units used in scaled graphs and data displays. • Use the context of a situation to determine the level of accuracy or precision used.	• significant digits • precision (refers to how much information is conveyed by a number (in terms of number of digits) • accuracy (the degree to which a measurement conforms to the correct value or a standard)	• Learners may not use the same units when a problem uses more than one unit. (ME) • Learners might use the incorrect angle measures on the protractor, e.g., reading a measure of 120° instead of 60°. (ME) • Some learners might expect a measurement written with a smaller unit to be more precise than one written with a larger unit, but this is not necessarily true. (ME) • Learners might confuse the rules for significant digits in calculations involving addition and subtraction with those for multiplication and division. (ME) • When estimating, learners may apply the rules for significant digits in calculated measurements to the original numbers instead of the rounded numbers. (ME)

Courses

Algebra I
Geometry
Integrated Mathematics I
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester I
Algebra I Semester II
Applied Geometry
Occupationally Applied Math

NINTH AND TENTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

9-10.AR.1 Use the structure of an expression (e.g., quadratic and exponential) to identify ways to rewrite it.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.NO.NS.3 8.AR.EE.1 8.AR.EE.6	9-10.NO.1 9-10.AR.2 9-10.AR.11	11-12.NO.1 11-12.NO.2 (+) 11-12.NO.11 (+) 11-12.NO.12 11-12.AR.1* 11-12.AR.2 11-12.AR.4* 11-12.AR.5 11-12.AR.6 11-12.AR.10 11-12.AR.13 11-12.AR.14 11.12-AR.15 (+) 11-12.AR.20 11-12.AR.F.3* 11-12.AR.F.6* 11-12.AR.F.14* 11-12.AR.F.15* 11-12.GM.1

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Simplify expressions by combining like-terms using distributive property and other operations with polynomials to write equivalent expressions. • Recognize patterns and structure in expressions. • Identify and write algebraic expressions. • Understand what linear expressions are, how to write them, and how to interpret them. • Use properties of exponents to write an equivalent form of an exponential expression. • Understand the concept of equivalent expressions.	• difference of squares • factor • combine like terms • distributive property • quadratic expression • exponential expression • polynomials	• Learners may not distribute the exponent to both the coefficient and variables inside the parentheses. (ME) • Some learners might not realize that the fraction bar is a grouping symbol and that they must simplify the numerator and denominator separately before dividing. (ME) • Learners may not distribute the negative sign when multiplying. (ME) • Learners may not distribute the coefficient to each term in parentheses. (ME) • When factoring, learners may not factor out the greatest common factor first. (ME)

Courses

Algebra I
Integrated Mathematics I
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester I
Algebra I Semester II

NINTH AND TENTH GRADE

Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*
9-10.AR.2 Rearrange formulas to isolate a quantity or variable(s) of interest using the same reasoning as in solving equations.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.AR.EE.5 8.GM.AV.1 8.DPS.D.3	9-10.AR.1 9-10.AR.6 9-10.AR.F.6* 9-10.AR.F.10* 9-10.GM.33	11-12.AR.1* 11-12.AR.2 11-12.AR.4* 11-12.AR.F.15* 11-12.AR.F.17
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Perform operations on rational numbers. • Apply the distributive property. • Apply the order of operations. • Identify and combine like terms. • Apply properties of equality to isolate the variable (inverse operations). • Apply strategies to isolate a variable when solving equations.	• literal equation • isolate quantity/variable • variable	• Some learners might overlook the specific variable they are solving for. (ME)

Courses
Algebra I Geometry Integrated Mathematics I College Ready English and Math – College Learning Lab Math 12 Algebra I Semester I Algebra I Semester II Applied Geometry

NINTH AND TENTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

9-10.AR.3* Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.AR.EE.2 7.AR.EE.3 8.AR.EE.2 8.AR.EE.6 8.AR.EE.7 8.AR.EE.8 8.AR.F.4 8.GM.GF.6 8.DPS.D.3	9-10.AR.4* 9-10.AR.6 9-10.AR.7* 9-10.AR.8 9-10.AR.9 9-10.AR.10 9-10.AR.F.6* 9-10.AR.F.8* 9-10.AR.F.9* 9-10.AR.F.10* 9-10.DPS.3*	11-12.AR.7* 11-12.AR.8* (+) 11-12.AR.10 11-12.AR.F.1* 11-12.AR.F.14*
Prerequisite Skills	**Key Vocabulary**	**Common Misconceptions**
• Write and solve one- and two-step linear inequalities where coefficients and solutions are integers and/or fractions and decimals. • Isolate a variable when solving a linear equation or inequality.	• linear equations • quadratic equations • exponential equations • inequalities • function	• When applying the Properties of Equality, learners may use the operation in the equation or inequality rather than the inverse operation. (ME) • Learners may not keep their equation or inequality balanced. What is done on one side of an equation or inequality must also be done on the other side. (ME) • When solving problems using division, some learners may automatically divide the larger number by the smaller number, e.g., a learner may try to solve $6x = 3$ by dividing both sides by 3. (ME) • Learners might not distribute a factor to all terms in the parentheses. (ME) • When the variable is on the right side of the inequality, some learners may have difficulty rewriting it with the variable on the left, e.g., a learner might rewrite $-6 > 3x$ as $x > -2$. (ME) • Learners might reverse the inequality symbol whenever there is multiplication or division in the problem or whenever there is a negative sign involved in the problem, e.g., learners may rewrite $2x < -10$ as $x > -5$. (ME) • When learners solve compound inequalities involving OR, some learners may write the statements as a connected chain of inequalities. (ME) • Learners might misunderstand the meanings of the words

		inclusive and exclusive as used in compound inequalities. (ME)

Courses
Algebra I Integrated Mathematics I (linear and exponential only) Integrated Mathematics II (include quadratics) College Ready English and Math – College Learning Lab Math 12 Algebra I Semester I Algebra I Semester II Occupationally Applied Math

NINTH AND TENTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

9-10.AR.4* Create linear and exponential equations in two or more variables to represent relationships between quantities.
Graph equations on coordinate axes with appropriate labels and scales.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.AR.EE.4 8.AR.F.4 8.AR.F.5 8.DPS.D.2 8.DPS.D.3	9-10.NO.3 9-10.AR.3* 9-10.AR.7* 9-10.AR.8 9-10.AR.9 9-10.AR.F.3* 9-10.AR.F.12 9-10.DPS.3*	11-12.NO.4 11-12.AR.3* 11-12.AR.4* 11-12.AR.7* 11-12.AR.8*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify variables and real number coefficients in context. • Apply strategies to isolate the variable when solving equations. • Create an equation to represent relationships between quantities. • Graph equations in two variables on a coordinate plane. • Appropriately label the axes and scales of a graph.	• coordinate plane • scale • linear equation • exponential equation • coordinate axes	• When graphing, learners might mistakenly begin the graph of a linear equation at the origin rather than the y-intercept. (ME) • Some learners might mistakenly put the y-intercept on the x-axis. (ME) • Learners may confuse a slope of 0 with an undefined slope. (ME) • When writing a function rule for an exponential function of the form f(x) = ab^x, some learners may not find the value of both a and b. (ME) • When generating ordered pairs for exponential functions in the form f(x) = ab^x, some students may multiply a by b and then raise that product to the x power. (ME) • Learners may struggle to raise a number to a negative power. (ME)

Courses

Algebra I
Integrated Mathematics I
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester II
Occupationally Applied Math

NINTH AND TENTH GRADE

Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*		
9-10.AR.5 Justify each step in solving a linear equation that may or may not have a solution.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.AR.EE.5	9-10.AR.10 9-10.AR.6 9-10.GM.27	(+) 11-12.AR.11 11-12.AR.F.8*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Perform operations on equations. • Apply the distributive property. • Identify variables and real number coefficients in context. • Combine like terms. • Apply strategies to isolate a variable when solving equations. • Solve linear equations. • Determine the number of solutions given a linear equation (infinite solutions, one solution, and no solutions).	• combine like terms • distributive property • additive inverse • linear equation	• Learners may struggle to manage situations where coefficients are zero. (ME)
Courses		
Algebra I Integrated Mathematics I College Ready English and Math – College Learning Lab Math 12 Algebra I Semester I Algebra I Semester II		

NINTH AND TENTH GRADE

Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*		
9-10.AR.6 Solve linear equations and inequalities (to include compound inequalities) in one variable.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.AR.NO.O.2 8.AR.EE.5 8.AR.EE.7	9-10.AR.2 9-10.AR.3 9-10.AR.5 9-10.AR.7* 9-10.AR.8 9-10.AR.9 9-10.AR.10	11-12.AR.1 11-12.AR.2 11-12.AR.7 11-12.AR.9* 11-12.AR.12 11-12.AR.17
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Perform operations on rational numbers. • Understand the meaning of the symbols < , >, ≥, and ≤. • Identify variables and real number coefficients in equations and inequalities. • Apply the properties of equality/inequalities to isolate the variable. • Simplify expressions by combining like terms and/or applying the distributive property. • Understand when the direction of inequality changes and when it does not. • Graph the solution set on a number line.	• compound inequality • combine like terms • distributive property • inverse operations • linear equations • linear inequalities • compound inequalities	• Learners might not reverse the inequality symbol when multiplying or dividing by a negative number. (ME) • When applying the Properties of Equality, learners might use the operation in the equation or inequality rather than the inverse operation. (ME) • Learners may not keep their equation or inequality balanced. What is done on one side of an equation or inequality must also be done on the other side. (ME) • When solving problems using division, some learners might automatically divide the larger number by the smaller number (e.g., a learner may try to solve 6x = 3 by dividing both sides by 3). (ME) • Learners may not distribute a factor to all terms in parentheses. (ME) • When the variable is on the right side of the inequality, some learners may have difficulty rewriting it with the variable on the left (e.g., a learner might rewrite 2 > x as x >2). (ME) • Learners might reverse the inequality symbol whenever there is multiplication or division in the problem or whenever there is a negative sign involved in the problem (e.g., a learner may rewrite 3x>-6 as x<-2). (ME) • When learners solve compound inequalities involving OR, some learners may write the statements as a connected chain of inequalities. (ME) • Learners might misunderstand

		the meanings of the words inclusive and exclusive as used in compound inequalities. (ME) • Learners may assume they need to change the inequality symbol when distributing a negative factor to a quantity in parentheses. (ME) • Learners may not understand that only compound inequalities involving AND can be written as a connected chain of inequalities. (ME)

Courses
Algebra I Integrated Mathematics I College Ready English and Math – College Learning Lab Math 12 Algebra I Semester I Algebra I Semester II Occupationally Applied Math

NINTH AND TENTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

9-10.AR.7* Solve a system of linear equations graphically and algebraically. Create and solve a system of linear equations in context.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.AR.EE.4 8.AR.EE.5	9-10.AR.3* 9-10.AR.4* 9-10.AR.6 9-10.AR.8 9-10.AR.F.12	11-12.AR.8* 11-12.AR.9* 11-12.AR.17 (+) 11-12.AR.19

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Perform operations on rational numbers. • Identify variables and real number coefficients in equations. • Identify, combine, and collect like terms. • Solve a linear equation in one variable. • Apply strategies to isolate the variable when solving equations.	• linear system • linear combination (elimination) • substitution • half-plane	• Some learners may assume that the solution to the inequality lies below the graphed boundary line if the given inequality contains a less than symbol (or above the boundary line if the given inequality contains a greater than sign). Learners can use the inequality symbol to determine the solution set only if the inequality is first solved for y. (ME) • When solving a system of equations algebraically, some learners may not substitute again to find the value of the second variable. (ME) • When using the elimination method to solve a system of equations, learners may not multiply each term on both sides of the equation by the same number to create opposites. (ME) • When using the elimination method to solve a system of equations, learners may not rearrange the terms so that the corresponding variables are vertically "stacked". (ME) • Learners might reverse the x- and y-coordinates in their solutions. (ME) • Learners might assume they solved a system of equations after finding the value of only one variable. (ME) • Learners may not put an expression in parentheses when substituting for a variable with a coefficient in front. (ME)

Courses

Algebra I
Integrated Mathematics I
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester II
Occupationally Applied Math

NINTH AND TENTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

9-10.AR.8 Graph the solution set to a two-variable system of linear inequalities.
Create and graph the solution set to a two-variable system of linear inequalities in context.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.AR.EE.7 8.AR.EE.8	9-10.AR.3* 9-10.AR.4* 9-10.AR.6 9-10.AR.7* 9-10.AR.F.12	11-12.AR.7* 11-12.AR.8* 11-12.AR.9* 11-12.AR.16 11-12.AR.17
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify an equation or inequality that models a situation in context. • Understand the meaning of the symbols < , >, ≥, and ≤. • Solve for a variable. • Understand when the direction of inequality changes and when it does not. • Explain the characteristics of a linear relationship, including identifying the slope and y-intercept. • Create a system of linear equations in a realistic context.	• half plane • viable solution • non-viable solution • linear system of inequalities • boundary • solution set	• Learners may confuse the solid or dashed lines when graphing. (ME) • Learners may not complete the shading on their graphs. (ME) • When solving systems of inequalities involving parallel lines, learners might assume that the solutions are the points that lie between the lines. Learners may need to be reminded that it is possible for a system to have no solutions or for the solutions to be the same as the solutions of one of the two inequalities. (ME)

Courses

Algebra I
Integrated Mathematics I
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester II
Occupationally Applied Math

NINTH AND TENTH GRADE

Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*
9-10.AR.9 Solve absolute value equations and inequalities in one or two variables.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.AR.EE.5 8.AR.EE.6 8.AR.EE.7 8.AR.EE.8	9-10.AR.3* 9-10.AR.4* 9-10.AR.6	11-12.AR.9* 11-12.AR.16 11-12.AR.F.10
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Understand the concept of real numbers and their placement on the number line. • Understand the concept of absolute value. • Recognize the symbol for absolute value (|5|, |-5|). • Find the absolute value of a real number. • Apply strategies to isolate the variable when solving equations. • Write equivalent equations. • Solve compound inequalities.	• absolute value • compound inequality	• When solving absolute value equations and inequalities, learners may not isolate the absolute value on one side before applying the definition of an absolute value. (ME) • Some learners might assume that if an absolute value equation does not have two solutions, then there must be no solution. (ME) • Learners may be confused about which type of compound statement to use, "and" or "or". (ME)

Courses
Algebra I Integrated Mathematics I College Ready English and Math – College Learning Lab Math 12 Algebra I Semester II

NINTH AND TENTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

9-10.AR.10 Solve quadratic equations in one variable by inspection (e.g., for $x^2 = 49$) taking square roots, the quadratic formula, and factoring, as appropriate to the initial form of the equation.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.NO.NS.2 8.NO.O.1 8.AR.EE.2 8.AR.EE.5 8.GM.GF.6	9-10.AR.3* 9-10.AR.5 9-10.AR.6	11-12.NO.6 11-12.NO.9 (+) 11-12.NO.13 11-12.AR.7* (+) 11-12.AR.10 (+) 11-12.AR.11 (+) 11-12.AR.12 11-12.AR.14 11-12.AR.16 11-12.AR.17 11-12.AR.F.3*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Perform operations on rational numbers. • Solve linear equations. • Factor an expression. • Solve an equation by inspection. • Determine appropriate methods for solving an equation in a given situation.	• quadratic formula • greatest common factor • factor • inspection • quadratic equation	• Learners may choose the constants in the binomial factors as the solutions when finding the zeros of a quadratic function, e.g., students might say that the zeros of f(x) = (x-2)(x-9) are -2 and -9. (ME) • When factoring a quadratic in the form ax^2 + bx + c, learners might confuse the order of the coefficients in the binomials. (ME) • Learners may not factor out the greatest common factor of a trinomial first. (ME) • Learners might apply the Zero Product Property before setting one side of the equation equal to zero. (ME) • Learners may not realize they need to isolate the x^2 term before taking square roots. They may also not realize that you get a positive and a negative value. (ME) • When using the Quadratic Formula, learners might make sign errors in finding -b or b^2 - 4ac when a, b, or c are negative numbers. (ME) • When using the Quadratic Formula, learners may not write the equation in standard form before finding the values of a, b, and c. (ME) • Learners may not write solutions involving radicals in the simplest form. (ME)

Courses
Algebra I
Integrated Mathematics II
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester II

NINTH AND TENTH GRADE

Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*		
9-10.AR.11 Add, subtract, and multiply polynomials.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.AR.EE.1 8.NO.O.2 8.AR.EE.1	9-10.AR.1	11-12.NO.7 11-12.NO.8 11-12.NO.9 (+) 11-12.NO.11 (+) 11-12.NO.12 (+) 11-12.NO.13 11-12.AR.5 11-12.AR.13
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Perform operations on rational numbers. • Identify the leading coefficient, constant, and like terms in a polynomial. • Combine like terms.	• polynomial • combine like terms • standard form (in order of decreasing exponents) • leading coefficient • constant • like bases	• When using the vertical format to add or subtract polynomials, learners may not rearrange like terms, so they are stacked vertically.(ME) • When subtracting a polynomial; enclosed in parentheses, learners may not realize they must subtract each term inside the parentheses. (ME) • When multiplying binomials, learners may be confused about when to multiply and when to add. (ME) • Learners might confuse the degree of a polynomial with the number of terms. (ME) • Some learners may arrange a polynomial using the values of the coefficients instead of the degrees of the terms. (ME) • When determining the degree of a polynomial, some learners may not count 1 for the exponent of a variable that has no visible exponent. (ME) • Learners may not realize that when they add or subtract like terms, the variable part does not change. (ME) • Learners may arrive at the incorrect answer to a real-world problem involving polynomial subtraction because they subtract the polynomials in the wrong order. (ME) • Learners might multiply the exponents of two factors instead of adding them. (ME) • Learners might not combine like terms when multiplying binomials. (ME) • When squaring a binomial,

		learners may only square the two terms of the binomial, losing the x-term of the trinomial. (ME) • Learners might not square the coefficients of terms when using the rules for special binomial products. (ME) • Learners may confuse the square of a difference, (p - q)^2, with the difference of two squares, p^2 - q^2. (ME)

Courses
Algebra I Integrated Mathematics II College Ready English and Math – College Learning Lab Math 12 Algebra I Semester I Algebra I Semester II

NINTH AND TENTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

Functions (F)

Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

9-10.AR.F.1 Determine whether a relationship is a function given a table, graph, or words, identifying x as an element of the domain and f(x) as an element in the range. Determine the domain and range of a function in context.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.AR.F.1 8.AR.F.4 8.AR.F.5	9-10.AR.F.2* 9-10.AR.F.3* 9-10.AR.F.4* 9-10.AR.F.6* 9-10.AR.F.10*	11-12.AR.F.1* 11-12.AR.F.5* 11-12.AR.F.7* 11-12.AR.F.8* 11-12.AR.F.9* 11-12.AR.F.10* (+) 11-12.AR.F.11* 11-12.AR.F.13* 11-12.AR.F.14*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify the x-values as inputs (independent variable, domain) into the function and y-values as outputs (dependent variable, range). • Describe the distinction between a relation and a function. • Graph linear, quadratic, and exponential functions. • Determine the domain and range of a function.	• function • domain (input) • range (output) • independent variable • dependent variable • vertical line test	• Learners may confuse the domains and/or ranges of functions. (ME) • Learners might have difficulty remembering the rule for determining whether a relation is a function. They may want to think of the x as a person and y as a place. Just as a person cannot be in more than one place at a time, an x-value in a function cannot correspond to more than one y-value. (ME) • Learners may use a horizontal line rather than a vertical line when using the Vertical Line Test. (ME)

Courses

Algebra I
Integrated Mathematics I
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester I
Algebra I Semester II
Occupationally Applied Math

NINTH AND TENTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

9-10.AR.F.2* Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in context.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.AR.F.1 8.AR.F.4 8.AR.F.5	9-10.AR.F.1 9-10.AR.F.4* 9-10.AR.F.10*	11-12.AR.F.1* 11-12.AR.F.3* 11-12.AR.F.4* 11-12.AR.F.9* 11-12.AR.F.13* 11-12.AR.F.14*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify the x-values as inputs (independent variable, domain) into the function and y-values as outputs (dependent variable, range). • Describe the distinction between a relation and a function. • Use the vertical line test to identify functions on a graph. • Distinguish between functions and relations given graphs, tables, and sets of ordered pairs.	• function notation • domain • vertical line test	• If learners have difficulty identifying the independent and dependent variables in a situation, consider having them use the words depends on, instead of is a function of. (IN) • Learners might mistakenly interpret the notation f(x) as meaning f times x. (ME)

Courses

Algebra I
Integrated Mathematics I
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester I
Algebra I Semester II
Occupationally Applied Math

NINTH AND TENTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

9-10.AR.F.3* Sketch the key features (to include intercepts, maximums, minimums, and lines of symmetry, where applicable) of linear, exponential, and quadratic functions modeling the relationship between two quantities using tables, graphs, written descriptions, and equations.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.AR.EE.4 8.AR.F.2 8.AR.F.4 8.AR.F.5	9-10.AR.4* 9-10.AR.F.1 9-10.AR.F.4* 9-10.AR.F.5* 9-10.AR.F.6* 9-10.AR.F.7* 9-10.AR.F.8* 9-10.AR.F.9* 9-10.AR.F.10* 9-10.AR.F.11* 9-10.AR.F.12	11-12.AR.F.4* 11-12.AR.F.7* 11-12.AR.F.8* 11-12.AR.F.10* (+) 11-12.AR.F.11*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Perform operations on rational numbers. • Identify line(s) of symmetry. • Rewrite equations to reveal key features (slope, intercepts, maximum, minimum). • Graph linear, quadratic, and exponential functions.	• intercepts • relative maximum • relative minimum • symmetry • linear function • exponential function • quadratic function	• Learners may assume that -x^2 is the same as (-x)^2. (ME) • When graphing a function, learners may try to draw a curve without plotting enough points to show its shape correctly. (ME) • Learners might confuse quadratic functions with exponential functions just because they both have exponents. (ME) • When using intercepts to graph an equation, some learners may plot the points on the wrong axes. (ME) • Learners might substitute zero for the wrong variable when finding intercepts. (ME) • Learners might assume that the zeros of a quadratic function can be found by substituting 0 for x in the function. (ME)

Courses

Algebra I
Integrated Mathematics I
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester I
Algebra I Semester II
Occupationally Applied Math

NINTH AND TENTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

9-10.AR.F.4* Relate the domain of a linear, quadratic, or exponential function to its graph and, where applicable, to the quantitative relationship it describes.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.AR.F.2 8.AR.F.3 8.AR.F.4 8.AR.F.5	9-10.NO.4* 9-10.AR.F.1 9-10.AR.F.2* 9-10.AR.F.3* 9-10.AR.F.5* 9-10.AR.F.7* 9-10.AR.F.10* 9-10.AR.F.11*	11-12.AR.F.1* 11-12.AR.F.2* 11-12.AR.F.3* 11-12.AR.F.4* 11-12.AR.F.5* 11-12.AR.F.6* 11-12.AR.F.7* 11-12.AR.F.8* 11-12.AR.F.9* 11-12.AR.F.12* 11-12.AR.F.13* 11-12.AR.F.14* (+) 11-12.AR.F.22
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify the x-values as inputs (independent variable, domain) into the function and y-values as outputs (dependent variable, range). • Describe the distinction between a relation and a function. • Rewrite equations to reveal key features (slope, intercepts, maximum, minimum). • Graph linear, quadratic, and exponential functions.	• domain • linear function • quadratic function • exponential function • quantitative relationship	• Learners might incorrectly assume that linear functions have restricted domains or ignore practical constraints that limit the domain (time, distance, quantities that cannot be negative). (ME) • Learners might confuse the domain (all possible x values) with the range (all possible y values) when working with quadratic functions. Learners may mistakenly limit it based on the graph's shape (parabola). (ME) • Learners may incorrectly assume the vertex or x-intercepts of a parabola limit the domain of a quadratic function when they actually pertain to specific values within an unrestricted domain. (ME) • Learners may incorrectly assume the domain of an exponential function is limited. (ME) • Learners might assume the horizontal asymptote of an exponential function restricts the domain when it actually influences the range. (ME) • Learners may not connect the algebraic form of the function to its graphical representation. (ME)

Courses
Algebra I
Integrated Mathematics I
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester I
Algebra I Semester II
Occupationally Applied Math

NINTH AND TENTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

9-10.AR.F.5* Calculate and interpret the rate of change of linear, quadratic, or exponential functions (presented algebraically or as a table) over specified intervals.
Estimate the rate of change from a graph.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.AR.EE.4 8.AR.F.5 8.DPS.D.2 8.DPS.D.3	9-10.AR.F.3* 9-10.AR.F.4* 9-10.AR.F.8* 9-10.GM.27	11-12.AR.F.2* 11-12.AR.F.11* 11-12.AR.F.12* (+) 11-12.AR.F.22
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Differentiate linear and nonlinear functions when represented on a graph. • Differentiate among constant, increasing, and decreasing functions when depicted on a graph. • Compute the slope using two given points.	• rate of change • average rate of change • interval • linear function • quadratic function • exponential function • slope	• Learners may not understand that a positive slope indicates an increasing function while a negative slope indicates a decreasing function. (ME) • Learners might mistake the average rate of change for the slope of a linear function without recognizing the variable nature of quadratic equations. (ME)

Courses

Algebra I
Integrated Mathematics III
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester I
Algebra I Semester II
Occupationally Applied Math

NINTH AND TENTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

9-10.AR.F.6* Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function.

a. Use appropriate forms of linear, quadratic, and exponential functions to show zeros, extreme values, and symmetry (where applicable) and interpret them in context.
b. Use the properties of an exponential function to classify it as growth or decay.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.AR.F.2 8.AR.F.3 8.AR.F.4 8.AR.F.5	9-10.NO.2 9-10.AR.1 9-10.AR.2 9-10.AR.3* 9-10.AR.F.1 9-10.AR.F.3* 9-10.AR.F.7* 9-10.AR.F.8*	11-12.NO.1 11-12.AR.4* 11-12.AR.15 11-12.AR.F.1* 11-12.AR.F.3* 11-12.AR.F.6* 11-12.AR.F.8* 11-12.AR.F.13* 11-12.AR.F.14* 11-12.AR.F.15*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Perform operations on rational numbers. • Identify the zeros of a function. • Identify the extreme values of a function (maximum, minimum). • Identify the lines of symmetry of a function. • Recognize the properties of an exponential function. • Identify the intercept(s) of a function.	• exponential growth • exponential decay • extreme values • line of symmetry • linear function • quadratic function • exponential function	• Learners may not find the value of both a and b when writing a function rule for an exponential function in the form f(x) = ab^x. (ME) • When generating ordered pairs for exponential functions of the form f(x) = ab^x, learners may multiply a by b and then raise that product to the power of x. (ME) • Learners may struggle to raise a number to a negative power. (ME) • Learners might not convert the percent growth/decay rate to decimal form. (ME) • Some learners might not add 1 to the rate of growth in the exponential growth model. (ME) • Some learners might not subtract the decay rate from 1 in the exponential decay model.

Courses

Algebra I
Integrated Mathematics II
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester II
Occupationally Applied Math

NINTH AND TENTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

9-10.AR.F.7* Compare key features of two linear, exponential, or quadratic functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.AR.F.2 8.AR.F.3 8.AR.F.4 8.AR.F.5	9-10.AR.F.1* 9-10.AR.F.3* 9-10.AR.F.12	11-12.AR.F.7* 11-12.AR.F.10* (+) 11-12.AR.F.11* 11-12.AR.F.13* 11-12.AR.F.14*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Interpret graphs, tables, verbal descriptions, and algebraic equations. • Identify key features of a linear function (slope, intercepts, domain, range). • Identify key features of a quadratic function (domain, range, vertex, intercept(s), axis of symmetry). • Identify the key features of an exponential function (asymptote, intercept, domain, range, growth, decay).	• linear function • quadratic function • exponential function	• Learners may need to be reminded that variables can have different meanings when they are used in different functions (e.g., the variable b does not mean the same thing in a slope-intercept form equation and an exponential function). (ME) • Learners might have difficulty differentiating between a vertical stretch or compression and a vertical translation. (ME) • Learners may incorrectly identify or use the endpoint of the interval. (ME) • Learners may not correctly draw or interpret the secant line connecting two points on the graph of a quadratic function. (ME) • Learners may not understand that exponential functions grow at a rate proportional to their current value. (ME) • Learners might make errors substituting points into the rate of change formula when working with exponential expressions. (ME)

Courses

Algebra I
Integrated Mathematics II
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester II
Occupationally Applied Math

NINTH AND TENTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

9-10.AR.F.8* Identify situations that can be modeled with linear, quadratic, and exponential functions. Justify the most appropriate model for a situation based on the rate of change over equal intervals. Include situations in which a quantity grows or decays.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.AR.F.3 8.AR.F.4 8.AR.F.5 8.DPS.D.2 8.DPS.D.3	9-10.AR.3* 9-10.AR.F.3* 9-10.AR.F.5* 9-10.AR.F.6* 9-10.AR.F.11*	11-12.AR.F.13* 11-12.AR.F.14*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify key features of linear, quadratic, and exponential functions. • Recognize the graph of a linear, quadratic, or exponential function given its equation. • Calculate the rate of change of a function over a specified interval. • Determine the growth or decay factor of an exponential function.	• linear function • quadratic function • exponential function • constant difference • constant multiplier	• Learners may not recognize situations with parabolic patterns. (ME) • Learners may not understand quadratic functions are characterized by a rate of change that changes at a constant rate (the second derivative is constant). (ME) • Learners might confuse situations with a constant additive rate of change for those with a multiplicative rate of change. (ME)

Courses

Algebra I
Integrated Mathematics II
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester II
Occupationally Applied Math
Consumer Mathematics

NINTH AND TENTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

9-10.AR.F.9* Identify the effect of transformations on the graph of a linear, absolute value, or quadratic function by replacing f(x) with af(x), f(x - h), and f(x) + k, for specific values of a, h, and k (both positive and negative). Find the value of a, h, and k given the graph of the function.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.GM.GF.2 8.GM.GF.3	9-10.AR.F.3* 9-10.AR.F.11* 9-10.GM.2 9-10.GM.3 9-10.GM.4 9-10.GM.5 9-10.GM.14 9-10.GM.15	11-12.AR.4* 11-12.AR.F.3* 11-12.AR.F.4* 11-12.AR.F.7*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Recognize a dilation as a vertical or horizontal stretch. • Recognize a compression as a vertical or horizontal shrink. • Recognize a translation as a horizontal or vertical slide. • Determine whether a figure exhibits a reflection. • Determine whether a figure exhibits a rotation. • Distinguish between rigid and non-rigid transformations.	• transformations (stretch/dilation, shrink/compression, translation/ slide, reflection, rotation) • linear function • absolute value • quadratic function	• Learners may confuse the direction of horizontal and vertical shifts based on the signs of h and k. (ME) • Learners might mix up the effects of horizontal and vertical shifts. (ME) • Learners might confuse the effect of scaling (multiply by a) with shifting (adding or subtracting h or k). (ME) • Learners may misjudge how steep the graph is or whether it has been reflected. (ME)

Courses

Algebra I
Integrated Mathematics III
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester II

NINTH AND TENTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

9-10.AR.F.10* Find the inverse of a linear function and describe the relationship between the domain, range, and graph of the function and its inverse in context.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.AR.F.2 8.AR.F.4 8.AR.F.5	9-10.AR.2 9-10.AR.F.1 9-10.AR.F.3* 9-10.AR.F.4*	11-12.AR.1* 11-12.AR.7* 11-12.AR.8* (+) 11-12.AR.18 11-12.AR.F.5* 11-12.AR.F.6* 11-12.AR.F.10* 11-12.AR.F.15* (+) 11-12.AR.F.22 (+) 11-12.AR.F.23*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Perform operations on rational numbers. • Recognize the characteristics of a linear relationship to include the slope and intercepts. • Determine the domain and range of a linear function. • Identify a one-to-one function. • Recognize that exchanging the x and y values leads to an inverse relationship.	• inverse function • domain • range • one-to-one function	• Learners may not switch the variables at the end of the process, leaving the inverse function incorrectly expressed. (ME) • Learners may not reverse the operations when solving for x. (ME) • When graphing, learners might confuse the domain of the function with the range of the inverse of the function. (ME) • Learners may not consider the constraints on the domain and range based on the context of the problem. (ME) • Learners might incorrectly plot the inverse function on the graph. (ME)

Courses

Algebra I
Integrated Mathematics II
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester II
Occupationally Applied Math

NINTH AND TENTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

9-10.AR.F.11* Interpret the parameters in a linear, quadratic, or exponential function in context.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.AR.EE.3 8.AR.F.2 8.AR.F.3 8.AR.F.4 8.AR.F.5 8.DPS.D.3	9-10.AR.F.3* 9-10.AR.F.4* 9-10.AR.F.7* 9-10.AR.F.8* 9-10.AR.F.9*	11-12.AR.F.4* 11-12.AR.F.7* 11-12.AR.F.8*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Perform operations on rational numbers. • Identify the parameters of a linear function (slope, intercepts, domain, range, constants, coefficients). • Identify the parameters of a quadratic equation (axis of symmetry, maximum, minimum, vertex, intercepts, domain, range). • Identify the parameters of an exponential function (initial value, growth or decay factor, domain, range, asymptote).	• coefficient • constant • parameter (a constant or a variable in a mathematical expression that distinguishes various specific cases)	• Learners may not understand that in a linear function, the slope “a” represents the rate of change. (ME) • Learners may not recognize that in a linear function, a positive “a” indicates an increasing function while a negative “a” indicates a decreasing function. (ME) • Learners may not understand that in a linear function, “b” represents the initial value of the function when x = 0. (ME) • In a quadratic function, learners may not understand that “a” determines whether the parabola opens upward (positive a) or downward (negative a) and affects the width of the parabola (larger |a| means narrower parabola). (ME) • In a quadratic function, learners may not understand how “b” affects the vertex and the axis of symmetry of the parabola. (ME) • In a quadratic function, learners might misinterpret “b” as a constant rate of change without realizing it modifies the symmetry and location of the vertex. (ME) • In a quadratic function, learners may not realize “c” is the y-intercept. (ME) • In an exponential function, learners might confuse the implications of b > 1 (growth) and 0 < b < 1 (decay). (ME) • When working with exponential functions, learners might misinterpret the base “b” as a linear line rather than an exponential rate. (ME)

Courses
Algebra I
Integrated Mathematics II
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester II
Occupationally Applied Math

NINTH AND TENTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

9-10.AR.F.12 Identify, using graphs or tables, the solution(s) to linear and exponential functions f(x) = g(x) as x-value(s) that result in equivalent y-values.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
	9-10.AR.4* 9-10.AR.7* 9-10.AR.8 9-10.AR.F.3* 9-10.AR.F.7*	11-12.AR.16
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Perform operations on rational numbers. • Identify the parameters of a linear function (slope, intercepts, domain, range, constants, coefficients)Identify the parameters of an exponential function (initial value, growth or decay factor, domain, range, asymptote). • Comprehend the distinction between a linear and exponential function.	• point of intersection • solution • equivalent	• Learners might incorrectly identify the points of intersection or overlook the scale or units on the axes when using graphs. (ME) • Learners might incorrectly identify x values where f(x) = g(x) by not accurately comparing the y values in the tables. (ME) • Learners may incorrectly assume linearity when interpolating between table values for exponential functions. (ME) • Learners may not recognize the trend in the table values indicating where the functions may intersect. (ME)

Courses

Algebra I
Integrated Mathematics II
College Ready English and Math – College Learning Lab Math 12
Algebra I Semester II

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*		
9-10.GM.1 Know precise definitions and notations of angle, circle, perpendicular line, parallel line, and line segment based on the undefined notions of point, line, and plane.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.GM.GF.1 8.GM.GF.2 8.GM.GF.3 8.GM.GF.4	9-10.GM.2 9-10.GM.4 9-10.GM.5 9-10.GM.11 9-10.GM.12 9-10.GM.22 9-10.GM.26 9-10.GM.27 9-10.GM.28 9-10.GM.29	(+) 9-10.GM.23 (+) 9-10.GM.24 11-12.GM.1 11-12.GM.2*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify the characteristics of points, lines, planes, angles, circles, perpendicular lines, parallel lines, and line segments.	• point • line • angle • circle • perpendicular lines • parallel lines • line segment • distance • arc length • endpoint	• Learners may struggle to understand angle measurement in degrees to distinguish between acute, obtuse, right, and straight angles. (ME) • Learners may not realize that all points on the circumference are equidistant from the center point. (ME) • Learners may not understand that perpendicular lines must intersect at a 90-degree angle.
Courses		
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics I Applied Geometry Informal Geometry Occupationally Applied Math		

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.2 Represent transformations in the plane. Describe transformations as functions taking points in the plane as inputs and giving other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., rigid vs. non-rigid motion).

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.AR.F.1 8.GM.GF.1 8.GM.GF.2 8.GM.GF.3	9-10.AR.F.9* 9-10.GM.1 9-10.GM.3 9-10.GM.4 9-10.GM.5 9-10.GM.6 9-10.GM.7 9-10.GM.15	(+) 11-12.NO.18 11-12.AR.F.4* 11-12.GM.1 11-12.GM.2*

Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Recognize reflection, rotation, translation, dilation, and compression. • Determine if the two figures are congruent or similar. • Recognize the x-coordinates as the inputs and the y-coordinates as the outputs when plotting points on the coordinate plane.	• points • plane • transformation • rigid transformation • translations • dilations • rotation • reflection • non-rigid motion • rigid motion	• Learners might describe a translation from one vertex of a triangle to a noncorresponding vertex of the triangle's image. (ME) • Learners may confuse the x- and y-axes when working with reflections. (ME) • Learners might confuse the length of CP with the length of CP' when working with dilations. (ME) • Learners may confuse clockwise with counterclockwise when working with rotations. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics I Applied Geometry Informal Geometry Occupationally Applied Math

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*		
9-10.GM.3 Describe the rotations and reflections of a triangle, rectangle, parallelogram, trapezoid, or regular polygon that map each figure onto itself or another figure.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.GM.GF.1 8.GM.GF.2 8.GM.GF.3	9-10.AR.F.1 9-10.AR.F.2 9-10.AR.F.9* 9-10.GM.2 9-10.GM.4 9-10.GM.5 9-10.GM.6 9-10.GM.7 9-10.GM.15 9-10.GM.16 9-10.GM.17	(+) 11-12.NO.18 11-12.AR.F.4* 11-12.GM.1 11-12.GM.2*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Perform single transformations to a figure on the coordinate plane. • Recognize the pre-image and image given a transformation. • Recognize reflection, rotation, translation, dilation, and compression. • Determine whether the two figures are congruent or similar.	• rotation • reflection • triangle • rectangle • parallelogram • trapezoid • regular polygon	• Learners might confuse the direction of a rotation (clockwise or counterclockwise). Teachers should remind learners that the direction is assumed to be counterclockwise unless otherwise stated. (ME) • Learners may rotate a figure around its center or around one of its vertices, not around a point P. (ME)
Courses		
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics I Applied Geometry Informal Geometry Occupationally Applied Math		

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.4 Develop or verify the characteristics of rotations, reflections, and translations in angles, circles, perpendicular lines, parallel lines, and line segments.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.GM.GF.1 8.GM.GF.2 8.GM.GF.3	9-10.AR.F.9* 9-10.GM.1 9-10.GM.2 9-10.GM.5 9-10.GM.6 9-10.GM.14 9-10.GM.15 9-10.GM.27 9-10.GM.29	(+) 11-12.NO.18 11-12.AR.F.4* 11-12.GM.1 11-12.GM.2*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Recognize reflection, rotation, translation, dilation, and compression. • Determine whether two figures are congruent or similar. • Describe transformations that result in congruent or similar shapes. • Perform individual transformations on a shape within the coordinate plane. • Distinguish between rigid and non-rigid motion.	• rotation • reflection • translation • angle • circle • perpendicular lines • parallel lines • segments	• Learners may not correctly identify or use the center of rotation. (ME) • Learners might misunderstand the angle of rotation, either rotating in the incorrect direction or by the incorrect angle. (ME) • Learners may draw the line of reflection incorrectly. (ME) • Learners might misinterpret or incorrectly apply the translation vector. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics I Informal Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.5 Draw the image of a figure that has undergone a series of transformations [rotation(s), reflection(s), or translation(s)] of a geometric figure using a variety of methods (e.g., graph paper, tracing paper, or geometry software).

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.GM.GF.1 8.GM.GF.2 8.GM.GF.3	9-10.AR.F.9* 9-10.GM.1 9-10.GM.2 9-10.GM.3 9-10.GM.4 9-10.GM.6 9-10.GM.12 9-10.GM.14 9-10.GM.29	(+) 11-12.NO.18 11-12.AR.F.4* 11-12.GM.1 11-12.GM.2*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Recognize reflection, rotation, translation, dilation, and compression. • Determine whether two figures are congruent or similar. • Perform individual transformations on a shape within the coordinate plane. • Describe transformations that result in congruent and similar shapes. • Distinguish between rigid and non-rigid motion. • Recognize the pre-image and image given a transformation.	• rotation • reflection • translation • transformation	• Learners might transform the original figure twice instead of transforming the first image to get the second and the second to get the third. (ME) • Learners may perform a combination of transformations in the incorrect order. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics I Applied Geometry Informal Geometry Occupationally Applied Math

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.6 Predict the effect of a specified rigid motion on a given figure using geometric descriptions of rigid motions. Determine whether two figures are congruent using the definition of congruence in terms of rigid motions.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.GM.GF.1 8.GM.GF.2 8.GM.GF.3	9-10.GM.2 9-10.GM.3 9-10.GM.4 9-10.GM.5 9-10.GM.7 9-10.GM.8 9-10.GM.9 9-10.GM.15 9-10.GM.16 9-10.GM.17 9-10.GM.34	(+) 11-12.NO.18 11-12.AR.F.4*

Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Recognize reflection, rotation, translation, dilation, and compression. • Determine whether two figures are congruent or similar. • Perform individual transformations on a shape within the coordinate plane. • Describe transformations that result in congruent and similar shapes. • Recognize the pre-image and image given a transformation. • Distinguish between rigid and non-rigid motion.	• congruent • rigid motion	• Learners might assume that if two figures are congruent, then there is one rigid motion that can map one figure to the other. Explain that it may take a sequence of rigid motions to map a figure to a congruent figure. (ME) • Learners might perceive that two angles cannot be congruent if the rays forming the angles have different lengths. (ME) • Learners might make an error when using computations to determine if a transformed figure is congruent or not congruent. Emphasize that a resulting figure with the sides crossing each other is an indication of an error, not necessarily a noncongruent figure. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics I Applied Geometry Informal Geometry Occupationally Applied Math

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.7 Use the definition of congruence, based on rigid motions, to show two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.GM.GF.1 8.GM.GF.2	9-10.GM.2 9-10.GM.3 9-10.GM.6 9-10.GM.8 9-10.GM.9 9-10.GM.10 9-10.GM.15 9-10.GM.16 9-10.GM.17	11-12.GM.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Recognize reflection, rotation, translation, dilation, and compression. • Determine whether two figures are congruent or similar. • Perform individual transformations on a shape within the coordinate plane. • Identify the characteristics of acute, right, scalene, and obtuse triangles. • Identify and label corresponding sides and angles of congruent triangles.	• corresponding sides • corresponding angles • congruent • rigid motions	• Learners might confuse congruence with equality. (ME) • When working with congruent triangles, learners may interchange the sides with 1, 2, or 3 tick marks or the angles with 1, 2, or 3 arcs. (ME) • When working with triangle proofs, learners may skip steps or have difficulty understanding which corresponding parts are congruent. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics I Informal Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*		
9-10.GM.8 Prove two triangles are congruent using the congruence theorems.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.GM.GF.1	9-10.GM.6 9-10.GM.7 9-10.GM.9 9-10.GM.10 9-10.GM.15 9-10.GM.16 9-10.GM.17	11-12.GM.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Recognize reflection, rotation, translation, dilation, and compression. • Determine whether two figures are congruent or similar. • Perform individual transformations on a shape within the coordinate plane. • Identify the characteristics of acute, right, scalene, and obtuse triangles. • Identify and label corresponding sides and angles of congruent triangles. • Arrange reasoning in logical sequence. • Support reasoning using appropriate justifications.	• ASA • SSS • SAS • congruent • congruence theorems	• When completing triangle proofs, learners may either omit steps or provide an inadequate explanation. (ME) • Learners might conclude they do not have sufficient information to prove triangles are congruent because they forgot to consider the common side or angle of two triangles. (ME) • Learners might assume the triangles can be proven congruent by SAS even though the angle is not between the two sides. (ME)
Courses		
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Informal Geometry		

NINTH AND TENTH GRADE

Geometry and Measurement (GM)

Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

9-10.GM.9 Prove and apply theorems about lines and angles.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.GM.GF.2 8.GM.GF.4 8.GM.GF.5 8.GM.GF.6	9-10.GM.6 9-10.GM.7 9-10.GM.8 9-10.GM.10 9-10.GM.11 9-10.GM.22 (+) 9-10.GM.23 9-10.GM.29	(+) 9-10.GM.19 (+) 9-10.GM.21 (+) 9-10.GM.23 (+) 9-10.GM.24 9-10.GM.25 9-10.GM.26 11-12.AR.F.16 11-12.GM.3 (+) 11-12.GM.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify lines and line segments. • Identify intersecting lines. • Identify parallel lines. • Identify complementary, supplementary, vertical, exterior, interior, and adjacent angles. • Perform operations on rational numbers. • Solve linear equations. • Apply postulates, definitions, properties, and theorems to identify or solve for angle measurements. • Arrange reasoning in logical sequence. • Support reasoning using appropriate justifications.	• transversal • alternate interior angles • alternate exterior angles • corresponding angles • perpendicular bisector • equidistant • vertical angles • parallel lines • linear pair • theorem • postulates	• Learners may confuse vertical angles with linear pairs. (Consider pointing out that the non-shared sides of the angles in a linear pair form a line.) (ME) • Learners may mistakenly believe that any pair of angles with a special name, such as alternate exterior angles, must be congruent. (Consider pointing out that pairs of angles do not have to be congruent to have a special name, such as same-side interior angles.) (ME) • Learners may either omit steps or provide an inadequate explanation. (ME) • Learners might confuse perpendicular bisectors with medians. (ME)

Courses

Geometry
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics II
Applied Geometry
Informal Geometry
Occupationally Applied Math

NINTH AND TENTH GRADE

Geometry and Measurement (GM)

Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

9-10.GM.10 Prove and apply theorems about triangles.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
8.GM.GF.4 8.GM.GF.5 8.GM.GF.6	9-10.GM.7 9-10.GM.8 9-10.GM.9 9-10.GM.11 9-10.GM.16 9-10.GM.18	(+) 9-10.GM.19 (+) 9-10.GM.21 (+) 9-10.GM.23 (+) 9-10.GM.24 11-12.AR.F.16 11-12.AR.F.17 11-12.GM.3 (+) 11-12.GM.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify complementary, supplementary, vertical, exterior, interior, and adjacent angles. • Recognize properties of triangles (base angles of an isosceles triangle, midpoint, midsegment, median, and centroid of a triangle). • Perform operations on rational numbers. • Arrange reasoning in logical sequence. • Support reasoning using appropriate justifications.	• isosceles triangle • base angles • midpoint • midsegment • median • centroid • Triangle Sum Theorem • theorem • postulate	• Remind learners that diagrams can be misleading because they are not always drawn to scale. (IN) • Learners may confuse congruent triangles with similar triangles. (ME) • Learners might not recognize a right triangle when it is not oriented with one of the legs horizontally. (ME) • Learners might try to measure the angles using a protractor rather than using the Triangle Sum Theorem and the Exterior Angle Theorem to calculate angle measures. (ME) • Learners may not be able to solve problems because they overlook the symbols for congruent angles or congruent sides. (ME) • Learners might mistakenly believe that the centroid in a diagram is halfway between the vertex and its opposite side. (ME)

Courses

Geometry
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics II
Applied Geometry
Informal Geometry
Occupationally Applied Math

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*		
9-10.GM.11 Prove and apply theorems about parallelograms.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.GM.GF.4	9-10.GM.1 9-10.GM.9 9-10.GM.10 9-10.GM.34	(+) 9-10.GM.23 (+) 11-12.NO.10
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify and distinguish between different types of parallelograms. • Identify the diagonal of a parallelogram. • Bisect a line segment or angle. • Apply postulates, definitions, properties, and theorems about parallelograms to solve problems. • Arrange reasoning in logical sequence. • Support reasoning using appropriate justifications.	• diagonal • bisect • rectangle • rhombus • quadrilateral • square • parallelogram • theorem • postulate	• Learners might assume that since the diagonals of a parallelogram bisect each other, the diagonals are also congruent. (ME)
Courses		
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry Informal Geometry		

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.12 Make basic geometric constructions (e.g., segment, angle, bisectors, parallel and perpendicular lines) with a variety of tools and methods.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.GM.GF.1	9-10.GM.1 9-10.GM.5 9-10.GM.27 9-10.GM.29	(+) 9-10.GM.13 (+) 9-10.GM.23 (+) 9-10.GM.24
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Recognize line segment and angle bisectors. • Recognize types of angles (acute, obtuse, right, scalene). • Appropriate use of ruler, protractor, compass, Mira, patty paper, technology.	• perpendicular bisector • segment • angle • parallel lines • perpendicular lines	• Teachers may need to remind learners of the techniques for accuracy in construction, including using a sharp pencil, lining up the compass tip and the pencil tip, and making sure the compass opening stays the same size. If a learner's compass settings are not tightly fixed, the compass setting may change without the learners' awareness. (IN) • When constructing the angle bisector, learners may change the compass setting when they draw the intersecting arcs from each side ray of the angle. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry Informal Geometry Occupationally Applied Math

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
(+) 9-10.GM.13 Apply basic constructions to create polygons such as equilateral triangles, squares, and regular hexagons inscribed in circles.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.GM.12		(+) 9-10.GM.23 (+) 9-10.GM.24
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Recognize polygons (equilateral triangle, square, regular hexagon). • Recognize line segment and angle bisectors. • Recognize types of angles (acute, obtuse, right, scalene). • Appropriate use of ruler, protractor, compass, Mira, patty paper, technology.	• inscribed • hexagon • regular • equilateral • polygons • triangles • hexagons • squares • circles	• Learners may incorrectly place or identify the center of the circle. (ME) • Learners may incorrectly estimate or measure the radius of the circle leading to misplacement of the vertices of the inscribed polygon. (ME) • Learners may not construct the vertices of the polygon at equal angular intervals along the circumference of the circle, resulting in a polygon with uneven sides. (ME) • Learners might omit a vertex or add an extra vertex resulting in a polygon with an incorrect number of sides. (ME) • Learners might not ensure each vertex of the polygon lies on the circumference of the circle violating the property of an inscribed polygon. (ME) • Learners might not ensure the sides of the polygon intersect the circle at exactly two points resulting in a polygon that is not inscribed. (ME) • Learners might misinterpret the placement of the vertices relative to the circle. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*		
9-10.GM.14 Verify experimentally and justify the properties of dilations given by a center and a scale factor.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.GM.GF.2	9-10.AR.F.9* 9-10.GM.5 9-10.GM.15	(+) 11-12.NO.18 (+) 11-12.NO.19 11-12.AR.F.4*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Differentiate a size change from a scale change. • Identify midpoint and scale factors. • Recognize when a figure has been dilated.	• dilation • center • scale factor • pre-image • image	• Learners may not understand whether a scale factor greater than 1 enlarges or reduces a figure and vice versa, leading to errors in verifying dilation properties. (ME) • Learners may not understand if the center of the dilation should be inside or outside the given figure. (ME) • Learners might overlook the need to preserve specific properties of figures under dilation (parallelism, angle measures, lengths). (ME) • Learners may not realize that dilations produce similar figures meaning corresponding angles are congruent and corresponding sides are proportional. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry Informal Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.15 Use transformations to decide if two given figures are similar. Apply the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.GM.GF.1 8.GM.GF.2 8.GM.GF.3	9-10.AR.F.9* 9-10.GM.2 9-10.GM.3 9-10.GM.4 9-10.GM.6 9-10.GM.7 9-10.GM.8 9-10.GM.14 9-10.GM.16 9-10.GM.17 9-10.GM.18	(+) 11-12.NO.18 11-12.AR.F.4*

Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish between similar and congruent figures. • Identify the characteristics of similar triangles. • Recognize that if a ratio remains constant, then the relationship is proportional. • Identify transformations (rotation, reflection, translation, dilation, compression).	• ratio • proportion • similar • transformations • corresponding angles • corresponding sides	• To avoid making errors in setting up proportions in similar triangles, learners may redraw overlapping triangles separately, labeling each one individually. (IN) • When working with similar polygons, learners might mismatch the corresponding angles when one figure of a pair is flipped or rotated. (ME) • Learners may have difficulty identifying the corresponding sides due to the orientations of the figures. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry Informal Geometry Occupationally Applied Math

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*		
9-10.GM.16 Prove similarity theorems about triangles.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.GM.GF.1 8.GM.GF.4	9-10.GM.3 9-10.GM.6 9-10.GM.7 9-10.GM.8 9-10.GM.10 9-10.GM.15 9-10.GM.17 9-10.GM.18	(+) 11-12.NO.18 11-12.AR.F.4*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish between similar and congruent figures. • Identify the characteristics of similar triangles. • Recognize that if a ratio remains constant, then the relationship is proportional. • Identify transformations (rotation, reflection, translation, dilation, compression) that produce similar images.	• ratio • triangles • triangle similarity theorems	• Learners might confuse congruent triangles with similar triangles. (ME) • Learners may have difficulty setting up proportions because they did not use the Segment Addition Postulate. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Informal Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*		
9-10.GM.17 Apply knowledge of congruence and similarity criteria for triangles to solve problems and to prove relationships in various geometric figures.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.GM.GF.1 8.GM.GF.4	9-10.GM.3 9-10.GM.6 9-10.GM.7 9-10.GM.8 9-10.GM.15 9-10.GM.16 9-10.GM.18 9-10.GM.28	(+) 11-12.NO.18 11-12.AR.F.4*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish between similar and congruent figures. • Identify the characteristics of similar or right triangles. • Recognize that if a ratio remains constant, then the relationship is proportional. • Identify transformations (rotation, reflection, translation, dilation, compression) that produce similar images. • Apply the similar triangle theorems (AA or AAA, SAS, SSS). • Apply the congruent triangle theorems (SSS, SAS, AAS, ASA, HL). • Arrange reasoning in a logical sequence. • Support reasoning using appropriate justification.	• ratio • scale factor • congruent triangle theorems (SSS, SAS, AAS, ASA, HL) • similar triangle theorems (AA or AAA, SAS, SSS) • postulates	• Learners may write proportions that do not compare corresponding parts of the figure. (ME) • Learners may not attend to the units in their final answers and ensure the lengths they find for authentic problems are reasonable. (ME) • Learners might overlook the possibility of finding the similarity ratio of two figures without knowing the lengths of the sides. They may need to be reminded that the perimeters have the same similarity ratio as the corresponding sides, and the areas have the same ratio as the squares of the corresponding sides. (ME) • Learners may find it puzzling that a leg in one triangle can become the hypotenuse in a related triangle. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry Informal Geometry Occupationally Applied Math

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.18 Recognize how the properties of similar right triangles allow the trigonometric ratios to be defined and determine the sine, cosine, and tangent of an acute angle in a right triangle.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
	9-10.GM.10 9-10.GM.15 9-10.GM.16 9-10.GM.17 9-10.GM.20*	(+) 9-10.GM.19 (+) 9-10.GM.21* 11-12.AR.F.16 11-12.AR.F.17 (+) 11-12.AR.F.18 (+) 11-12.AR.F.19 (+) 11-12.AR.F.24 11-12.GM.3 (+) 11-12.GM.4
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Determine when two angles are complementary. • Identify the properties of right triangles. • Apply appropriate trigonometric ratios to find the measure of acute angles and side lengths of right triangles.	• complementary angles • sine • cosine • tangent • right triangles • acute angle • adjacent • opposite • hypotenuse	• If learners use calculators to compute the trig ratios of an angle, teachers should remind them to check that their calculators are in degree mode. (IN) • Learners might have the misconception that the opposite leg is always a vertical side of the triangle and the adjacent leg is always a horizontal side of the triangle. Teachers should remind learners that the opposite and adjacent sides are determined by the location of the associated angle, not by the orientation of the triangle. (ME) • Learners might use the incorrect ratio for sine and cosine by interchanging them. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Informal Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*		
(+) 9-10.GM.19 Explain and use the relationship between the sine and cosine of complementary angles.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.GM.10 9-10.GM.18	9-10.GM.20*	11-12.AR.F.16 11-12.AR.F.17 (+) 11-12.AR.F.18 (+) 11-12.AR.F.19 11-12.GM.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Determine when two angles are complementary. • Identify the properties of right triangles. • Apply appropriate trigonometric ratios to find the measure of acute angles and side lengths of right triangles.	• complementary angles • sine • cosine	• Learners might mistakenly assume the sine and cosine of complementary angles are equal. (ME) • Learners might not clearly distinguish between sine and cosine functions and their values for complementary angles. (ME) • Learners might incorrectly apply the trigonometric functions without considering the relationship between the angles involved. (ME) • Learners may not understand what it means for angles to be complimentary. (ME) • Learners might overlook or misunderstand trigonometric identities. (ME)
Courses		
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics III		

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.20* Solve applied problems involving right triangles using trigonometric ratios, the Pythagorean Theorem, and special right triangles (30°-60°-90° and 45°-45°-90°).

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.GM.GF.5 8.GM.GF.6	9-10.GM.18 (+) 9-10.GM.19	(+) 9-10.GM.21 11-12.AR.F.16 11-12.AR.F.17 11-12.GM.3 (+) 11-12.GM.4
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify the attributes of a right triangle (hypotenuse, right angle, legs, sum of other two interior angles is 90 degrees). • Perform operations on rational numbers. • Solve equations for missing variables. • Apply the Pythagorean Theorem to find unknown side lengths. • Apply trigonometric ratios to find the measure of acute angles and side lengths of right triangles. • Apply inverse trigonometric ratios in right triangles.	• special right triangles • Pythagorean Theorem • trigonometric ratios	• Learners may have difficulty identifying their reference angle when using SOHCAHTOA. (ME) • Learners may not simplify their radical expressions completely. (ME) • When solving right triangles, learners might confuse the hypotenuse with the adjacent side as they write ratios. (ME) • Learners may confuse the ratios for sine and cosine. (ME) • Learners might set up the equations incorrectly when they use trigonometric ratios to find side lengths. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry Informal Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
(+) 9-10.GM.21* Solve unknown sides and angles of non-right triangles using the Laws of Sines and Cosines.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.GM.18 9-10.GM.20*	9-10.GM.18	11-12.AR.F.16 11-12.AR.F.17 11-12.GM.3
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Perform operations on rational numbers. • Apply trigonometric ratios to find unknown sides or angles in a right triangle. • Use the Pythagorean Theorem to find the measure of an angle in a right triangle.	• Law of Sines • Law of Cosines • non-right triangles • trigonometric ratios	• Learners might make errors finding the unknown measure for triangles that are not shown in a diagram. (ME) • Learners might round intermediate answers when applying the Law of Sines, which may result in an incorrect final answer. (ME) • When using the Law of Cosines to determine the side length of a triangle, some learners may solve for the square on the side length and then forget to take the positive square root of each side of the equation. (ME)

Courses
Geometry Trigonometry Trigonometry/Analytic Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics III Applied Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.22 Apply theorems about relationships between line segments and circles or angles and circles formed by radii, diameter, secants, tangents, and chords to find unknown lengths or angles.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.GM.AV.1	9-10.GM.1 9-10.GM.9 9-10.GM.25 9-10.GM.26 9-10.GM.31	(+) 9-10.GM.23 (+) 9-10.GM.24 (+) 11-12.AR.F.18 11-12.GM.1 11-12.GM.2*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify line segments, angles, and circles. • Identify and distinguish between the radii, diameter, secant, tangent, and chord of a circle. • Determine the relationship between the segments of chords, secants, and tangents. • Identify and determine the relationship between central angles, inscribed/circumscribed angles, arc measures, and arc lengths.	• radii • diameter • secant • tangent • chord • inscribed/circumscribed angle • central angle • arc • theorem	• Learners might need to be reminded that if arcs are not on the same circle or on congruent circles, then their arc lengths are not equal. (IN) • Learners may confuse the diameter with the radius. (ME) • Learners may confuse chords with secants. (ME) • When working with a line tangent to a circle, learners might assume the central angle is a right angle. (ME) • Learners may confuse minor and major arcs. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
(+) 9-10.GM.23 Construct the incenter and circumcenter of a triangle. Relate the incenter and circumcenter to the inscribed and circumscribed circles.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.GM.1 9-10.GM.11 9-10.GM.12 9-10.GM.22	9-10.GM.1 9-10.GM.5 9-10.GM.9 (+) 9-10.GM.13 (+) 9-10.GM.24	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Calculate ratios. • Identify and distinguish between perpendicular and angle bisectors of triangles. • Distinguish between and apply bisector theorems. • Identify the incenter/ circumcenter of a triangle. • Identify the inscribed angle and chords of a circumscribed circle. • Apply appropriate circle theorems.	• inscribed circle • circumscribed circle • incenter • circumcenter • chords • perpendicular bisector • angle bisector • median • ratio	• Learners may use the compass or straight edge incorrectly, resulting in inaccurate angle bisectors and an incorrectly placed incenter. (ME) • Learners might mistakenly identify the intersection of incorrect angle bisectors as the incenter due to confusion about the vertices of the triangle. (ME) • Learners may not accurately locate the perpendicular bisectors of the triangle's sides, leading to an incorrectly placed circumcenter. (ME) • Learners might confuse the incenter with the circumcenter and vice versa. (ME) • Learners may not understand which center corresponds to the inscribed circle (incenter) vs. the circumscribed circle (circumcenter). (ME) • Learners may not understand that the radius of the inscribed circle equals the perpendicular distance from the incenter to any side, while the radius of the circumscribed circle equals half the length of any side of the triangle. (ME) • Learners might have misconceptions about the inscribed circle touching the sides of the triangle tangently at the points of contact. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*		
(+) 9-10.GM.24 Construct a tangent line from a point outside a given circle to the circle.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.GM.1 9-10.GM.12 9-10.GM.22	9-10.GM.22 (+) 9-10.GM.13 9-10.GM.23	11-12.AR.F.2*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Calculate the slope of a line. • Graph linear functions. • Identify tangent lines of circles.	• tangent line	• Learners may not accurately identify the point where the arc intersects the circle resulting in a tangent line that does not touch the circle at a single point. (ME) • Learners might confuse tangent lines with secant lines, constructing lines that intersect the circle at two points instead of one. (ME) • Learners might misinterpret the direction of the tangent line resulting in lines that do not touch the circle as intended. (ME)
Courses		
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry		

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*		
9-10.GM.25 Explain and use the formulas for arc length and area of sectors of circles.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.GM.AV.1	9-10.GM.22 9-10.GM.26 9-10.GM.31	(+) 11-12.AR.F.18 (+) 11-12.AR.F.19
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Calculate the slope of a line. • Calculate the circumference of a circle. • Explain the concept of arc length. • Identify the sector of a circle. • Calculate the area of a circle.	• sector • area • radian • circumference • arc length	• Point out that the m in the formula for the length of an arc represents the arc measure, not the central angle. (IN) • Learners might confuse arc measure and arc length. Teachers might want to point out the difference between these terms. (ME) • Learners may not realize that an answer that contains the pi symbol is an appropriate answer, and they might automatically give an appropriate decimal answer rather than leaving the pi symbol in their answer, which would be the exact answer. (ME) • Learners may substitute the length of the diameter to the area instead of the length of the radius. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry Informal Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.26 Recognize that the radian measure of an angle is the ratio of the length of the arc to the length of the radius of a circle.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.AR.RP.4 7.GM.AV.1	9-10.GM.1 9-10.GM.22 9-10.GM.25	11-12.AR.F.16 (+) 11-12.AR.F.18 (+) 11-12.AR.F.19
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Calculate ratios. • Calculate the arc length of a circle. • Calculate the length of the radius of a circle.	• radians • angle • arc length • radius length	• Learners might not convert degrees to radians when measuring arc length. (ME) • Learners might mistakenly use the diameter of the circle instead of the radius when calculating radian measure. (ME) • Learners may not realize the radius length remains constant in all parts of the circle, regardless of the size or position of the arc. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics III

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*		
9-10.GM.27 Develop and verify the slope criteria for parallel and perpendicular lines. Apply the slope criteria for parallel and perpendicular lines to solve problems.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.AR.EE.3 8.AR.EE.4	9-10.AR.5 9-10.AR.F.5* 9-10.GM.1 9-10.GM.4 9-10.GM.12 9-10.GM.28	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify the characteristics of parallel and perpendicular lines. • Calculate the slope of a line. • Write an equation for a line parallel or perpendicular to a given line passing through a given point.	• parallel lines • perpendicular lines • slope	• Learners might make the common error when working with converses of the parallel lines postulate or theorems of assuming that the lines are already parallel. (ME) • When working with perpendicular lines, learners may assume lines are perpendicular if they look perpendicular. (ME) • When finding the slopes of perpendicular lines, learners may use the same sign for both slopes. (ME)

Courses
Algebra I Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II College Ready English and Math – College Learning Lab Math 12 Algebra I Semester I Algebra I Semester II Applied Geometry Informal Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.28 Verify simple geometric theorems algebraically using coordinates. Verify algebraically, using coordinates, that a given set of points produces a particular type of triangle or quadrilateral.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.GM.GF.1 8.AR.EE.3 8.GM.GF.6	9-10.GM.1 9-10.GM.17 9-10.GM.27 9-10.GM.29 9-10.GM.30*	(+) 11-12.NO.15
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Plot points on the coordinate plane. • Calculate the slope given two points. • Apply the distance formula to find a length. • Identify the characteristics of acute, right, scalene, and obtuse triangles. • Identify the characteristics of parallel and perpendicular lines.	• coordinate geometry • coordinates • points • triangle (types) • quadrilateral (types)	• Learners might apply incorrect theorems or properties. (ME) • Learners might not verify if a given set of points form a valid quadrilateral, e.g., ensure points do not lie on the same line or consecutive points form sides. (ME) • Learners might make errors in calculating the lengths or slopes of diagonals leading to misidentification of quadrilaterals. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.29 Determine the midpoint or endpoint of a line segment using coordinates. (+) Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.AR.RP.4	9-10.GM.1 9-10.GM.4 9-10.GM.5 9-10.GM.9 9-10.GM.12 9-10.GM.28	(+) 11-12.AR.F.18
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish between a line segment and a line or ray. • Apply the midpoint formula. • Use coordinates to identify the midpoint and endpoints of line segments on the coordinate plane. • Identify directed line segments. • Calculate ratios. • Apply the distance formula to find a length.	• midpoint • endpoint • segment • directed line segment	• Learners may subtract coordinates rather than add them when finding the coordinates of the midpoint. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II College Ready English and Math – College Learning Lab Math 12 Applied Geometry Informal Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.30* Compute perimeters of polygons and areas of triangles, parallelograms, trapezoids, and kites using coordinates.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.GM.AV.2 8.GM.GF.6	9-10.GM.28 9-10.GM.31 9-10.GM.32 9-10.GM.33 9-10.GM.35* 9-10.GM.36*	11-12.GM.3

Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Calculate the perimeter and area of rectangles and triangles. • Distinguish between perimeter and area. • Identify the coordinates for polygons, triangles, parallelograms, trapezoids, or kites. • Estimate the perimeter and area of irregular shapes. • Apply the distance formula to find a length.	• distance formula • perimeter • area • polygon • trapezoid • kite • parallelogram	• Learners might misread a measure as applying to only part of a side instead of correctly reading that it is the measure of the entire side. (ME) • When calculating the area of a triangle, learners may first halve the measure of the base, then multiply this measure by the height, and multiply the result by one-half. (ME) • When finding the area of a rhombus, learners might have answers that are exactly double the correct answers. These learners did not multiply by one-half when finding the area of the rhombus. (ME) • Learners may assume that doubling the dimensions will double the area but doubling the dimensions will quadruple the area of a geometric figure. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry Informal Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.31 Explain derivations of the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.GM.AV.1 7.GM.AV.3 8.GM.AV.1	9-10.GM.22 9-10.GM.25 9-10.GM.30* 9-10.GM.32 9-10.GM.33 9-10.GM.35* 9-10.GM.36*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Find the circumference of a circle. • Use area and volume formulas for a cylinder, pyramid, and cone appropriately in the context of the problem.	• volume • area • cylinder • pyramid • cone • Cavalier's Principle (2D) • Cavalier's Principle (3D)	• Learners might confuse diameter and radius. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*		
9-10.GM.32 Calculate the surface area for prisms, cylinders, pyramids, cones, and spheres to solve problems.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.GM.AV.2	9-10.GM.30* 9-10.GM.31 9-10.GM.33 9-10.GM.34 9-10.GM.35* 9-10.GM.36*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish between prisms, cylinders, pyramids, cones, and spheres based on their attributes. • Calculate the area of a rectangle or square, triangle, and/or circle. • Distinguish between area and surface area. • Investigate the surface area of prisms, cylinders, pyramids, cones, and spheres using nets.	• prism • cylinder • pyramid • cone • sphere • surface area • nets	• When calculating the volume of a prism, remind learners that the base is not always rectangular. (IN) • When finding the surface area of cylinders, learners may not include the area of both bases. (ME) • When finding the surface area of cylinders, learners tend to use diameter instead of radius. (ME) • When finding the surface area of cylinders, learners may use the squares of the radius. (ME) • When finding the surface area of cylinders, learners tend to multiply the height by the area of a base instead of by its circumference. (ME) • When finding the surface area of a pyramid or cone, learners may use the height instead of the slant height. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry Informal Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.33 Know and apply volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.GM.AV.3 8.GM.AV.1	9-10.AR.2 9-10.GM.30* 9-10.GM.31 9-10.GM.32 9-10.GM.34 9-10.GM.35*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish between prisms, cylinders, pyramids, cones, and spheres based on their attributes. • Calculate the area of a rectangle or square, triangle, and/or circle. • Identify the dimensions of a polygon. • Decompose a figure to determine an appropriate method or formula for calculating volume.	• volume • prism • cylinder • cone • pyramid • sphere	• When calculating the volume of a cone or pyramid, learners tend to use the slant height rather than the height. (ME) • When working with spheres, learners might confuse the formulas for volume and surface area. (ME) • Learners may confuse finding the surface area or volume of a hemisphere with the entire sphere. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry Informal Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*		
9-10.GM.34 Identify the shapes of two-dimensional cross-sections of three-dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.GM.GF.1 8.GM.GF.2 8.GM.GF.3	9-10.GM.6 9-10.GM.11 9-10.GM.32 9-10.GM.33	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish between two-dimensional and three-dimensional shapes and their attributes. • Identify lines of symmetry in two-dimensional shapes. • Distinguish between parallel and perpendicular lines. • Identify intersecting planes.	• cross-section • rotations • lines of symmetry • horizontal cross-section • vertical cross-section • parallel • perpendicular • intersecting planes	• Learners may not understand which axis the two-dimensional object is being rotated around, leading to errors in visualizing the resulting cross-sections. (ME) • Learners might align the two-dimensional object with the rotation axis inaccurately, leading to misidentified cross-sectional shapes. (ME) • Learners might overlook how the rotation of a two-dimensional object affects its orientation and shape in three dimensions, resulting in misconceptions about the resulting cross sections. (ME) • Learners might incorrectly assume the cross sections will always exhibit symmetry or mirror the original object's shape. (ME) • Learners may not understand how angles and orientations change during rotation. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*		
9-10.GM.35* Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.AR.RP.1 7.GM.AV.1 7.GM.AV.2 7.GM.AV.3 8.GM.AV.1	9-10.NO.3 9-10.NO.4 9-10.NO.5 9-10.GM.30 9-10.GM.31 9-10.GM.32 9-10.GM.33 9-10.GM.36*	11-12.NO.4* 11-12.NO.5*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Apply appropriate formulas to find area and volume. • Apply unit analysis to convert measurements and find unit rates.	• density • area • volume • unit analysis	• Learners might mistakenly equate density with mass or weight.
Courses		
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry		

NINTH AND TENTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
9-10.GM.36* Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; scaling a model).

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.AR.RP.4 7.GM.AV.2 7.GM.AV.3 8.GM.AV.1	9-10.NO.3 9-10.NO.4 9-10.NO.5 9-10.GM.30 9-10.GM.31 9-10.GM.32 9-10.GM.35*	11-12.NO.4* 11-12.NO.5*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Apply unit analysis to convert measurements and find unit rates. • Interpret and define the scale and unit rates used in descriptive modeling. • Determine the level of accuracy/ precision appropriate when reporting quantities.	• constraint • scale model • unit analysis	• Learners may not maintain consistent scaling leading to disproportionate results. (ME) • Learners might not convert units appropriately when scaling. (ME) • Learners might overlook geometric constraints imposed by the design problem. (ME) • Learners might misapply geometric principles, e.g., symmetry, congruence, similarity, proportionality. (ME)

Courses
Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II Applied Geometry Informal Geometry Occupationally Applied Math

NINTH AND TENTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*
9-10.DPS.1* Represent data with plots on the real number line (dot plots, histograms, and box plots).

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
6.DPS.D.4	9-10.NO.3 9-10.NO.4* 9-10.NO.5 9-10.DPS.2* 9-10.DPS.3*	11-12.NO.4* 11-12.NO.5* 11-12.DPS.3* 11-12.DPS.4*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Display numerical data as plots on a number line, including dot plots and histograms. • Describe patterns of data, such as gaps, clusters, and skews. • Select the level of accuracy or precision when reporting quantities.	• dot plot • histogram • box plot • quartiles • interquartile range • maximum • minimum • cluster • skew	• Learners may include the median in the upper and lower halves of the data set when finding the first quartile and the third quartile. (ME) • Learners might try to draw a histogram with bars representing intervals of different sizes. (ME) • When comparing box plots, learners may draw each box plot using different scales on their number lines. (ME)

Courses
Algebra I Integrated Mathematics I College Ready English and Math – College Learning Lab Math 12 Algebra I Semester I Algebra I Semester II Occupationally Applied Math Applied Math

NINTH AND TENTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*
9-10.DPS.2* Compare the center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets using statistics appropriate to the shape of the data distribution.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.DPS.D.2	9-10.DPS.1* 9-10.DPS.3* 9-10.DPS.4* 9-10.DPS.10*	11-12.DPS.1* 11-12.DPS.2* 11-12.DPS.3* 11-12.DPS.4* (+) 11-12.DPS.8*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Read and interpret dot plots, histograms, box plots, and tables. • Calculate measures of center and differentiate between median and mean. • Calculate measures of spread (use technology to calculate standard deviation) and differentiate between range and standard deviation.	• median • mean • interquartile range • standard deviation • data distribution • skewed distribution • symmetric distribution • data set • statistics	• Technology is used to calculate standard deviation. (IN) • The process of calculating standard deviation is not addressed in prior standards. Learners will need to be given the algorithm for finding standard deviation before using technology to develop an understanding of the meaning of the concept. (IN) • Learners may include the median in the upper and lower halves of the data set when finding the first quartile and the third quartile. (ME) • Learners might try to determine the median of a data set without listing the values in numerical order. (ME) • When calculating standard deviation, learners may not take the square root of the mean of the squared deviations. (ME) • Learners might confuse the terms skewed to the left and skewed to the right. (ME)

Courses
Algebra I Probability and Statistics Integrated Mathematics I College Ready English and Math – College Learning Lab Math 12 Algebra I Semester I Algebra I Semester II Occupationally Applied Math Applied Math

NINTH AND TENTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*
9-10.DPS.3* Represent data on two quantitative variables on a scatter plot and describe how the variables are related. a. Fit a linear function to the data (with or without technology) if appropriate. b. Compute (using technology) and interpret the correlation coefficient of a linear fit. c. Interpret the meaning of the slope and y-intercept of the linear model in context. d. Interpolate and extrapolate the linear model to predict values.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.AR.F.4 8.AR.F.5 8.DPS.D.1 8.DPS.D.2 8.DPS.D.3	9-10.AR.3* 9-10.AR.4* 9-10.AR.F.11* 9-10.DPS.1* 9-10.DPS.2* 9-10.DPS.4* 9-10.DPS.10*	(+) 11-12.AR.F.21 11-12.DPS.4* (+) 11-12.DPS.5*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Graph and interpret data on dot plots, histograms, scatter plots, and box plots. • Identify the slope and y-intercept of a linear relationship. • Represent linear relationships using tables, graphs, and equations.	• scatter plot • linear model • correlation coefficient • slope • y-intercept • interpolate • extrapolate	• Learners may confuse the correlation coefficient for a scatter plot with the slope of a linear function. Learners might need to be reminded that the correlation coefficient measures how close the points are to the line of fit, not the steepness of the line. (ME) • Learners may have difficulty drawing a line of fit on a scatter plot if they are trying to include actual data points on the line. (ME)

Courses
Algebra I Probability and Statistics Integrated Mathematics I College Ready English and Math – College Learning Lab Math 12 Algebra I Semester II Occupationally Applied Math Applied Math

NINTH AND TENTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*		
9-10.DPS.4* Distinguish between correlation and causation.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
8.DPS.D.1 8.DPS.D.2 8.DPS.D.3 8.DPS.D.4	9-10.DPS.2* 9-10.DPS.3*	11-12.DPS.3* 11-12.DPS.4*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Describe patterns of data, such as gaps, clusters, and skews. • Represent bivariate categorical data using a table, graph, or equation.	• correlation • causation	• Learners may confuse the correlation coefficient for a scatter plot with the slope of a linear function. Learners might need to be reminded that the correlation coefficient measures how close the points are to the line of fit, not the steepness of the line. (ME) • Learners might assume that a positive r-value indicates a stronger correlation than a negative r-value. (Learners may need to be reminded that the value for the correlation coefficient, r, will be close to 1 if the function is a good fit for the data set. (ME)

Courses
Algebra I Probability and Statistics Integrated Mathematics I College Ready English and Math – College Learning Lab Math 12 Algebra I Semester II Occupationally Applied Math Applied Math

NINTH AND TENTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*		
9-10.DPS.5* Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes or as unions, intersections, or complements of other events (“or,” “and,” “not”).		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.DPS.P.1 7.DPS.P.2	9-10.DPS.6* 9-10.DPS.7* 9-10.DPS.8* 9-10.DPS.9* 9-10.DPS.10*	(+) 11-12.DPS.10* (+) 11-12.DPS.11*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Determine the sample space and potential outcomes in context. • Apply the counting principles of "and," "or," and "not" to determine the number of outcomes in context.	• sample space • union • intersection • complementary events • mutually exclusive events • theoretical probability • experimental probability • outcomes • Addition Counting Principle • Multiplication Counting Principle	• There are no prior standards providing mean to "and," "or," or "not" in compound events. Teachers will need to familiarize learners with the mathematical meaning of "and" and "or" in the context of probability. (IN) • Learners may not define all possible outcomes/elements or incorrectly define the sample space. (ME) • Learners might misinterpret the characteristics or properties of events or provide vague descriptions of events. (ME) • Learners might misuse union or intersection operations when describing events. (ME)
Courses		
Probability and Statistics Integrated Mathematics II Occupationally Applied Math		

NINTH AND TENTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*
9-10.DPS.6* Recognize that event A is independent of event B if the probability of event A does not change in response to the occurrence of event B. Apply the formula P(A and B) = P(A)·P(B) given that events A and B are independent.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.DPS.P.1 7.DPS.P.2	9-10.DPS.5* 9-10.DPS.7* 9-10.DPS.8* 9-10.DPS.9* 9-10.DPS.10*	(+) 11-12.DPS.10* (+) 11-12.DPS.11* (+) 11-12.DPS.12* (+) 11-12.DPS.13* (+) 11-12.DPS.14* (+) 11-12.DPS.15* (+) 11-12.DPS.16* (+) 11-12.DPS.17* (+) 11-12.DPS.18*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Determine the sample space and potential outcomes in context. • Apply the Addition and Multiplication Counting Principles in context. • Determine the sample space and potential outcomes in context. • Apply the counting principles of "and," "or," and "not" to determine the number of outcomes in context.	• independent events • intersection • counting principle in probability	• Learners may mistakenly believe independence implies mutual exclusivity between events A and B and overlook the fact that independent events can still occur together. (ME) • Learners may not understand the concept of independence and think it implies equal probabilities for events A and B or that one event excludes the other. (ME) • Learners may not understand the meaning of joint probability (P(A and B)). (ME) • Learners may not understand the multiplication rule and always assume P(A and B) always equals P(A) x P(B) regardless of the independence of events A and B. • Learners might not update probabilities after the occurrence of one event when calculating joint probabilities, leading to inaccuracies. (ME)

Courses
Probability and Statistics Integrated Mathematics II Occupationally Applied Math

NINTH AND TENTH GRADE

Data, Probability, and Statistics (DPS)

Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.

9-10.DPS.7* Recognize that the conditional probability of an event A given B is the probability that event A will occur given the knowledge that event B has already occurred.
Calculate the conditional probability of A given B and interpret the answer in context.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.DPS.P.1 7.DPS.P.2	9-10.DPS.5* 9-10.DPS.6* 9-10.DPS.8* 9-10.DPS.9* 9-10.DPS.10*	(+) 11-12.DPS.10* (+) 11-12.DPS.11* (+) 11-12.DPS.12* (+) 11-12.DPS.13* (+) 11-12.DPS.14* (+) 11-12.DPS.15* (+) 11-12.DPS.16* (+) 11-12.DPS.17* (+) 11-12.DPS.18*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Determine the sample space and potential outcomes in context. • Apply the Addition and Multiplication Counting Principles in context. • Determine the sample space and potential outcomes in context. • Apply the counting principles of "and," "or," and "not" to determine the number of outcomes in context.	• conditional probability • Multiplication Counting Principle • independent events	• Learners should understand that P(A|B) is not the same as P(B|A), and they should not be confused in the formula P(A and B) = P(A) * P(B|A). (IN) • Learners may not recognize that conditional probability is used when events are dependent and assume independence when calculating conditional probabilities. (ME) • Learners might not understand that conditional probability represents the probability of an event occurring given specific information and conditions. (ME) • Learners may not consider the entire sample space when calculating the probability of event B. (ME) • Learners might not include all relevant information or conditions when determining the probability of event A given B. (ME) • Learners may not consider reverse probabilities when analyzing relationships between events. (ME)

Courses

Probability and Statistics
Integrated Mathematics II
Occupationally Applied Math

NINTH AND TENTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*		
9-10.DPS.8* Apply the formula P(A or B) = P(A) + P(B) – P(A and B) and interpret the answer in context.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.DPS.P.1 7.DPS.P.2	9-10.DPS.5* 9-10.DPS.6* 9-10.DPS.7* 9-10.DPS.9* 9-10.DPS.10*	(+) 11-12.DPS.10* (+) 11-12.DPS.11* (+) 11-12.DPS.12* (+) 11-12.DPS.13* (+) 11-12.DPS.14* (+) 11-12.DPS.15* (+) 11-12.DPS.16* (+) 11-12.DPS.17* (+) 11-12.DPS.18*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Determine the sample space and potential outcomes in context. • Apply the Addition and Multiplication Counting Principles in context. • Determine the sample space and potential outcomes in context. • Apply the counting principles of "and," "or," and "not" to determine the number of outcomes in context. • Determine whether events are disjoint (mutually exclusive).	• disjoint events • mutually exclusive events • Addition Counting Principle	• Learners might mistakenly use the formula for events that are mutually exclusive. (ME) • Learners may not examine the possibility of overlapping outcomes between events A and B, resulting in errors in calculating the probability of their union. (ME) • Learners might incorrectly apply the complement rule instead of the inclusion-exclusion principle when calculating the probability of the union of events A and B. (ME) • Learners may not subtract the probability of the intersection P(A and B) from the sum of P(A) and P(B). (ME) • Learners may not look for dependencies between events A and B (conditional probabilities, sequential events). (ME)
Courses		
Probability and Statistics Integrated Mathematics II Occupationally Applied Math		

NINTH AND TENTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*		
9-10.DPS.9* Determine the number of outcomes using permutations and combinations in context.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
7.AR.RP.4 7.DPS.P.1 7.DPS.P.2	9-10.DPS.5* 9-10.DPS.6* 9-10.DPS.7* 9-10.DPS.8* 9-10.DPS.10*	(+) 11-12.DPS.10* (+) 11-12.DPS.11* (+) 11-12.DPS.14* (+) 11-12.DPS.17* (+) 11-12.DPS.18*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Determine the sample space and potential outcomes in context. • Apply the Addition and Multiplication Counting Principles in context. • Determine the sample space and potential outcomes in context. • Apply the counting principles of "and," "or," and "not" to determine the number of outcomes in context. • Determine whether events are disjoint (mutually exclusive).	• permutation • combination • conditional events • independent events	• Learners might make errors when simplifying with factorials. Teachers should encourage learners to write out the product a factorial represents until they are sure which numbers to cancel. (ME) • Learners may have difficulty using combinations to find probability. Teachers should have learners break down the probability problem into parts. First, find the size of the sample space. Then, find the number of outcomes associated with the event. Finally, write the ratio. (ME) • Learners may confuse permutations and combinations. They might not recognize which should be applied or they may apply the incorrect formula. Teachers should have learners begin by deciding if the order is important. Then have them look up the formula. (ME) • Learners may attempt to simplify permutations or combinations by canceling factors. Teachers should remind learners of the meaning of factorials, and suggest they write out the multiplication to determine which factors cancel. (ME)
Courses		
Probability and Statistics Integrated Mathematics II Occupationally Applied Math		

NINTH AND TENTH GRADE

Data, Probability, and Statistics (DPS)

Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.

9-10.DPS.10* Construct and interpret two-way frequency tables of data for two categorical variables. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
7.AR.RP.2 7.DPS.P.2 8.DPS.D.4	9-10.DPS.2* 9-10.DPS.3* 9-10.DPS.5* 9-10.DPS.6* 9-10.DPS.7* 9-10.DPS.8* 9-10.DPS.9*	11-12.AR.F.8* (+) 11-12.DPS.14* (+) 11-12.DPS.17* (+) 11-12.DPS.18*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Interpret and analyze data in tables and graphs. • Create and utilize tables depicting ratio relationships and use them to solve problems. • Use tables and graphs to analyze dependent and independent relationships.	• two-way frequency table (contingency table) • categorical variables	• Learners may incorrectly treat categorical variables as numerical variables. (ME) • Learners might overlook the need to normalize frequencies to compute conditional probabilities accurately, leading to misinterpretations of the relationship between the variables. (ME) • Learners might focus only on cell frequencies in the two-way table and ignore marginal frequencies, which provide important context for interpreting relationships between variables. (ME) • Learners might assume that events are independent based only on the frequencies observed in the two-way table without considering other factors or contextual information. (ME) • Learners may interpret the absence of association as evidence of independence without recognizing independence requires more rigorous criteria. (ME)

Courses

Probability and Statistics
Integrated Mathematics I (Only construct and interpret two-way tables; conditional probabilities are optional for this course.)
Integrated Mathematics II
Occupationally Applied Math

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

11-12.NO.1.Rewrite complex expressions involving radicals and rational exponents using the properties of exponents.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.1 9-10.NO.2 9-10.AR.1 9-10.AR.F.6*	11-12.NO.2 11-12.NO.3 11-12.NO.7 11-12.NO.8 11-12.AR.2 11-12.AR.4* 11-12.AR.5 11-12.AR.6 11-12.AR.12 11-12.AR.F.3* 11-12.AR.F.6* 11-12.AR.F.15*	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Distinguish between the hierarchal numbers in the complex number system. • Apply properties of exponents to rewrite and simplify expressions. • Perform operations on expression in the real number system.	• rational • irrational • exponent • radical • properties of exponents • complex expressions • hierarchy of the complex number system	• Learners may not distinguish between the power and the root in a rational exponent and/or a radical expression. (ME) • Learners may inappropriately apply the exponent properties when simplifying a rational expression. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics II

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

11-12.NO.2 Perform operations on complex radical expressions and simplify radicals to write equivalent expressions.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.1 9-10.NO.2 9-10.AR.1	11-12.NO.1 11-12.NO.3 11-12.NO.7 11-12.NO.8 11-12.AR.2 11-12.AR.4* 11-12.AR.5 11-12.AR.6 11-12.AR.12 11-12.AR.F.3* 11-12.AR.F.6* 11-12.AR.F.15*	(+) 11-12.NO.11

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Write equivalent expressions in the real number system. • Write radical expressions as rational exponents and vice-versa. • Apply properties of exponents to rewrite and simplify expressions.	• radicals • rationalizing the denominator • simplify • equivalent • complex radical	• Learners may inappropriately combine like terms or combine terms with different indexes. (ME) • Learners may apply exponent properties incorrectly when simplifying a rational expression. (ME) • Learners may not rationalize a denominator when simplifying an expression. (ME) • Learners might not multiply by the conjugate of the denominator when dividing by a radical expression. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics II

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

11-12.NO.3 Demonstrate that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.1 9-10.NO.2	11-12.NO.1 11-12.NO.2 11-12.NO.7 11-12.NO.8 11-12.AR.5	(+) 11-12.NO.11

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Distinguish between the hierarchal numbers in the complex number system. • Write equivalent expressions in the real number system. • Write radical expressions as rational exponents and vice-versa. • Apply properties of exponents to rewrite and simplify expressions.	• closure property • irrational number • rational number • hierarchy of the complex number system	• Learners might not distinguish between rational and irrational numbers. (ME) • Learners may apply operations on rational and irrational numbers incorrectly. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics II
Abstract Algebra

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

11-12.NO.4* Use units to understand problems and to guide the solution of multi-step problems (e.g., unit analysis).
Choose and interpret units consistently in formulas.
Choose and interpret the scale and the units in graphs and data displays.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.3 9-10.NO.4 9-10.NO.5* 9-10.AR.4* 9-10.GM.35* 9-10.GM.36* 9-10.DPS.1*	11-12.NO.5*	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Apply unit analysis to convert measurements and find unit rates.	• unit analysis • scale • unit rate	• Learners might not apply correct equivalencies among units when performing unit analysis. (ME) • Learners may not place the units in the appropriate part of the fraction. (ME) • Learners might assume the conversion between two different lengths is the same conversion between two different areas. (ME)

Courses

Algebra II
College Algebra
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics for Computer Science/Information Technology
Integrated Mathematics II
Analytic Geometry

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO) *Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.*		
11-12.NO.5* Choose a level of accuracy or precision appropriate to limitations on measurement when reporting quantities.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.NO.3 9-10.NO.4 9-10.NO.5* 9-10.GM.35* 9-10.GM.36* 9-10.DPS.1*	11-12.NO.4* 11-12.AR.8* 11-12.AR.9*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Apply unit analysis to convert measurements and find unit rates. • Apply appropriate significant figures to reflect the precision of a value.	• significant digits • precision • accuracy	• Learners may not use appropriate scales when graphing. (ME) • Learners might not use the appropriate unit for measurement in context. (ME) • Learners may not distinguish between precision and accuracy. (ME) • Learners might have errors in rounding and/or significant digits. (ME)

Courses
Algebra II College Algebra Trigonometry/Analytic Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics for Computer Science/Information Technology Integrated Mathematics II Analytic Geometry

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

11-12.NO.6 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
Understand the hierarchal relationships among subsets of the complex number system.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.2 9-10.AR.10	11-12.NO.2 11-12.NO.3 11-12.NO.7 11-12.NO.8 11-12.NO.9	(+) 11-12.NO.11 (+) 11-12.NO.12 (+) 11-12.NO.13 (+) 11-12.AR.11

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Distinguish between the hierarchal numbers in the complex number system. • Recognize connections among subsets (whole numbers, natural numbers, integers, rational, irrational, real, imaginary) within the complex number system.	• whole number • natural number • integer • rational number • irrational number • real number • imaginary number • complex number	• Learners may not distinguish between the hierarchical relationships among subsets of the complex number system, e.g., may not distinguish an integer from an irrational number. (ME) • Learners might misrepresent an imaginary number with a negative number. (ME) • Learners may not represent the square root of a negative number appropriately. (ME) • Learners might multiply two imaginary numbers in radical form together before rewriting them as complex numbers in a + bi form. (ME)

Courses

Algebra II
College Algebra
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics II
Abstract Algebra
Analytic Geometry

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

11-12.NO.7 Use the definition $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.11	11-12.NO.1 11-12.NO.2 11-12.NO.3 11-12.NO.6 11-12.NO.8 11-12.NO.9 11-12.AR.12 11-12.AR.13	(+) 11-12.NO.11 (+) 11-12.NO.12 (+) 11-12.NO.13 (+) 11-12.AR.11

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Perform operations on complex numbers. • Distinguish between the hierarchal numbers in the complex number system.	• distributive property • commutative property • associative property	• Learners might apply PEMDAS inappropriately when simplifying a complex expression. (ME) • Learners may combine terms that are not like, e.g., real with imaginary. (ME) • Learners might cancel terms inappropriately when dividing complex expressions. (ME) • Learners might neglect to create a common denominator when simplifying an expression. (ME)

Courses

Algebra II
College Algebra
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics II
Abstract Algebra
Analytic Geometry

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

11-12.NO.8 Use conjugates to find quotients of complex numbers.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.11	11-12.NO.1 11-12.NO.2 11-12.NO.3 11-12.NO.6 11-12.NO.7 11-12.NO.9 11-12.AR.13	(+) 11-12.NO.11 (+) 11-12.NO.12 (+) 11-12.NO.13 (+) 11-12.AR.11

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Perform operations on complex numbers. • Distinguish between the hierarchal numbers in the complex number system.	• complex conjugate • complex numbers	• Learners might not apply the conjugate of the denominator when dividing complex numbers. (ME) • Learners might not apply the conjugate of the denominator to the numerator and denominator. (ME) • Learners may have computational errors when performing operations on complex numbers. (ME) • Learners may not recognize that multiplying a complex number by its conjugate results in a real number. (ME)

Courses

Algebra II
College Algebra
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics II
Abstract Algebra
Analytic Geometry

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

11-12.NO.9 Apply the Fundamental Theorem of Algebra to determine the number of zeros for polynomial functions.

Find all solutions to a polynomial equation.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.10 9-10.AR.11	11-12.NO.6 11-12.NO.7 11-12.NO.8 11-12.AR.14 11-12.AR.15 11-12.AR.F.3* 11-12.AR.F.10*	(+) 11-12.NO.11 (+) 11-12.NO.12 (+) 11-12.NO.13 (+) 11-12.AR.10 (+) 11-12.AR.11 (+) 11-12.AR.F.11*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Perform operations on complex numbers. • Distinguish between the hierarchal numbers in the complex number system. • Write a zero of a polynomial as a factor and vice versa. • Solve an equation by factoring, quadratic formula, polynomial division, and synthetic division.	• Fundamental Theorem of Algebra • zeros • degree • polynomial • solutions • roots • factors	• Learners might assume the Fundamental Theorem of Algebra finds roots. (ME) • Learners may factor inappropriately when finding the roots of a polynomial. (ME) • Learners may not recognize roots as complex. (ME) • Learners might not find the negative root when solving an even-powered function. (ME) • Learners might not determine the number of real vs. non-real roots. (ME) • Learners may not recognize when roots come in complex conjugate pairs. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics III
Abstract Algebra

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

(+) 11-12.NO.10 Represent complex numbers on the complex plane in rectangular, trigonometric, and polar forms.
Find the modulus (absolute value) of a complex number.
Explain why the rectangular, trigonometric, and polar forms of a given complex number represent the same number.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.GM.11 11-12.AR.4* 11-12.AR.6	(+) 11-12.NO.11 (+) 11-12.NO.12 (+) 11-12.NO.13 (+) 11-12.NO.14 (+) 11-12.NO.15 (+) 11-12.NO.16 (+) 11-12.NO.17 (+) 11-12.NO.18	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Perform operations on complex numbers. • Distinguish between the hierarchal numbers in the complex number system. • Distinguish between the rectangular coordinate plane and the complex plane. • Represent polar coordinates in the polar coordinate plane. • Represent complex numbers in the complex plane.	• complex plane • polar coordinate plane • complex numbers • polar form of a complex number • rectangular form of a complex number • binomial form of a complex number • trigonometric form of a complex number • modulus (absolute value, radius)	• There are no prior standards for graphing polar coordinates in the polar coordinate plane. (IN) • There are no prior standards for graphing complex numbers in the complex plane. (IN) • Learners may not recognize the differences between numbers in rectangular, trigonometric, and polar forms. (ME) • Learners might not convert between the different forms of a number appropriately. (ME) • Learners may not make the connection between numbers expressed in polar form and right triangle trigonometry. (ME)

Courses

College Algebra
Precalculus
Trigonometry
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
AP Precalculus
Abstract Algebra
Analytic Geometry

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

(+) 11-12.NO.11 Represent addition, subtraction, multiplication, conjugation, powers, and roots of complex numbers geometrically on the complex and/or polar plane; use properties of this representation for computation.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.1 9-10.NO.2 9-10.AR.1 9-10.AR.11 11-12.NO.2 11-12.NO.3 11-12.NO.6 11-12.NO.7 11-12.NO.8 11-12.NO.9 11-12.AR.5 11-12.AR.6 11-12.AR.12 11-12.AR.13	(+) 11-12.NO.10 (+) 11-12.NO.12 (+) 11-12.NO.13 (+) 11-12.NO.14 (+) 11-12.NO.15 (+) 11-12.NO.16 (+) 11-12.NO.17 (+) 11-12.NO.18 (+) 11-12.AR.11	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Distinguish between the hierarchal numbers in the complex number system. • Perform operations on complex numbers. • Distinguish between the rectangular coordinate plane and the complex plane. • Represent polar coordinates in the polar coordinate plane. • Represent complex numbers in the complex plane. • Write complex numbers in binomial, rectangular, polar, and trigonometric form.	• argument (angle measure) • absolute value (radius) • conjugation • powers • roots • complex numbers • complex plane • polar coordinate plane • De Moivre's Theorem • polar form of a complex number • rectangular form of a complex number • binomial form of a complex number • trigonometric form of a complex number	• Learners may not recognize the appropriate forms a complex number must be in when performing operations on complex numbers, e.g., a complex number must be in polar form to perform powers and roots. (ME)

Courses

College Algebra
Precalculus
Trigonometry
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
AP Precalculus
Abstract Algebra
Analytic Geometry

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

(+) 11-12.NO.12 Extend polynomial identities to the complex numbers.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.1 9-10.AR.11 11-12.AR.2 11-12.NO.6 11-12.NO.7 11-12.NO.8 11-12.NO.9 11-12.AR.4* 11-12.AR.5 11-12.AR.6 11-12.AR.13 11-12.AR.14	(+) 11-12.NO.10 (+) 11-12.NO.11 (+) 11-12.NO.13 (+) 11-12.AR.20	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Distinguish between the hierarchal numbers in the complex number system. • Perform operations on complex numbers. • Apply the sum or difference of cubes to factor a polynomial. • Apply sum or difference of squares to factor a polynomial. • Recognize recurring patterns within a polynomial to devise a method for factoring.	• polynomial identities • complex numbers	• Learners might perform operations on complex numbers incorrectly. (ME) • Learners may not recognize the sum and difference of squares and cubes when factoring complex numbers. (ME) • Learners may not recognize patterns when determining a process for factoring a polynomial. (ME)

Courses
College Algebra Precalculus AP Precalculus Abstract Algebra

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

(+) 11-12.NO.13 Apply the Fundamental Theorem of Algebra to find all roots of a polynomial equation and determine the nature (e.g., integer, rational, irrational, real, complex) of the roots.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.1 9-10.AR.10 9-10.AR.11 11-12.NO.6 11-12.NO.7 11-12.NO.8 11-12.NO.9 11-12.AR.4 11-12.AR.12 11-12.AR.14 11-12.AR.15	(+) 11-12.NO.10 (+) 11-12.NO.11 (+) 11-12.NO.12 (+) 11-12.AR.10 (+) 11-12.AR.11	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Distinguish between the hierarchal numbers in the complex number system. • Perform operations on complex numbers. • Apply the sum or difference of cubes to factor a polynomial. • Apply the sum or difference of squares to factor a polynomial. • Recognize recurring patterns within a polynomial to devise a method for factoring.	• Fundamental Theorem of Algebra • zeros • degree • complex • real • rational • irrational • integer • root	• Learners may not recognize the nature of the roots. (ME) • Learners may not recognize every root as complex, e.g., can be written in the form a + bi. (ME) • Learners might not determine all roots of a polynomial. (ME)

Courses

College Algebra
Precalculus
AP Precalculus
Integrated Mathematics III
Abstract Algebra

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

(+) 11-12.NO.14 Recognize vector quantities as having both magnitude and direction, writing them in polar form.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
		(+) 11-12.NO.10 (+) 11-12.NO.11 (+) 11-12.NO.15 (+) 11-12.NO.16 (+) 11-12.NO.17 (+) 11-12.NO.18 (+) 11-12.AR.F.18

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Apply the Pythagorean Theorem to solve problems. • Interpret and write functions to describe the relationship between two quantities. • Solve problems using trigonometric ratios. • Perform operations on complex numbers. • Distinguish between the rectangular coordinate plane and the complex plane. • Represent polar coordinates in the polar coordinate plane.	• vector • magnitude • direction • polar form • polar axis	• Learners may not recognize the length of the vector as the magnitude and the direction as the angle measure. (ME) • Learners may not recognize the form a vector must be in to perform operations on it, e.g., component form vs. polar form. (ME)

Courses

Precalculus
Trigonometry/Analytic Geometry
AP Precalculus
Linear Algebra
Abstract Algebra
Analytic Geometry

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

(+) 11-12.NO.15 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.GM.28	(+) 11-12.NO.10 (+) 11-12.NO.11 (+) 11-12.NO.14 (+) 11-12.NO.16 (+) 11-12.NO.17 (+) 11-12.NO.18 (+) 11-12.AR.F.18	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Apply the Pythagorean Theorem to solve problems. • Interpret and write functions to describe the relationship between two quantities. • Solve problems using trigonometric ratios. • Perform operations on complex numbers. • Distinguish between the rectangular coordinate plane and the complex plane. • Represent polar coordinates in the polar coordinate plane. • Identify a vector as a magnitude and direction.	• initial point • terminal point • vector • magnitude • components	• Learners might not put the vector in the appropriate form when performing operations. (ME)

Courses

Precalculus
Trigonometry/Analytic Geometry
AP Precalculus
Linear Algebra
Abstract Algebra
Analytic Geometry

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO) *Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.*		
(+) 11-12.NO.16 Solve problems involving magnitude and direction that can be represented by vectors.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
	(+) 11-12.NO.10 (+) 11-12.NO.11 (+) 11-12.NO.14 (+) 11-12.NO.15 (+) 11-12.NO.17 (+) 11-12.NO.18 (+) 11-12.AR.F.18	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Apply the Pythagorean Theorem to solve problems. • Interpret and write functions to describe the relationship between two quantities. • Solve problems using trigonometric ratios. • Perform operations on complex numbers. • Distinguish between the rectangular coordinate plane and the complex plane. • Represent polar coordinates in the polar coordinate plane. • Calculate the magnitude and direction of a vector as represented on a graph.	• velocity • magnitude • direction • vectors	• Learners may not recognize when to add or subtract vectors in context. (ME)
Courses		
Precalculus Trigonometry/Analytic Geometry AP Precalculus Linear Algebra Abstract Algebra Analytic Geometry		

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

(+) 11-12.NO.17 Add and subtract vectors.

a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Know that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
c. Understand that vector subtraction **v**-**w** is defined as **v**+(-**w**), where -**w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction.

Represent vector subtraction graphically by connecting the tips in the appropriate order and using the components to perform vector subtraction.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
11-12.AR.5 11-12.AR.6	(+) 11-12.NO.10 (+) 11-12.NO.11 (+) 11-12.NO.14 (+) 11-12.NO.15 (+) 11-12.NO.16 (+) 11-12.NO.18 (+) 11-12.AR.F.18	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Apply the Pythagorean Theorem to solve problems. • Interpret and write functions to describe the relationship between two quantities. • Solve problems using trigonometric ratios. • Perform operations on complex numbers. • Distinguish between the rectangular coordinate plane and the complex plane. • Represent polar coordinates in the polar coordinate plane. • Calculate the magnitude and direction of a vector as represented on a graph.	• resultant • vectors • parallelogram rule • magnitude • direction	• Learners may not put the vector in component form first when performing operations (addition, subtraction, scalar). (ME)

Courses

Precalculus
Trigonometry/Analytic Geometry
AP Precalculus
Linear Algebra
Abstract Algebra
Analytic Geometry

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

(+) 11-12.NO.18 Multiply a vector by a scalar.

a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction. Use the components to perform scalar multiplication (e.g., as $c(v_x, v_y) = (cv_x , cv_y)$).

b. Compute the magnitude of a scalar multiple c**v** using $||c\mathbf{v}|| = |c|\mathbf{v}$.

c. Compute the direction of cv knowing that when $|c|\mathbf{v} \neq 0$, the direction of c**v** is either along **v** (for $c > 0$) or against **v** (for $c < 0$).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.GM.2 9-10.GM.3 9-10.GM.4 9-10.GM.5 9-10.GM.6 9-10.GM.14 9-10.GM.15 9-10.GM.16 9-10.GM.17 9-10.GM.29 11-12.AR.5 11-12.AR.6	(+) 11-12.NO.10 (+) 11-12.NO.11 (+) 11-12.NO.14 (+) 11-12.NO.15 (+) 11-12.NO.16 (+) 11-12.NO.17 (+) 11-12.AR.F.18	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Solve problems using trigonometric ratios. • Perform operations on complex numbers. • Distinguish between the rectangular coordinate plane and the complex plane. • Represent polar coordinates in the polar coordinate plane. • Calculate the magnitude and direction of a vector as represented on a graph. • Apply ratio relationships to determine the scale of a drawing. • Justify why magnitudes cannot be negative. • Define the magnitude and change of a vector as represented on a graph. • Determine the impact of multiplying positive and negative scalars to a vector.	• scalar • scalar multiple • vector • direction (argument) • magnitude (radius, absolute value)	• Learners may not put the vector in component form first when performing operations (addition, subtraction, scalar). (ME)

Courses

Precalculus
Trigonometry/Analytic Geometry
AP Precalculus
Linear Algebra
Abstract Algebra
Analytic Geometry

ELEVENTH AND TWELFTH GRADE

Number and Operations (NO)

Learners will develop a foundational understanding of the number system, operations, and computational fluency to create connections and solve problems within and across concepts.

(+) 11-12.NO.19 Represent data in a matrix.
Perform operations (e.g., addition, subtraction, multiplication) on matrices of appropriate dimensions to solve problems and in context.
Know that matrix multiplication is not commutative.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.GM.14	(+) 11-12.AR.18 (+) 11-12.AR.19	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Use arrays to model repeated addition and multiplication problems. • Perform operations on real numbers. • Solve systems of equations in two variables. • Represent data in a table.	• matrix	• Learners might reverse the rows and columns in a matrix when performing operations on matrices. (ME) • Learners may not check whether the number of columns in the first matrix is equal to the number of rows in the second matrix before determining whether the matrices can be multiplied together. (ME)

Courses

College Algebra
Precalculus
Trigonometry/Analytic Geometry
AP Precalculus
Integrated Mathematics for Computer Science/Information Technology
Linear Algebra
Abstract Algebra

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

11-12.AR.1* Rearrange multi-variable formulas to highlight a quantity of interest.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.1 9-10.AR.2 9-10.AR.6 9-10.AR.F.1*	11-12.AR.2 11-12.AR.3* 11-12.AR.4* 11-12.AR.5 11-12.AR.6 11-12.AR.F.3* 11-12.AR.F.6* 11-12.AR.F.14*	(+) 11-12.AR.10 (+) 11-12.AR.F.23* (+) 11-12.AR.F.24

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Perform operations on rational numbers to generate equivalent expressions. • Rewrite expressions by factoring, combining like terms, using factoring techniques, applying properties of operations, recognizing patterns, and recognizing structures in expressions. • Apply strategies to isolate a variable when solving equations.	• literal equation	• Learners might not recognize patterns or structure when rewriting expressions. (ME) • Learners may not perform the correct inverse operations when solving for a specific variable. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics for Computer Science/Information Technology
Integrated Mathematics II

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

11-12.AR.2 Use the structure of an expression (to extend to polynomial and rational expressions) to identify ways to rewrite it.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.1 9-10.NO.2 9-10.AR.1 9-10.AR.2	11-12.NO.1 11-12.NO.2 11-12.AR.1* 11-12.AR.4* 11-12.AR.5 11-12.AR.6 11-12.AR.F.3* 11-12.AR.F.4* 11-12.AR.F.5* 11-12.AR.F.6* 11-12.AR.F.14*	(+) 11-12.NO.12 (+) 11-12.AR.20

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Evaluate numerical and algebraic expressions noting exponents, integers, coefficients, and grouping symbols (order of operations). • Rewrite expressions by factoring, combining like terms, using factoring techniques, applying properties of operations, recognizing patterns, and recognizing structures in expressions.	• expression • polynomial • rational	• Learners might not consider appropriate methods for rewriting expressions, e.g., factoring, patterns, and structure. (ME) • Learners might not consider applying powering properties to rewrite an expression or to manipulate an equation to make it easier to solve. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics II

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

11-12.AR.3* Interpret expressions that represent a quantity in context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.3* 9-10.AR.4* 9-10.AR.F.11*	11-12.AR.1 11-12.AR.4* 11-12.AR.5 11-12.AR.7* 11-12.AR.8* 11-12.AR.9* 11-12.AR.F.3*	(+) 11-12.AR.10

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Evaluate numerical and algebraic expressions, noting exponents, integers, variables, and grouping symbols (order of operations). • Rewrite expressions by factoring, combining like terms, using factoring techniques, applying properties of operations, recognizing patterns, and recognizing structures in expressions.	• expression • term • factor • coefficient	• Learners might not recognize the structure in an expression when attempting to simplify it. (ME) • Learners may not use advanced factoring strategies, expanding the expression rather than simplifying it. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics for Computer Science/Information Technology
Integrated Mathematics II

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

11-12.AR.4* Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Use the properties of exponents to transform exponential expressions.
c. Complete the square in a quadratic expression to produce an equivalent expression.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.1 9-10.NO.2 9-10.AR.1 9-10.AR.4* 9-10.AR.F.6* 9-10.AR.F.9*	11-12.NO.1 11-12.NO.2 11-12.NO.3 11-12.NO.6 11-12.NO.7 11-12.NO.8 11-12.NO.9 11-12.AR.1* 11-12.AR.2* 11-12.AR.3* 11-12.AR.5 11.12.AR.6 11-12.AR.7 11-12.AR.13 11-12.AR.14 11-12.AR.15 11-12.AR.F.3* 11-12.AR.F.4* 11-12.AR.F.5* 11-12.AR.F.15* 11-12.GM.1 11-12.GM.2*	(+) 11-12.NO.10 (+) 11-12.NO.12 (+) 11-12.NO.13 (+) 11-12.AR.10 (+) 11-12.AR.11 (+) 11-12.AR.F.23*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Evaluate numerical and algebraic expressions, noting exponents, integers, variables, and grouping symbols (order of operations). • Identify terms, factors, constants, and coefficients in an expression. • Recognize equivalent expressions. • Determine the key features of a given expression and recognize the form of the expression (vertex form of a quadratic). • Identify the zeros of a function by factoring. • Factor expressions and combine like terms. • Apply the properties of exponents to rewrite expressions and equations. • Complete the square to rewrite a quadratic equation. • Identify exponential growth or decay in a given exponential	• equivalent form • quadratic function • zero of a function • complete the square • maximum • minimum • vertex • exponent • exponential • rate of growth or decay	• Learners may need to find zeros using multiple strategies (graphing, factoring, long division). (ME) • Learners may not recognize when an exponential expression or equation can be rewritten in such a way that factoring becomes an option for simplifying or solving. (ME)

expression. • Identify the initial value and growth/decay factor in a given exponential expression.		

Courses
Algebra II College Algebra Geometry/Trigonometry/Advanced Algebra Integrated Mathematics II

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

11-12.AR.5 Add, subtract, multiply, and divide rational expressions.
Understand that rational expressions form a system analogous to rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.1 9-10.NO.2 9-10.AR.1 9-10.AR.11	11-12.NO.1 11-12.NO.2 11-12.NO.3 11-12.AR.1 11-12.AR.2 11-12.AR.4* 11-12.AR.6 11-12.AR.13 11-12.AR.15 11-12.AR.F.3* 11-12.AR.F.4*	(+) 11-12.NO.11 (+) 11-12.NO.12 (+) 11-12.NO.17 (+) 11-12.NO.18 (+) 11-12.AR.F.24

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Distinguish numbers within the complex number system. • Add, subtract, multiply, and divide rational numbers. • Recognize equivalent expressions. • Recognize how closure applies when simplifying an expression.	• rational expression • nonzero rational expression • closure	• Learners may perform operations on rational expressions incorrectly, e.g., neglecting a common denominator when adding or subtracting. (ME) • Learners may not rewrite the square root of a negative number in an a + bi form before simplifying an expression. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

11-12.AR.6 Rewrite simple rational expressions in different forms. Write a(x)/b(x) in the form of q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or technology for the more complicated examples.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.1	11-12.NO.1 11-12.NO.2 11-12.AR.1 11-12.AR.2 11-12.AR.4* 11-12.AR.5 11-12.AR.13 11-12.AR.14 11-12.AR.15 11-12.AR.F.3* 11-12.AR.F.5*	(+) 11.12.NO.10 (+) 11-12.NO.11 (+) 11-12.NO.12 (+) 11-12.NO.17 (+) 11-12.NO.18 (+) 11-12.AR.F.24

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Distinguish numbers within the complex number system. • Add, subtract, multiply, and divide rational numbers. • Recognize equivalent expressions. • Recognize how closure applies when simplifying an expression. • Apply strategies (factoring, completing the square) to simplify rational expressions. • Apply properties of exponents to simplify a rational expression.	• rational expression • polynomial	• When using long division to divide polynomials, it may be advantageous to “change the signs and add” when subtracting like terms throughout the division process. (IN) • Learners may not subtract like terms from each other when performing long division. (ME) • Learners may not use the Quotient Remainder Theorem in determining the quotient, remainder, or dividend when given two of the three. (ME) • Learners might not recognize when the degree of their remainder is larger than the degree of the divisor, resulting in an incorrect remainder. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*
11-12.AR.7* Create equations and inequalities and use them to solve problems. Include equations arising from linear and quadratic equations and simple rational and exponential equations.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.AR.3* 9-10.AR.4* 9--10.AR.6 9-10.AR.8 9-10.AR.10 9-10.AR.F.10*	11-12.AR.2 11-12.AR.3* 11-12.AR.8* 11-12.AR.9* 11-12.AR.12 11-12.AR.F.5* 11-12.AR.F.6* 11-12.AR.F.15* 11-12.GM.1 11-12.GM.3	(+) 11-12.AR.11
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish numbers within the complex number system. • Solve equations and inequalities in one variable. • Identify equations by their type (linear, quadratic, exponential, polynomial).	• linear equations • quadratic equations • rational equations • exponential equations • inequalities	• Learners may not use the test point method when determining solutions to inequalities that are not linear. (ME) • Learners may not distinguish between a quadratic and an exponential model given data. (ME)

Courses
Algebra II College Algebra Geometry/Trigonometry/Advanced Algebra Integrated Mathematics for Computer Science/Information Technology Integrated Mathematics II Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

11-12.AR.8* Create equations in two or more variables to represent relationships between quantities. Graph equations on coordinate axes with proper labels and scales.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.3 9-10.NO.4* 9-10.NO.5 9-10.AR.3* 9-10.AR.4* 9-10.AR.7* 9-10.AR.8 9-10.AR.F.10*	11-12.AR.3* 11-12.AR.7* 11-12.AR.8* 11-12.AR.9* 11-12.AR.12 11-12.AR.17 11-12.AR.F.1* 11-12.AR.F.5* 11-12.AR.F.6* 11-12.GM.1	(+) 11-12.AR.F.21

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify equations by their type (linear, quadratic, exponential, polynomial). • Write linear, quadratic, exponential, and polynomial equations. • Graph simple equations in one or two variables.	• coordinate plane • scale	• Use of graphing tools or software will help verify hand-drawn graphs. (IN) • Learners may not correctly understand the relationships between the quantities, causing errors in the formulation (e.g., $y = 2^x$ may be perceived as $y=2x^{2)}$. (ME) • Learners may not ensure the units of both sides of the equation match, leading to errors in their equations. (ME) • Learners may not label the x-axis and y-axis, making it unclear what the graph represents. (ME) • Learners might distort graphs by selecting a scale that is too large or small. (ME) • Learners might incorrectly assume relationships are linear. (ME) • When graphing, learners may not plot enough points to accurately capture the shape of the graph. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics for Computer Science/Information Technology
Integrated Mathematics II
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

11-12.AR.9* Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.3 9-10.NO.4* 9-10.NO.5 9-10.AR.6 9-10.AR.7* 9-10.AR.8 9-10.AR.9	11-12.NO.5* 11-12.AR.3* 11-12.AR.7* 11-12.AR.8* 11-12.AR.16 11-12.AR.17 11-12.AR.F.7* 11-12.AR.F.8* 11-12.AR.F.9*	(+) 11-12.AR.18 (+) 11-12.AR.19 (+) 11-12.AR.F.11*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify equations by their type (linear, quadratic, exponential, polynomial). • Write linear, quadratic, exponential, and polynomial equations. • Graph simple equations in one or two variables. • Solve linear and quadratic equations and inequalities.	• linear programming • constraint • feasible region	• Learners may confuse the inequality symbols. (ME) • Learners might exclude the boundary points, e.g., ≤ or ≥, in cases where inequalities should include boundary points. (ME) • Learners might overlook additional constraints given in the problem, leading to incomplete systems of equations or inequalities. (ME) • Learners may assign the incorrect variables to quantities leading to incorrect constraints. (ME) • Learners might make errors in the steps of elimination or substitution methods used to solve systems of equations. (ME) • Learners might make errors in shading the correct region for inequalities or fail to identify and shade the feasible region satisfying all constraints. (ME) • Learners might not check whether the solutions make sense in the context of the problem. (ME) • Learners may not recognize when a system of equations or inequalities has no solution. (ME) • Learners may not consider practical limitations such as maximum capacities or minimum requirements. (ME) • Learners might misinterpret the units of the quantities involved, e.g., square units vs. linear

		units. (ME)

Courses
Algebra II College Algebra Geometry/Trigonometry/Advanced Algebra Integrated Mathematics for Computer Science/Information Technology Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*
(+) 11-12.AR.10 Derive the quadratic formula from the form $0 = ax^2 + bx + c$.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.AR.1 9-10.AR.3* 9-10.AR.10 11-12.NO.9 11-12.AR.1* 11-12.AR.3* 11-12.AR.4* 11-12.AR.7* 11-12.AR.12	(+) 11-12.NO.13 (+) 11-12.AR.11	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Calculate the square root of rational numbers. • Solve a quadratic equation by inspection. • Apply the quadratic formula to solve quadratic equations. • Complete the square to solve a quadratic equation.	• completing the square • quadratic formula	• Learners may not factor (or divide out) the a-term when preparing to complete the square. (ME) • Learners may complete the square incorrectly. (ME) • Learners might not maintain the property of equality upon completing the square. (ME) • Learners may not use common denominators when combining terms. (ME) • Learners might omit the ± when taking the square root of both sides. (ME) • Learners might make errors in isolating the variable x, e.g., simplification or sign errors. (ME)

Courses
College Algebra Geometry/Trigonometry/Advanced Algebra Precalculus Integrated Mathematics II

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*
(+) 11-12.AR.11 Solve quadratic equations with real coefficients that have solutions of the form a + bi and a - bi.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.AR.5 9-10.AR.10 11-12.NO.2 11-12.NO.3 11-12.NO.6 11-12.NO.7 11-12.NO.8 11-12.NO.9 11-12.AR.4* 11-12.AR.7*	(+) 11-12.NO.11 (+) 11-12.NO.13 (+) 11-12.AR.10 11-12.AR.14 11-12.AR.15	

Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish numbers within the complex number system. • Perform operations on complex numbers. • Recognize complex conjugate numbers. • Calculate the square root of rational numbers. • Solve a quadratic equation by inspection. • Apply the quadratic formula to solve quadratic equations. • Complete the square to solve a quadratic equation.	• completing the square • quadratic formula • coefficients	• Learners might misinterpret the discriminant or fail to recognize that a negative discriminant indicates complex solutions. (ME) • Learners might incorrectly simplify the square root of a negative number, often forgetting the imaginary unit i. (ME) • Learners might make computational errors in applying the quadratic formula. (ME) • Learners might make errors while performing operations on complex numbers. (ME) • Learners might omit the ± in the solutions. (ME)

Courses
Algebra II College Algebra Geometry/Trigonometry/Advanced Algebra Abstract Algebra

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

11-12.AR.12 Solve simple rational and radical equations in one variable and identify extraneous solutions.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.1 9-10.NO.2 9-10.AR.6 9-10.AR.10	11-12.NO.1 11-12.NO.2 11-12.NO.7 11-12.AR.7* 11-12.AR.8* 11-12.AR.14 11-12.AR.15	(+) 11-12.AR.10 (+) 11-12.NO.11 (+) 11-12.NO.13

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Distinguish numbers within the complex number system. • Perform operations on complex numbers. • Recognize complex conjugate numbers. • Differentiate between rational and radical equations. • Apply properties of exponents to determine when it is appropriate to use rational exponents or radicals when simplifying expressions or solving equations. • Rearrange formulas to isolate a quantity or variable of interest. • Identify extraneous solutions.	• extraneous solutions • rational equation • radical equation	• Learners may not consider domain restrictions when using cross-multiplication as a method of solution. (ME) • Learners may not consider non-reversible steps when solving, resulting in extraneous solutions. (ME) • Learners may not check if the solutions satisfy the original equation. (ME) • Learners may not properly isolate the radical before squaring both sides when solving a radical equation. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

11-12.AR.13 Add, subtract, and multiply polynomials beyond quadratics.
Understand that polynomials form a system comparable to integers, namely, they are closed under the operations of addition, subtraction, and multiplication.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.1 9-10.AR.11	11-12.NO.7 11-12.NO.8 11-12.AR.4* 11-12.AR.5 11-12.AR.6 11-12.AR.14 11-12.AR.15 11-12.AR.F.10*	(+) 11-12.NO.11 (+) 11-12.NO.12 (+) 11-12.AR.20

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Perform operations on complex numbers. • Identify coefficients, exponents, constants, and bases in an expression or equation. • Apply strategies (combine like terms, isolate a variable, distribute, apply properties of exponents) to rewrite expressions and equations.	• closure • polynomial • quadratic • integer • system	• Learners may not identify and combine like terms when performing operations on polynomials. (ME) • Learners may not distribute the negative sign across all terms of a polynomial that is being subtracted from another polynomial. (ME) • Learners might apply the exponent properties incorrectly when multiplying polynomials. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*
11-12.AR.14 Identify zeros of polynomial equations when suitable factorizations are available. Use the zeros to construct a rough graph of the function defined by the polynomial.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.AR.1 9-10.AR.10	11-12.NO.9 11-12.AR.4* 11-12.AR.5 11-12.AR.6 11-12.AR.11 11-12.AR.12 11-12.AR.13 11-12.AR.15 11-12.AR.F.4* 11-12.AR.F.10*	(+) 11-12.NO.12 (+) 11-12.NO.13 (+) 11-12.AR.F.11*

Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Perform operations on complex numbers. • Identify coefficients, exponents, constants, bases and the degree in an expression or equation. • Apply strategies (combine like terms, isolate a variable, distribute, factor, apply properties of exponents) to rewrite expressions and equations. • Recognize zeros of a polynomial. • Use the leading coefficient and degree to determine the end behavior of a polynomial. • Identify the zeros of a polynomial as simple or having a multiplicity.	• zeros of a polynomial • polynomial equations • multiplicity	• Learners might incorrectly factor the polynomial by missing a factor, using the incorrect factor, or failing to completely factor the polynomial. (ME) • Learners might misapply the zero-product property by not setting each factor equal to zero. (ME) • Learners may not recognize and account for zeros with multiplicities. (ME) • Learners may not consider roots that are complex conjugate pairs when the polynomial has real coefficients. (ME)

Courses
Algebra II College Algebra Geometry/Trigonometry/Advanced Algebra Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

11-12.AR.15 Apply the Factor and Remainder Theorems to determine efficiently whether a linear expression is a factor of a polynomial equation.
Apply the Remainder Theorem in context.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.1 9-10.AR.F.6*	11-12.NO.9 11-12.AR.4* 11-12.AR.6 11-12.AR.11 11-12.AR.12 11-12.AR.13 11-12.AR.14 11-12.AR.F.3	(+) 11-12.NO.13

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Perform operations on complex numbers. • Identify coefficients, exponents, constants, bases and the degree in an expression or equation. • Apply strategies (combine like terms, isolate a variable, distribute, factor, apply properties of exponents) to rewrite expressions and equations. • Recognize zeros of a polynomial.	• Remainder Theorem • zero of a polynomial • linear expression	• Ensure learners have a clear understanding of the Factor and Remainder Theorems and are able to distinguish between them (e.g., the Factor Theorem tells us that if a is a zero of polynomial f(x), then x-a is a factor of f(x); the Remainder Theorem tells us for any polynomial f(x) if you divide it by the binomial x−a, the remainder is equal to the value of f(a)). (IN) • Learners may attempt to use the Factor Theorem and Remainder Theorem interchangeably. (ME) • Learners might substitute an incorrect value into the polynomial when using the theorems. (ME) • Learners might incorrectly perform polynomial division when using the Remainder Theorem to find the remainder. (ME) • Learners may check only one potential factor and conclude the factorization is complete. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Precalculus
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

11-12.AR.16 Using graphs, technology, tables, or successive approximations, show that the solution(s) to the equation f(x) = g(x) are the x- value(s) that result in the y-values of f(x) and g(x) being the same.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.8 9-10.AR.9 9-10.AR.10 9-10.AR.F.12	11-12.AR.9 11-12.AR.F.1* 11-12.AR.F.3* 11-12.AR.F.4* 11-12.AR.F.7* 11-12.AR.F.10*	(+) 11-12.AR.F.11*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Determine whether a relation is a function. • Interpret function notation when problem-solving. • Compare key features (domain, range, asymptotes, zeros, intercepts, end behavior) of functions represented algebraically, numerically in tables, or graphically.	• intersection • relation • function	• Learners may misinterpret where the graphs of f(x) and g(x) intersect. (ME) • Learners may not consider the domain and range of the functions leading to missing intersection points or including extraneous points. (ME) • Learners might misuse graphing calculators or software or input data incorrectly leading to incorrect graphs or data tables. (ME) • Learners may create tables of values for f(x) and g(x) incorrectly leading to wrong conclusions about the solutions. (ME) • Learners might misinterpret the data in tables. (ME) • Learners may not refine the x values sufficiently when using successive approximations. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics for Computer Science/Information Technology
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

11-12.AR.17 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.6 9-10.AR.7* 9-10.AR.8 9-10.AR.10	11-12.AR.8* 11-12.AR.9*	(+) 11-12.AR.18 (+) 11-12.AR.19

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Represent linear relationships algebraically, graphically, and numerically in tables. • Write a linear equation in standard and slope-intercept form. • Write a quadratic equation in standard and vertex form. • Identify coefficients, exponents, constants, bases, and the degree in an expression or equation. • Solve and graph linear and quadratic equations algebraically and graphically.	• system of equations • linear equation • quadratic equation	• Learners might not substitute correctly when isolating one variable from the linear equation and substituting it into the quadratic equation. (ME) • Learners might make simplification errors such as incorrectly expanding or combining like terms. (ME) • Learners may not set the resulting quadratic equation to zero before solving. (ME) • Learners might misapply the quadratic formula. (ME) • Learners might ignore one of the two possible solutions of the quadratic equation. (ME) • Learners may not correctly identify where the linear and quadratic graphs intersect. (ME) • When graphs intersect at more than one point, learners may miss one or more of the intersection points. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics II

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*
(+) 11-12.AR.18 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.AR.F.10 11-12.AR.8* 11-12.AR.9 11-12.AR.17 11-12.AR.F.5*	(+) 11-12.NO.19 (+) 11-12.AR.19	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Solve systems of linear equations. • Find the inverse of a linear function. • Represent data in a matrix. • Perform operations (addition, subtraction, scalar) on matrices with and without technology.	• coefficient matrix • inverse matrix • system of linear equations	• Learners might neglect to compute the determinant when finding a 2x2 inverse matrix. (ME) • Learners may not verify the computed matrix to ensure it is the inverse. (ME) • Learners may not recognize when a matrix has no inverse. (ME) • Learners might not switch the a and d values and/or change the signs on the b and c values when determining a 2x2 inverse matrix. (ME)

Courses
College Algebra Precalculus Linear Algebra

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR) *Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.*
(+) 11-12.AR.19 Solve a system of equations in three or more variables with matrices (using technology).

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.AR.7* 11-12.AR.8* 11-12.AR.9 11-12.AR.17 11-12.AR.F.5*	(+) 11-12.NO.19 (+) 11-12.AR.18	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Solve systems of linear equations. • Find the inverse of a linear function. • Represent data in a matrix. • Perform operations (addition, subtraction, scalar) on matrices with and without technology. • Calculate the determinant of a square matrix. • Find the inverse of a square matrix (if it exists).	• inverse matrices • determinant • system	• Learners might incorrectly input the coefficient and/or constant matrix into the software or calculator. (ME) • Learners may not use the inverse of the coefficient matrix when solving a system using matrices. (ME) • Learners may not understand how to use the specific functions or commands for solving matrix equations. (ME) • Learners may not recognize when a system results in a degenerate or singular matrix with no unique solution. (ME) • Learners might misinterpret the output provided by technology, especially when it involves complex matrices or solutions. (ME)

Courses
College Algebra Precalculus Linear Algebra

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

(+) 11-12.AR.20 Apply the Binomial Theorem for the expansion of $(ax + by)^n$ in powers of x and y for a positive integer n and integers a and b.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.1* 11-12.AR.2 11-12.AR.13	(+) 11-12.NO.12	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Perform operations on complex numbers and polynomials. • Apply powering properties to simplify expressions. • Apply the nth row of Pascal's Triangle to determine binomial coefficients.	• Pascal's Triangle • Binomial Theorem	• Learners might misapply the Binomial Theorem and Pascal's Triangle when expanding $(ax + by)^n$. (ME) • Learners may calculate binomial coefficients incorrectly or may not recognize their role in the expansion. (ME) • Learners may not consider all terms. (ME) • Learners may not simplify terms fully after expanding. (ME) • Learners might misapply properties of exponents when expanding terms involving powers of ax and by. (ME)

Courses

College Algebra
Probability and Statistics
Precalculus
AP Statistics
Discrete Mathematics

ELEVENTH AND TWELFTH GRADE

Algebraic Reasoning (AR)

Learners will look for, generate, and make sense of patterns, relationships, and algebraic symbols to represent mathematical models while adopting approaches and solutions in novel situations.

Functions (F)

Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.1* Write a function that describes a relationship between two quantities.

a. Combine standard function types using arithmetic operations.
b. Compose functions.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9.AR.3* 9-10.AR.F.1 9-10.AR.F.2* 9-10.AR.F.4* 9-10.AR.F.6*	11-12.AR.8* 11-12.AR.16 11-12.AR.F.3* 11-12.AR.F.4* 11-12.AR.F.7* 11-12.AR.F.10* 11-12.AR.F.13* 11-12.AR.F.14* 11-12.AR.F.16 11-12.AR.F.17	(+) 11-12.AR.F.11*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Determine whether a relation is a function. • Interpret function notation when problem-solving. • Perform operations on expressions.	• composition of functions • function	• Learners might reverse the order of composition or may misunderstand that the output of one function becomes the input of the other. (ME) • Learners may not recognize the composition of functions given different types of notations, e.g., f(g(x)) vs. $(f \circ g)(x)$). (ME) • Learners might assume the composition of two increasing functions is always increasing. • Learners might misinterpret the relationship between the two quantities, e.g., assume a linear relationship that is actually exponential. (ME) • Learners might overlook restrictions on the domain and range when performing a composition of two functions. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics for Computer Science/Information Technology
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.2* Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.F.4* 9-10.AR.F.5* (+) 9-10.GM.24	11-12.AR.F.8* 11-12.AR.F.13* 11-12.AR.F.14*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify slope as the average rate of change of the secant line between two points. • Identify the slope and y-intercept in tables, graphs, equations, and descriptions. • Analyze the graph of a function to determine if the function is constant, increasing, decreasing, linear, nonlinear, discrete, or continuous.	• rate of change • interval • secant line	• Learners may select the incorrect interval for calculating the average rate of change. (ME) • Learners might leave the fractional or decimal expressions unsimplified when calculating the average rate of change. (ME) • Learners may not recognize that secant lines on a graph represent average rates of change over intervals. (ME) • Learners might omit units of measure when interpreting the rate of change. (ME)

Courses

Algebra II
College Algebra
Precalculus
Calculus
Geometry/Trigonometry/Advanced Algebra
AP Calculus AB
AP Calculus BC
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.3* Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum, and symmetry of the graph, and interpret these in terms of context.
b. Use the properties of exponents to interpret expressions for exponential functions.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.1 9-10.NO.2 9-10.AR.1 9-10.AR.4* 9-10.AR.10 9-10.AR.F.2* 9-10.AR.F.4* 9-10.AR.F.6* 9-10.AR.F.9*	11-12.NO.1 11-12.NO.2 11-12.NO.9 11-12.AR.1* 11-12.AR.2 11-12.AR.3* 11-12.AR.4* 11-12.AR.5 11-12.AR.6 11-12.AR.15 11-12.AR.16 11-12.AR.F.1* 11-12.AR.F.6* 11-12.AR.F.7* 11-12.AR.F.8* 11-12.AR.F.10* 11-12.DPS.4*	(+) 11-12.AR.F.11* (+) 11-12.AR.F.21

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Interpret the structure of an expression to determine how to rewrite it. • Perform operations on functions. • Apply strategies (factoring, completing the square) to find key features (vertex, zeros, maximums, minimums, symmetry) of a quadratic function. • Apply properties of exponents to simplify expressions. • Distinguish between linear, exponential, and quadratic functions. • Interpret exponential functions as growth or decay.	• exponential growth • exponential decay • extreme values • minimum • maximum • quadratic function • factoring • completing the square • symmetry	• Learners may not recognize equivalent forms of a function or may misunderstand how different representations are related. (ME) • Learners might leave expressions unsimplified or not fully factored when attempting to reveal all properties. (ME) • Learners may not complete the square correctly, leading to errors in identifying the vertex or axis of symmetry. (ME) • Learners might incorrectly factor quadratic expressions. (ME) • Learners might misinterpret the significance of zeros and the extrema of the function in relation to the graph. (ME) • Learners might misinterpret the rules of exponents when simplifying exponential expressions or equations. (ME) • Learners might misinterpret the behavior of exponential functions when observing growth and decay rates. (ME)

Courses
Algebra II
College Algebra
Precalculus
Calculus
Geometry/Trigonometry/Advanced Algebra
AP Calculus AB
AP Calculus BC
Integrated Mathematics II

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.4* Identify the effect of transformations on the graph of a function by replacing f(x) with af(x), f(bx), f(x - h), and f(x) + k, for specific values of a, h, and k (both positive and negative).
Find the value of a, b, h, and k given the graph of the function.

Recognize even and odd functions from their graphs and equations.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.F.2* 9-10.AR.F.3* 9-10.AR.F.4* 9-10.AR.F.9* 9-10.AR.F.11* 9-10.GM.2 9-10.GM.3 9-10.GM.4 9-10.GM.5 9-10.GM.6 9-10.GM.14 9-10.GM.15 9-10.GM.16 9-10.GM.17	11-12.AR.2 11-12.AR.4* 11-12.AR.5 11-12.AR.14 11-12.AR.16 11-12.AR.F.1* 11-12.AR.F.10*	(+) 11-12.AR.F.11* (+) 11-12.AR.F.19 (+) 11-12.AR.F.21

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Recognize function notation for reflection, translation, size change, and scale change. • Recognize transformations of functions given the parent graph. • Recognize transformations of a function given its graph. • Apply reflections to a graph of a function. • Apply size and scale factors to a graph of a function. • Apply translations to the graph of a function.	• transformation • even functions • odd functions	• Learners may confuse the representation of a, b, c, and d in the equation of a function with its graph. (ME) • Learners may not recognize the leading coefficient's value in determining a reflection in addition to a stretch/ compression. (ME) • Learners might incorrectly translate a graph vertically or horizontally instead of stretching/compressing. (ME) • Learners may confuse horizontal translations (right or left). (ME) • Learners may confuse vertical translations (up or down). (ME) • Learners might misinterpret horizontal translations as vertical and vice versa. (ME) • Learners might misinterpret the scale factor and incorrectly determine the magnitude of a stretch or compression. (ME) • Learners might not determine reflections across the x or y-axis (e.g., not look at the sign of a and/or b-values). (ME) • Learners may not recognize symmetry and mistakenly identify an even function as odd or neither. They may not recognize rotation symmetry

		about the origin and mistakenly identify an odd function as even or neither. (ME)

Courses
Algebra II
College Algebra
Precalculus
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.5* Find inverse functions.
a. Verify by composition that one function is the inverse of another.
b. Recognize that the graph of a function and its inverse are reflection images over the line y = x.
c. Produce an invertible function from a non-invertible function by restricting the domain.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.F.1 9-10.AR.F.4* 9-10.AR.F.10*	11-12.AR.2 11-12.AR.6 11-12.AR.7* 11-12.AR.F.8* 11-12.AR.F.4* 11-12.AR.F.6*	(+) 11-12.AR.18 (+) 11-12.AR.19 (+) 11-12.AR.F.22 (+) 11-12.AR.F.23*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Analyze the relationship between independent and dependent variables in a function. • Find the domain and range of a function. • Compose functions. • Identify a one-to-one function using the horizontal line test on a given function. • Solve for a specific variable given a function.	• inverse functions • independent variable • dependent variable • one-to-one function • invertible function • composition	• Learners might verify only one composition and neglect the other. (ME) • Learners may not recognize or accurately graph the reflection image over the line y=x. (ME) • Learners might mistake other forms of symmetry, e.g., about the y-axis or origin, in determining whether two functions are inverses. (ME) • Learners might misidentify the corresponding points between the function and its inverse. (ME) • Learners may choose an incorrect domain in attempting to produce a 1-1 function. (ME) • Learners may restrict the domain without ensuring the function is strictly increasing or decreasing. (ME) • Learners may not adjust the range accordingly when the domain is restricted. (ME)

Courses
Algebra II
College Algebra
Precalculus
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics for Computer Science/Information Technology
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.6* Apply the inverse relationship between exponents and logarithms to solve problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.1 9-10.AR.F.4* 9-10.AR.F.6* 9-10.AR.F.10*	11-12.NO.1 11-12.NO.2 11-12.AR.1* 11-12.AR.2 11-12.AR.7* 11-12.AR.8* 11-12.AR.F.3* 11-12.AR.F.5* 11-12.AR.F.10* 11-12.AR.F.15*	(+) 11-12.AR.F.23*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Apply properties of logarithms to rewrite expressions and equations. • Isolate the base term in an exponential equation to create equivalent equations. • Apply the properties of exponents to rewrite expressions and equations. • Determine the inverse of a function. • Identify the base number and exponent. • Rewrite logarithms in exponential form and vice-versa. • Evaluate logarithmic expressions.	• logarithm • exponents • inverse	• Learners might make mistakes when converting between logarithmic and exponential forms. (ME) • Learners might misapply logarithmic properties to include the product rule, the quotient rule, the power rule, and the change of base rule. (ME) • Learners may confuse the bases of logarithms and exponents. (ME) • Learners might input log and ln values incorrectly into a calculator. (ME) • Learners might overlook that logarithms are undefined for non-positive numbers and try to compute logarithms of zero and negative numbers. (ME) • Learners might ignore the domains of the functions involved, especially when dealing with the inverse nature of logarithms and exponents. (ME) • Learners might not isolate the exponential or logarithmic term before applying the inverse operation when solving equations. (ME)

Courses

Algebra II
College Algebra
Precalculus
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.7* Compare key features of two functions each represented in a different way (algebraically, graphically, numerically, in tables, or by verbal descriptions).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.F.1 9-10.AR.F.3* 9-10.AR.F.4* 9-10.AR.F.7* 9-10.AR.F.9* 9-10.AR.F.11*	11-12.AR.9* 11-12.AR.16 11-12.AR.F.1* 11-12.AR.F.3* 11-12.AE.F.8* 11-12.AR.F.9* 11-12.AR.F.10* 11-12.AR.F.12* 11-12.AR.F.13* 11-12.AR.F.14*	(+) 11-12.AR.F.11* (+) 11-12. AR.F.20 (+) 11-12.AR.F.21*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Determine whether a relation is a function. • Identify the key features (domain, range, intercepts, rate of change, interval increase/decrease, maximum/ minimum, symmetry, end behavior, periodicity) of functions.	• domain • range • intercepts • symmetry • end behavior • periodicity	• Learners might incorrectly identify features such as intercepts, slopes, maxima, minima, or asymptotes from a graph. (ME) • Learners may misunderstand the different forms of a function and the features each reveals. (ME) • Learners may struggle to find and compare key features of two functions represented in different ways. (ME) • Learners might ignore the domain and range restrictions of the functions when comparing them. (ME) • Learners might misidentify or overlook key features such as intercepts, intervals of increase/decrease, maximum/ minimum values, asymptotic behavior, and end behavior. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics for Computer Science/Information Technology
Integrated Mathematics II
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.8* Use tables, graphs, verbal descriptions, and equations to interpret and sketch the key features of a function modeling the relationship between two quantities.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.3 9-10.NO.4 9-10.AR.5 9-10.AR.F.1 9-10.AR.F.3* 9-10.AR.F.4* 9-10.AR.F.6* 9-10.AR.F.11* 9-10.DPS.10*	11-12.AR.9* 11-12.AR.F.2* 11-12.AR.F.3* 11-12.AR.F.7* 11-12.AR.F.9* 11-12.AR.F.10* 11-12.AR.F.12* 11-12.AR.F.13* 11-12.AR.F.14* 11-12.GM.1 11-12.GM.2*	(+) 11-12.AR.F.11* (+) 11-12.AR.F.21* (+) 11-12.AR.F.22

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Determine whether a relation is a function. • Identify the key features (domain, range, intercepts, rate of change, interval increase/decrease, maximum/ minimum, symmetry, end behavior, periodicity) of functions given tables, graphs, verbal descriptions, and equations. • Use inequality or interval notation to indicate where functions are increasing, decreasing, positive, or negative.	• intercepts • relative maximum • relative minimum • end behavior • periodicity • symmetry • domain • range	• Learners might incorrectly find or plot the x-intercepts (roots) and y-intercepts. (ME) • Learners might miscalculate the vertex of a quadratic function. (ME) • Learners might misinterpret the direction a parabola opens. (ME) • Learners may neglect to find the vertical or horizontal asymptotes, especially for rational functions. (ME) • Learners might misidentify where a function is increasing or decreasing. (ME) • Learners may not consider a function's domain and range. (ME) • Learners may not recognize symmetry in functions. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics II
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.9* Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.3 9-10.NO.4* 9-10.AR.F.1 9-10.AR.F.2* 9-10.AR.F.4*	11-12.AR.9 11-12.AR.F.7* 11-12.AR.F.8* 11-12.AR.F.13* 11-12.AR.F.14*	(+) 11-12.AR.F.11* (+) 11-12.AR.F.22

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Determine whether a relation is a function. • Apply the Vertical Line Test to a graph to determine whether it is a function. • Determine the domain and range of a function. • Identify the key features given a graph of a function.	• domain • range	• Learners may not recognize x-values where the function is undefined, e.g., removable and non-removable discontinuities. (ME) • Learners may not consider the context of the problem, which could impose additional domain restrictions. (ME) • Learners may not correctly identify the domain of a piecewise function. (ME) • Learners might overlook the need to consider the domain of each function involved in a composition of functions. (ME) • Learners may not recognize implicit domain restriction for some functions, e.g., logarithmic and square root functions. (ME)

Courses

Algebra II
College Algebra
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics for Computer Science/Information Technology
Integrated Mathematics II
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.10* Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases.

a. Graph square root, cube root, piecewise-defined, step, and absolute value functions.
b. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior.
c. Graph exponential and logarithmic functions, showing intercepts and end behavior.
d. Graph f(x) = sin x and f(x) = cos x as representations of periodic phenomena.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.3 9-10.NO.4 9-10.AR.9 9-10.AR.F.1 9-10.AR.F.3 9-10.AR.F.7 9-10.AR.F.10	11-12.NO.9 11-12.AR.13 11-12.AR.14 11-12.AR.16 11-12.AR.F.1* 11-12.AR.F.3* 11-12.AR.F.4* 11-12.AR.F.6* 11-12.AR.F.7* 11-12.AR.F.8* 11-12.AR.F.12* 11-12.AR.F.13* 11-12.AR.F.15* 11-12.GM.1 11.12.GM.2*	(+) 11-12.AR.F.11 (+) 11-12.AR.F.20 (+) 11-12 AR.F.21*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Recognize square root, cube root, piecewise-defined, step, polynomial, exponential, logarithmic, sine and cosine, and absolute value function equations and graphs. • Apply the vertical line test to identify functions on a graph. • Add, subtract, and multiply polynomials. • Identify zeros of polynomials.	• square root function • cube root function • piecewise-defined function • step function • absolute value function • polynomial function • exponential function • logarithmic function • asymptote • period • midline • amplitude	• Learners may not recognize domain restrictions on the square-root function. (ME) • Learners may not recognize the cube root function as symmetrical about the origin and defined for all real numbers. (ME) • Learners might misinterpret or incorrectly apply the domain values for each part of a piecewise-defined function. (ME) • Learners may not correctly graph jumps or holes given the domain intervals of a piecewise function. (ME) • Learners might use open or closed circles incorrectly to indicate inclusive and exclusive endpoints when graphing step functions. (ME) • Learners might incorrectly identify the vertex of an absolute value function. (ME) • Learners might misidentify the zeros (roots) and incorrectly show the end behavior when graphing a polynomial function. (ME) • Learners may not correctly

		identify and graph horizontal and vertical asymptotes and end behavior of logarithmic or exponential functions. (ME) • Learners might incorrectly identify the period, amplitude, phase shift, and vertical shift of a trigonometric function. (ME) • Learners may not find the period of a trigonometric function given the frequency, and vice versa. (ME) • Learners may not apply phase and vertical shifts correctly, and may also misplace key points such as maxima, minima, and intercepts. (ME)

Courses
Algebra II College Algebra Precalculus Geometry/Trigonometry/Advanced Algebra Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

(+) 11-12.AR.F.11* Analyze and graph functions expressed symbolically (by hand in simple cases and using technology for more complicated cases), identifying key features of the graph.

a. (+) Graph rational functions, identifying domain, range, asymptote(s), removable and non-removable discontinuities, intercepts, behavior at the asymptote(s), and end behavior.
b. (+) Graph trigonometric functions, showing period, midline, phase shift, and amplitude.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.3 9-10.NO.4 9-10.AR.F.1 9-10.AR.F.3* 9-10.AR.F.5* 9-10.AR.F.7* 11.12.NO.9 11-12.AR.9* 11-12.AR.14* 11-12.AR.16 11-12.AR.F.1* 11-12.AR.F.3 11-12.AR.F.4* 11-12.AR.F.7 11-12.AR.F.8 11-12.AR.F.9 11-12.AR.F.10 11-12.AR.F.12*	11-12.AR.F.13* 11-12.AR.F.15* (+) 11-12.AR.F.18 (+) 11-12.AR.F.20 (+) 11-12 AR.F.21 (+) 11-12.AR.F.22	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify asymptotes (horizontal, vertical, slant, or oblique) of a rational function on a graph. • Identify key features of functions (domain, range, zeros, intercepts, end behavior). • Graph y=sin x and y=cos x. • Interpret and use information from graphs to identify key features of rational and trigonometric functions.	• rational functions • asymptotes • trigonometric functions • period • midline • phase shift • amplitude • removable and non-removable discontinuities • end behavior • domain • range • asymptotes	• Learners might incorrectly determine vertical, horizontal, or slant asymptotes when analyzing and graphing rational functions. (ME) • Learners might incorrectly identify the domain or range of the function leading to incorrect graphing. (ME) • Learners might incorrectly determine transformations by mis-applying translations, reflections, stretches, and compressions. (ME) • Learners may confuse removable and non-removable discontinuities or fail to recognize them. (ME) • Learners might incorrectly determine the horizontal asymptote or plot it incorrectly, resulting in errors in the end behavior of the function. (ME) • Learners might overlook the function's behavior near the asymptotes and removable discontinuities. (ME) • Learners may confuse the period and frequency of trigonometric functions. (ME)

Courses
College Algebra
Precalculus
Trigonometry
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
AP Precalculus

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.12* Compare the end behavior of linear, quadratic, and exponential functions using graphs and/or tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing as a linear or quadratic function.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.F.4* 9-10.AR.F.5*	11-12.AR.F.7* 11-12.AR.F.8* 11-12.AR.F.10*	(+) 11-12.AR.F.11*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Analyze graphs to describe the functional relationship between two quantities (constant, increasing, decreasing, linear, nonlinear, discrete, continuous). • Identify key features of functions (domain, range, zeros, intercepts, end behavior).	• linear function • quadratic function • exponential function	• Teachers should reinforce that linear functions increase at a constant rate; quadratic functions increase at an accelerating rate and exponential functions increase at a multiplicative rate. (IN) • Learners may not recognize the different rates of increase for linear, quadratic, and exponential functions. (ME) • Learners might misinterpret values in tables and fail to see how exponential growth eventually surpasses linear and quadratic growth. (ME) • Learners may not recognize the impact of coefficients and constants on the function's growth rate and end behavior. (ME) • Learners may not apply knowledge of limits to determine the end behavior of linear, quadratic, and exponential functions. (ME)

Courses

Algebra II
College Algebra
Precalculus
Calculus
Geometry/Trigonometry/Advanced Algebra
AP Calculus AB
AP Calculus BC
Integrated Mathematics II

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.13* Determine whether a linear, quadratic, polynomial, exponential, logarithmic, or trigonometric model fits a situation.
Determine an appropriate mathematical model in context (with or without technology).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.F.1 9-10.AR.F.2* 9-10.AR.F.4* 9-10.AR.F.6* 9-10.AR.F.7* 9-10.AR.F.8*	11-12.AR.F.1* 11-12.AR.F.2* 11-12.AR.F.7* 11-12.AR.F.8* 11-12.AR.F.9* 11-12.AR.F.10* 11-12.AR.F.15* 11-12.DPS.4*	(+) 11-12.AR.F.11 (+) 11-12.AR.F.23* (+) 11-12.DPS.5*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Describe the functional relationship between two quantities modeled through graphs, tables, or equations. • Identify and compare key features of linear, quadratic, exponential, logarithmic, polynomial, or trigonometric functions. • Create and analyze a scatter plot.	• constant difference • constant multiplier • periodic • linear model • quadratic model • polynomial model • exponential model • logarithmic model • trigonometric model	• Learners may over rely on familiar models rather than considering the behavior of the data and considering the context of the problem. (ME) • Learners might select a model without thoroughly analyzing the data or considering alternative options and then fail to verify if their chosen model's assumptions are met by the data. (ME) • Learners might disregard outliers or atypical data points that may affect the suitability of a model. (ME) • Learners might misinterpret trends or patterns in data leading to the selection of an inappropriate model. (ME) • Learners may not understand the properties and behaviors of different types of functions. (ME) • Learners may assume a linear relationship when the data exhibits nonlinear behavior. (ME)

Courses
Algebra II
College Algebra
Precalculus
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.14* Write arithmetic and geometric sequences both recursively and with an explicit formula and convert between the two forms.
Use sequences to model situations.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.1 9-10.AR.3 9-10.AR.F.1* 9-10.AR.F.2* 9-10.AR.F.4* 9-10.AR.F.6* 9-10.AR.F.7* 9-10.AR.F.8*	11-12.AR.1* 11-12.AR.2 11-12.AR.3 11-12.AR.F.1* 11-12.AR.F.2* 11-12.AR.F.7* 11-12.AR.F.8* 11-12.AR.F.9*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify a sequence as arithmetic, geometric, or neither. • Generate, extend, and describe patterns based on a given rule. • Model a function between two quantities by creating a table, graph, or equation.	• sequence • recursive formula • explicit formula • arithmetic sequence • geometric sequence	• Learners might misinterpret the pattern or rule governing the sequence which leads to incorrect recursive or explicit formulas. (ME) • Learners may incorrectly determine the first term when writing recursive formulas. (ME) • Learners might use incorrect notation and/or subscripts, e.g., indexes when writing the explicit and/or recursive formula for a sequence. (ME) • Learners may incorrectly determine the constant difference in an arithmetic sequence or common ratio in a geometric sequence. (ME) • Learners may not determine whether a sequence is arithmetic, geometric, or neither when modeling. (ME)

Courses

Algebra II
College Algebra
Precalculus
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics III
Discrete Mathematics

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.15* Use properties of logarithms to express the solution to $ab^{ct} = d$ where a, c, and d are real numbers and b is a positive real number. Evaluate the logarithm using technology when appropriate.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.1 9-10.AR.2 9-10.AR.F.6* 9-10.AR.F.10*	11-12.NO.1 11-12.NO.2 11-12.AR.4* 11-12.AR.7* 11-12.AR.F.6* 11-12.AR.F.10* 11-12.AR.F.13*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Rewrite formulas to highlight a quantity of interest. • Write a function that describes a relationship between two quantities. • Calculate common and natural logarithms. • Apply properties of logarithms to simplify expressions.	• natural logarithm • common logarithm	• Learners might incorrectly apply logarithmic properties, e.g., product rule, quotient rule, power rule, change of base rule. (ME) • Learners may not isolate the variable before applying the power rule. (ME) • Learners may not apply the power rule for logarithms when solving an exponential equation. (ME) • Learners may not determine whether it is more efficient to use common logarithms vs. natural logarithms (or vice versa) when solving an exponential equation. (ME)

Courses

Algebra II
College Algebra
Precalculus
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.16 Extend right triangle trigonometry and apply knowledge of the unit circle to determine values of sine, cosine, and tangent for multiples of π/3, π/4, and π/6.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.GM.9 9-10.GM.10 9-10.GM.18 (+) 9-10.GM.19 9-10.GM.20 (+) 9-10.GM.21 9-10.GM.26	11-12.AR.F.1 11-12.AR.F.17 11-12.GM.3	(+) 11-12.AR.F.18 (+) 11-12.AR.F.19 (+) 11-12.AR.F.20 (+) 11-12.AR.F.21* (+) 11-12.AR.F.24 (+) 11-12.GM.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Interpret positive angles as counterclockwise rotations and negative angles as clockwise rotations. • Use degree and radian forms to describe angle measures. • Find a reference angle and sketch the right triangle in the appropriate quadrant. • Apply the Pythagorean Theorem and knowledge of special right triangles to find unknown side lengths, angle measures, and coordinates on the unit circle.	• special right triangles • reference angle • right triangle • trigonometry • unit circle • Sine • Cosine • tangent	• Learners may not identify the reference angle in the appropriate quadrant and use it to determine the sign of trigonometric ratios. (ME) • Learners might use incorrect side lengths in special right triangles when determining values of sine, cosine, or tangent. (ME) • Learners might incorrectly label side lengths in special right triangles, resulting in determining incorrect ratios when finding values of sine, cosine, or tangent. (ME) • Learners might incorrectly convert from degrees to radians (or vice-versa) when finding values of sine, cosine, or tangent. (ME) • Learners may not rotate clockwise when given negative angle measures. (ME)

Courses

Algebra II
Precalculus
Trigonometry
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

11-12.AR.F.17 Use the Pythagorean Identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to find sin (θ), cos (θ), or tan (θ) given sin (θ), cos (θ), or tan (θ) and the quadrant of the angle.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.2 9-10.GM.10 9-10.GM.18 (+) 9-10.GM.19 9-10.GM.20 (+) 9-10.GM.21	11-12.AR.F.1* 11-12.AR.F.16 11-12.GM.3	(+) 11-12.AR.F.18 (+) 11-12.AR.F.19 (+) 11-12.AR.F.20 (+) 11-12.AR.F.21* (+) 11-12.AR.F.24 (+) 11-12.GM.4

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Apply the Pythagorean Theorem and knowledge of special right triangles to find unknown side lengths, angle measures, and coordinates on the unit circle. • Rearrange multi-variable formulas to highlight a quantity of interest.	• Pythagorean Identity • unit circle	• Learners must understand the relationship between the Pythagorean Theorem and the Pythagorean Identity as they relate to side lengths and angle measures of right triangles on the unit circle. (IN) • Learners might confuse the signs of sin θ or cos θ in different quadrants. (ME) • Learners may incorrectly substitute a given value for sin (θ) or cos (θ) into the Pythagorean Identity (given sin θ =0.5, learners may write "(sin 0.5)²" in the equation instead of $(0.5)^2$. (ME) • Learners may not use knowledge of the relationship between tan θ and sin θ/cos θ to determine tangent values. (ME)

Courses

Algebra II
Precalculus
Trigonometry
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

(+) 11-12.AR.F.18 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.GM.18 (+) 9-10.GM.19 9-10.GM.22 9-10.GM.25 9-10.GM.26 11-12.AR.F.16 11-12.AR.F.17	(+) 11-12.AR.F.19 (+) 11-12.AR.F.20 (+) 11-12.AR.F.21 (+) 11-12.AR.F.22 (+) 11-12.AR.F.23 (+) 11-12.AR.F.24 (+) 11-12.NO.14 (+) 11-12.NO.15 (+) 11-12.NO.16 (+) 11-12.NO.17 (+) 11-12.NO.18 (+) 11-12.NO.19	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Apply the Pythagorean Theorem and knowledge of special right triangles to find unknown side lengths, angle measures, and coordinates on the unit circle. • Recognize the unit circle as a tool that provides reference when working with trig functions and angle measurements.	• special right triangles • unit circle • trigonometric function • radian measure	• Learners may incorrectly convert between radian measure and degree measure (and vice versa). (ME) • Learners may place angles incorrectly on the unit circle and fail to recognize the x- and y-coordinates of the point on the circle as the cosine of the angle and sine of the angle. (ME)

Courses

Precalculus
Trigonometry
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
AP Precalculus

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

(+) 11-12.AR.F.19 Use the unit circle to express the values of sine, cosine, and tangent for π - x, π + x, and 2π - x in terms of their values for x, where x is any real number.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.GM.18 (+) 9-10.GM.19 9-10.GM.25 9-10.GM.26 11-12.AR.F.4*	(+) 11-12.AR.F.18 (+) 11-12.AR.F.20 (+) 11-12.AR.F.21 (+) 11-12.AR.F.22 (+) 11-12.AR.F.24	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Apply the Pythagorean Theorem and knowledge of special right triangles to find unknown side lengths, angle measures, and coordinates on the unit circle. • Recognize the unit circle as a tool that provides reference when working with trig functions and angle measurements.	• special right triangles • unit circle	• Learners might not recognize the symmetry properties of sine and cosine functions with respect to the quadrants and axes on the unit circle. (ME) • Learners may not understand the periodic nature of trigonometric functions and the effect on values for angles outside the interval [0,2π]. (ME) • Learners might not recognize Supplement's Theorem, Complement's Theorem, and Periodicity Theorem when expressing values of sine, cosine, and tangent. (ME)

Courses

Precalculus
Trigonometry
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
AP Precalculus

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

(+) 11-12.AR.F.20 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
11-12.AR.F.7 11-12.AR.F.10 11-12.AR.F.16 11-12.AR.F.17	(+) 11-12.AR.F.11* (+) 11-12.AR.F.18 (+) 11-12.AR.F.19 (+) 11-12.AR.F.21* (+) 11-12.AR.F.22 (+) 11-12.AR.F.23* (+) 11-12.AR.F.24	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Apply the Pythagorean Theorem and knowledge of special right triangles to find unknown side lengths, angle measures, and coordinates on the unit circle. • Recognize the unit circle as a tool that provides reference when working with trig functions and angle measurements. • Use the unit circle to find the values of sine, cosine, or tangent. • Recognize periodicity as recurrence at regular intervals.	• odd function • even function • periodicity • unit circle • trigonometric function	• Learners may not distinguish between odd and even trigonometric functions and their properties. (ME) • Learners may not understand the concept of periodicity and its implications for trigonometric functions. (ME) • Learners may not recognize that even functions require f(-x)=f(x) and odd functions require -f(x)=f(-x). (ME) • Learners might not realize even functions are symmetric about the y-axis and odd functions are symmetric about the origin. (ME)

Courses

Precalculus
Trigonometry
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
AP Precalculus

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

(+) 11-12. AR.F.21 Create a trigonometric function to model periodic phenomena.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.DPS.3 11-12.AR.8* 11-12.AR.F.4 11-12.AR.F.7 11-12.AR.F.8 11-12.AR.F.10 11-12.AR.F.16 11-12.AR.F.17 11-12.DPS.4*	(+) 11-12.AR.F.11 (+) 11-12.AR.F.18 (+) 11-12.AR.F.19 (+) 11-12.AR.F.20 (+) 11-12.AR.F.21 (+) 11-12.AR.F.22 (+) 11-12.AR.F.23* (+) 11-12.DPS.5*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Analyze a scatter plot to determine data trends (linear, quadratic, exponential, periodic). • Collect and record periodic data using tables and/or graphs. • Determine the amplitude, period, phase shift, and vertical shift given a transformed trigonometric function.	• periodic • amplitude • frequency • midline (centerline, vertical shift) • phase shift • trigonometric function	• Learners may struggle with analyzing the characteristics of the phenomenon they are modeling and select an inappropriate trigonometric function to model that periodic phenomenon. (ME) • Learners may not consider any phase shift present in the periodic phenomenon leading to an inaccurate trigonometric model. (ME) • Learners may not identify the maximum or minimum values of the phenomenon causing errors in calculating the amplitude and period of the trigonometric function. (ME) • Learners may not convert the period of the trigonometric function to a frequency value. (ME)

Courses

Precalculus
Trigonometry
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
AP Precalculus

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

(+) 11-12. AR.F.22 Restrict the domain of a trigonometric function to construct its inverse.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.F.4* 9-10.AR.F.5 9-10.AR.F.10* 11-12.AR.F.5* 11-12.AR.F.8* 11-12.AR.F.9*	(+) 11-12.AR.F.11 (+) 11-12.AR.F.18 (+) 11-12.AR.F.19 (+) 11-12.AR.F.20 (+) 11-12.AR.F.21 (+) 11-12.AR.F.23	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify a function and its key features using tables, graphs, written descriptions, and equations. • Find the inverse of a function describing the relationship between the domain, range, and graph of the function and its inverse. • Describe an invertible function and the criteria that render the function invertible.	• inverse function • trigonometric function	• Learners may not determine the correct range of the trigonometric function. (ME) • Learners may not verify whether the trigonometric function is 1-1 within the restricted domain. (ME) • Learners might disregard the domain restrictions imposed by the original function when defining the inverse function. (ME)

Courses

Precalculus
Trigonometry
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
AP Precalculus

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

(+) 11-12. AR.F.23* Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions and interpret them in context.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.AR.F.10* 11-12.AR.1* 11-12.AR.4* 11-12.AR.F.5* 11-12.AR.F.6* 11-12.AR.F.13*	(+) 11-12.AR.F.18 (+) 11-12.AR.F.20 (+) 11-12.AR.F.21 (+) 11-12.AR.F.22	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify a function and its key features using tables, graphs, written descriptions, and equations. • Find the inverse of a function describing the relationship between the domain, range, and graph of the function and its inverse. • Describe an invertible function and the criteria that render the function invertible. • Limit the domain of a function to ensure the resulting inverse is also a function. • Solve a trigonometric equation.	• inverse function • trigonometric equation	• Learners might misapply inverse trigonometric functions without considering their domains or restrictions. (ME) • Learners may not consider domain restrictions when using inverse trigonometric functions. (ME) • Learners may not identify and discard extraneous solutions that do not satisfy the original equation. (ME) • Learners might use the incorrect trigonometric ratio when solving a trig equation in context. (ME) • Learners may not consider using the Law of Sines or the Law of Cosines when solving for side lengths or angle measures in non-right triangles (in context). (ME)

Courses

Precalculus
Trigonometry
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
AP Precalculus

ELEVENTH AND TWELFTH GRADE

Functions (F)
Learners will develop a foundational knowledge of functions and use them to model relationships between quantities.

(+) 11-12. AR.F.24 Know and apply the addition and subtraction formulas for sine, cosine, and tangent to solve problems.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.GM.18 11-12.AR.1 11-12.AR.5 11-12.AR.6 11-12.AR.F.16 11-12.AR.F.17	(+) 11-12.AR.F.18 (+) 11-12.AR.F.19 (+) 11-12.AR.F.20	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Apply the Pythagorean Theorem and knowledge of special right triangles to find unknown side lengths, angle measures, and coordinates on the unit circle. • Recognize the unit circle as a tool that provides reference when working with trig functions and angle measurements. • Use the unit circle to find the values of sine, cosine, or tangent. • Apply the Pythagorean Identities in rewriting trigonometric equations.	• addition identities • subtraction identities • sine • cosine • tangent	• Learners might misapply the addition and subtraction formulas for sine, cosine, and tangent. (ME) • Learners may not check domain restrictions when applying formulas. (ME) • Learners may struggle to simplify expressions obtained by applying the addition/subtraction formulas leading to unnecessarily complex solutions. (ME)

Courses

Precalculus
Trigonometry
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
AP Precalculus

ELEVENTH AND TWELFTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
11-12.GM.1 Write the equation of a conic section given its special features. Convert between the standard form and general form equations of conic sections.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.AR.1 9-10.GM.1 9-10.GM.2 9-10.GM.3 9-10.GM.4 9-10.GM.5 9-10.GM.22	11-12.AR.4 11-12.AR.7 11-12.AR.8 11-12.AR.F.8* 11-12.AR.F.10* 11-12.GM.2	

Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Recognize that a conic section is a curve formed by the intersection of a plane with a cone. • Define and differentiate between the types of conic sections (circle, ellipse, parabola, hyperbola).	• conic section (circle, ellipse, parabola, hyperbola) • center • radius • directrix • focus (foci) • major and minor axis • eccentricity • vertices • co-vertices • asymptotes	• Learners may incorrectly identify the type of conic section (circle, ellipse, parabola, hyperbola). (ME) • Learners may not use the property of equality when completing the square to convert a conic section in general form to standard form. (ME) • Learners might confuse the standard and general form of a conic section. (ME) • Learners may not correctly identify the special features of each conic nor find the special features. (ME) • Learners might confuse the formulas used for finding the foci of an ellipse and a hyperbola, in addition to neglecting to use the correct formula to find the focus and directrix of a parabola. (ME)

Courses
Algebra II College Algebra Precalculus Trigonometry/Analytic Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics III Analytic Geometry

ELEVENTH AND TWELFTH GRADE

Geometry and Measurement (GM)

Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.

11-12.GM.2* Identify key features of a conic section given its equation.
Apply properties of conic sections in context.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.GM.1 9-10.GM.2 9-10.GM.3 9-10.GM.4 9-10.GM.5 9-10.GM.22	11-12.AR.4* 11-12.AR.F.8* 11-12.AR.F.10* 11-12.GM.1	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Recognize that a conic section is a curve formed by the intersection of a plane with a cone. • Define and differentiate between the types of conic sections (circle, ellipse, parabola, hyperbola). • Convert between standard and general forms of equations of conic sections. • Identify the properties of conic sections (vertex, focus, center of cone, directrix, eccentricity). • Graph conic sections to identify their special features.	• conic section (circle, ellipse, parabola, hyperbola) • center • radius • directrix • focus (foci) • major and minor axis • eccentricity • vertices • co-vertices • asymptotes	• Learners might make mistakes in identifying the center and radius of a circle or ellipse if the equation is not in standard form. (ME) • Learners might overlook the foci of ellipses and hyperbolas or fail to identify the asymptotes of hyperbolas. (ME) • Learners might misinterpret the eccentricity of conic sections and their significance in determining the shape of the ellipses and hyperbolas. (ME) • Learners may confuse the directrix and vertex of a parabola or fail to identify them correctly. (ME)

Courses

Algebra II
College Algebra
Precalculus
Trigonometry/Analytic Geometry
Geometry/Trigonometry/Advanced Algebra
Integrated Mathematics III
Analytic Geometry

ELEVENTH AND TWELFTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
11-12.GM.3 Determine and apply appropriate formulas to solve right and non-right triangle problems in context.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.GM.7 9-10.GM.8 9-10.GM.9 9-10.GM.10 9-10.GM.18 (+) 9-10.GM.19 9-10.GM.20 (+) 9-10.GM.21 9-10.GM.30*	11-12.AR.7 11.AR.F.16 11.AR.F.17	(+) 11-12.GM.4
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish between right and non-right triangles. • Apply the Pythagorean Theorem and knowledge of trigonometric ratios and special right triangles to find unknown side lengths, angle measures, and coordinates on the unit circle. • Recognize the unit circle as a tool that provides reference when working with trig functions and angle measurements. • Recognize angles of elevation and depression in a triangle. • Apply the inverse of sine, cosine, or tangent in right triangles.	• Law of Sines • Law of Cosines • right triangle	• Learners may not recognize ambiguous cases (possibility of multiple solutions, no solution) when using the Law of Sines. (ME) • Learners may not understand when to apply the Law of Sines or Law of Cosines given a problem in context. (ME) • Learners might add a^2+b^2-2ab together when using the Law of Cosines, neglecting to isolate the 2abcosC term on one side of the equation. (ME) • Learners may not include units in final answers. (ME) • Learners may not round or truncate their answers appropriately based on the context and precision of the data. (ME)

Courses
Algebra II Precalculus Trigonometry Trigonometry/Analytic Geometry Geometry/Trigonometry/Advanced Algebra Integrated Mathematics III Analytic Geometry Applied Geometry

ELEVENTH AND TWELFTH GRADE

Geometry and Measurement (GM) *Learners will use visualization, spatial reasoning, geometric modeling, and measurement to investigate the characteristics of figures, perform transformations, and construct logical arguments.*
(+) 11-12.GM.4 Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.GM.9 9-10.GM.10 9-10.GM.18 9-10.GM.20 9-10.GM.30* 11-12.AR.F.16 11-12.AR.F.17 11-12.GM.3		
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Apply the formula to find the area of a triangle. • Apply the Pythagorean Theorem and knowledge of trigonometric ratios and special right triangles to find unknown side lengths, angle measures, and coordinates on the unit circle. • Apply the Laws of Sines and Cosines to solve unknown angles and sides of triangles.	• vertex • auxiliary line	• Learners might confuse A=0.5absinC with the Law of Sines. (ME) • Learners may incorrectly identify angle C as one of the other angles in the triangle or use an incorrect angle measure. (ME) • Learners may not recognize that a and b are the sides of the triangle that form angle C. (ME) • Learners might confuse sine with cosine in the area formula. (ME) • Learners might misinterpret the diagram of the triangle or mislabel the sides relative to the given angle. (ME)

Courses
Precalculus Trigonometry Trigonometry/Analytic Geometry Geometry/Trigonometry/Advanced Algebra AP Precalculus Analytic Geometry

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS)

Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.

11-12.DPS.1* Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.DPS.2*	11-12.DPS.2* 11-12.DPS.3*	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Calculate the measures of center (mean, median, mode) and spread (range, interquartile range, standard deviation) given a set of data. • Calculate outliers in a given set of data. • Create a box plot for a set of data. • Create a histogram using a data set and identify its shape (skewed, normal distribution).	• outlier • measures of center • measures of spread • normal distribution • data set • skewed	• Learners might misinterpret the skewness of the data distribution, leading to inaccurate descriptions of its shape. (ME) • Learners might disregard outliers in the data set when describing its center and spread. (ME) • Learners might misinterpret specific measures of spread (range, standard deviation, IQR) in the presence of outliers of non-normal data distributions. (ME)

Courses

Algebra II
Probability and Statistics
Precalculus
Geometry/Trigonometry/Advanced Algebra
AP Statistics
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*		
11-12.DPS.2* Use the mean and standard deviation of a data set to fit it to a normal distribution and estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.NO.3 9-10.NO.4 9-10.DPS.2	11-12.DPS.1* 11-12.DPS.3*	(+) 11-12.DPS.8*
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Calculate the measures of center (mean, median, mode) and spread (range, interquartile range, standard deviation) given a set of data. • Calculate outliers in a given set of data. • Create a box plot for a set of data. • Create a histogram using a data set and identify its shape (skewed, normal distribution). • Determine the effect of outliers on the measures of center and spread. • Analyze data sets to identify center, variability, spread, and potential outliers. • Recognize that the properties of the normal distribution curve (symmetric; unimodal (has a single peak value), bell-shaped, the mean, median, and mode are equal and follow the empirical rule (68-95-99.7)).	• normal distribution • mean • standard deviation • Empirical Rule • unimodal	• Learners might assume data follows a normal distribution without verifying it using graphical or statistical methods. (ME) • Learners may incorrectly apply the empirical rule to estimate population percentages for non-normal data. (ME) • Learners may not identify or address outliers causing a distorted estimate of population percentages. (ME)

Courses
Algebra II Probability and Statistics Precalculus Geometry/Trigonometry/Advanced Algebra AP Statistics Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS)

Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.

11-12.DPS.3* Evaluate reports based on data.

a. Identify and explain misleading use of data, recognize when claims based on data confuse correlation and causation.

b. Recognize and describe how graphs and data can be distorted to support different points of view.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.3 9-10.NO.4 9-10.DPS.1* 9-10.DPS.4*	11-12.DPS.1* 11-12.DPS.2*	(+) 11-12.DPS.5* (+) 11-12.DPS.6* (+) 11-12.DPS.7* (+) 11-12.DPS.8* (+) 11-12.DPS.9*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Identify advantages and disadvantages of data representations (scatter plot, circle graph, bar graph, line graph). • Calculate the measures of center (mean, median, mode) and spread (range, interquartile range, standard deviation) given a set of data. • Determine the effect of outliers on the measures of center and spread. • Analyze data sets to identify center, variability, spread, and potential outliers. • Recognize the properties of the normal distribution curve (symmetric; unimodal (has a single peak value), bell-shaped, the mean, median, and mode are equal and follow the empirical rule (68-95-99.7)). • Use the mean and standard deviation to create a normal distribution curve. • Determine the correlation between two variables and distinguish between correlation and causation as related to data. • Differentiate between probability and odds.	• correlation • causation • probability • odds	• Learners might not examine the context of the data (how it was collected, population sampled, variables measured). (ME) • Learners may ignore the variability in the data and draw overly broad conclusions based on a single data point or small sample size. (ME) • Learners might confuse correlation with causation leading to erroneous conclusions about the cause-and-effect relationship. (ME) • Learners may selectively choose data supporting a predetermined conclusion and ignore contradictory evidence. (ME) • Learners might misinterpret statistical measures (averages, percentages) without understanding their meaning or limitations. (ME) • Learners may overgeneralize findings from a specific data set to a broader population without considering the limitations of the sample or the ability to generalize the results. (ME)

Courses

Algebra II
Probability and Statistics
Precalculus
Geometry/Trigonometry/Advanced Algebra
AP Statistics
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS)

Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.

11-12.DPS.4* Represent data on a scatter plot for two quantitative variables and describe how the variables are related.

a. Fit a function to the data (with or without technology) and interpret the special features (e.g., meaning of a and b in the exponential function $y = ab^x$) of the function in context.
b. Use functions fitted to data to solve problems in the context of the data.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.NO.3 9-10.NO.4 9-10.DPS.1* 9-10.DPS.2* 9-10.DPS.3*	11-12.AR.F.3 11-12.AR.F.13	(+) 11-12.AR.F.21* (+) 11-12.DPS.5* (+) 11-12.DPS.8*

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Represent data on a scatter plot and recognize the trend (linear, quadratic, exponential). • Understand the concept of a trend line; draw and justify the fit of a trend line. • Interpret patterns of bivariate data (cluster outliers, association - positive, negative, linear, nonlinear) represented on scatter plots.	• scatter plot	• Learners might not identify or describe the relationship between the two variables based on the scatter plot. (ME) • Learners may overgeneralize trends observed in the scatter plot or assume causation based solely on correlation. (ME) • Learners might select an inappropriate function to fit the data or misinterpret the special features of the function. (ME) • Learners might misinterpret the meaning of the parameters (coefficients) of a fitted function in the context of the data. (ME) • Learners may not validate predictions made using the fitted function against additional data. (ME)

Courses

Algebra II
Probability and Statistics
Precalculus
Geometry/Trigonometry/Advanced Algebra
AP Precalculus
AP Statistics
Integrated Mathematics III

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*		
(+) 11-12.DPS.5* Informally assess the fit of a function by plotting and analyzing residuals.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.DPS.3* 11-12.AR.F.13* 11-12.DPS.3* 11-12.DPS.4*	11-12.AR.F.13* (+) 11-12.AR.F.21 (+) 11-12.DPS.7*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Represent data on a scatter plot and recognize the trend (linear, quadratic, exponential). • Understand the concept of a trend line; draw and justify the fit of a trend line. • Interpret patterns of bivariate data (cluster outliers, association - positive, negative, linear, nonlinear) represented on scatter plots. • Create a function to fit a set of data with or without technology.	• scatter plot • residual • residual plot	• Learners may not identify patterns or trends in the residual plot indicating systematic errors or lack of fit. (ME) • Learners might overinterpret random variation in the residual plot as evidence of a lack of fit or systematic error. (ME) • Learners may incorrectly interpret the size of residuals as an indicator of the quality of fit. (ME) • Learners may compare residuals from different models without considering differences in model complexity or appropriateness. (ME)

Courses
Probability and Statistics AP Precalculus AP Statistics

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*		
(+) 11-12.DPS.6* Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
11-12.DPS.3*	(+) 11-12.DPS.7* (+) 11-12.DPS.8* (+) 11-12.DPS.9*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Calculate the mean of a data set. • Distinguish between population mean and population proportion and how they are calculated. • Distinguish between random sampling and simulation models and identify the strengths and weaknesses of each method. • Calculate the margin of error and state what it represents.	• margin of error • mean • random sampling • simulation model • population mean • population proportion	• Learners may not account for variability in sample estimates and how it affects the margin of error. (ME) • Learners might confuse sample statistics and population parameters in population estimation. (ME) • Learners may use simulation parameters incorrectly. (ME) • Learners might over-rely on simulation results without considering their limitations or verifying their accuracy. (ME) • Learners might confuse the margin of error with the width of a confidence interval and fail to see the relationship between them. (ME) • Learners might misinterpret the confidence level. (ME)
Courses		
Probability and Statistics AP Statistics		

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS)

Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.

(+) 11-12.DPS.7* Understand the process of making inferences about population parameters based on a random sample from that population.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
11-12.DPS.3*	(+) 11-12.DPS.5* (+) 11-12.DPS.6* (+) 11-12. DPS.8* (+) 11-12.DPS.9*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish between population mean and population proportion and how they are calculated. • Distinguish between random sampling and simulation models and identify the strengths and weaknesses of each method. • Calculate the margin of error and state what it represents. • Apply the method of random sampling to collect population data.	• random sample • population • population parameters	• Learners might consider or account for sampling bias. (ME) • Learners may confuse population parameters with sample statistics or do not understand the relationship between them. (ME) • Learners may incorrectly interpret the confidence levels. (ME) • Learners may confuse confidence levels with hypothesis testing or misinterpret the purpose of constructing a confidence level. (ME) • Learners may ignore or underestimate the variability in sample statistics, failing to recognize its impact on the precision of estimates. (ME) • Learners may over-rely on the Central Limit Theorem. (ME) • Learners may confuse Type I and Type II errors and their consequences in hypothesis testing. (ME)

Courses
Probability and Statistics AP Statistics

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*
(+) 11-12.DPS.8* Decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation).

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
11-12.DPS.2* 11-12.DPS.3* 11-12.DPS.4*	(+) 11-12.DPS.6* (+) 11-12.DPS.7* (+) 11-12.DPS.9*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Calculate the mean of a data set. • Distinguish between population mean and population proportion and how they are calculated. • Distinguish between random sampling and simulation models and identify the strengths and weaknesses of each method. • Calculate the margin of error and state what it represents. • Apply appropriate formulas to make statistical inferences (hypothesis testing, confidence intervals, margin of error, and regression modeling).	• simulation • sampling methods • statistical inferences	• Learners may not verify if the assumptions of the specified model are met by the data-generating process. (ME) • Learners may be confused between correlation and causation. (ME) • Learners might misapply the hypothesis test to assess the fit of a specified model to the data. (ME) • Learners might over-rely on P-Values as the sole criterion to determine if a specified model is consistent with the data. (ME) • Learners might misinterpret the output of simulation models or misunderstand the implications for model consistency. (ME) • Learners might confuse sample and population. (ME)

Courses
Probability and Statistics AP Statistics

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*		
(+) 11-12.DPS.9* Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
11-12.DPS.3*	(+) 11-12.DPS.6* (+) 11-12.DPS.7* (+) 11-12.DPS.8*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Identify the different methods of collecting quantitative data (experiments, surveys, observation) and their strengths (objectivity, rapid analysis, replication, tested hypothesis, validation of theories) and limitations (context, researcher expertise required, variability, confirmation bias).	• sample survey • experiment • observational study	• Learners may not recognize experimental studies and mistake observational studies for experiments. (ME) • Learners might misunderstand the purpose and importance of randomization in different study designs. (ME) • Learners may not recognize randomization in observational studies (random sampling, randomized controlled trials). (ME) • Learners might misinterpret the objectives of small surveys, experiments, and observational studies and fail to understand the strengths and weaknesses of the different study types. (ME) • Learners may struggle to distinguish between causal and associational inferences (conclusions drawn about the relationships or correlations between variables based on observed data). (ME)
Courses		
Probability and Statistics AP Statistics		

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS)

Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.

11-12.DPS.10* Determine when the order in counting matters and use permutations and combinations to compute probabilities of events accordingly.
Determine probability situations as conditional, “or” (union), or “and” (intersection), and determine the probability of an event.

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.DPS.5* 9-10.DPS.6* 9-10.DPS.7* 9-10.DPS.8* 9-10.DPS.9*	(+) 11-12.DPS.11* (+) 11-12.DPS.12* (+) 11-12.DPS.13* (+) 11-12.DPS.14* (+) 11-12.DPS.15* (+) 11-12.DPS.16* (+) 11-12.DPS.17* (+) 11-12.DPS.18*	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Distinguish between conditional, union, or intersection probability situations. • Construct a tree diagram to find the number of outcomes in an event or the probability of an event. • Apply the addition counting principle to determine the probability of an event. • Apply the multiplication counting principle to determine the probability of an event. • Apply conditional probability appropriately to determine the probability of an event. • Apply the combination formula to determine the probability of an event. • Apply the permutation formula to determine the probability of an event.	• permutation • combination • intersection • union • conditional • probability • tree diagram	• Learners may confuse permutations (arrangements of objects where order matters) with combinations (selections of objects where order doesn't matter). (ME) • Learners may struggle to determine when a situation can be modeled by a permutation vs. a combination. (ME) • Learners may struggle with recognizing situations where the probability of an event depends on the occurrence of another event (conditional probability). (ME) • Learners might confuse "or" (union) events with "and" (intersection events) when determining probabilities. (ME) • Learners may not subtract the intersection when applying the addition counting principle. (ME)

Courses

Algebra II
Probability and Statistics
Precalculus
Geometry/Trigonometry/Advanced Algebra
AP Statistics
Integrated Mathematics for Computer Science/Information Technology
Integrated Mathematics II
Discrete Mathematics

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*
(+) 11-12.DPS.11* Use permutations and combinations to compute probabilities of compound events and solve problems.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.DPS.6* 9-10.DPS.7* 9-10.DPS.8* 9-10.DPS.9*	(+) 11-12.DPS.10*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish between conditional, union, or intersection probability situations. • Construct a tree diagram to find the number of outcomes in an event or the probability of an event. • Apply the addition counting principle to determine the probability of an event. • Apply the multiplication counting principle to determine the probability of an event. • Apply conditional probability appropriately to determine the probability of an event.	• permutation • combination • compound events	• Learners might confuse permutations (arrangements of objects where order matters) with combinations (selections of objects where order doesn't matter). (ME) • Learners may not recognize situations involving multiple events or compound events and incorrectly treat them as independent or mutually exclusive. (ME) • Learners may have difficulty determining when a situation can be modeled by a permutation vs. a combination. (ME)

Courses
Probability and Statistics Precalculus AP Statistics Integrated Mathematics for Computer Science/Information Technology Discrete Mathematics

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*
(+) 11-12.DPS.12* Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space. Graph the corresponding probability distribution using the same graphical displays as for data distributions.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.DPS.6* 9-10.DPS.7* 9-10.DPS.8*	(+) 11-12.DPS.10* (+) 11-12.DPS.13* (+) 11-12.DPS.14* (+) 11-12.DPS.15* (+) 11-12.DPS.16* (+) 11-12.DPS.17* (+) 11-12.DPS.18*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Construct organized lists, tree diagrams, area models, and simulations given data. • Determine whether a discrete, continuous, or mixed random variable is used given the event. • Calculate the probability of a random variable. • Calculate the expected value for different types of random variables.	• random variable • theoretical probability • empirical probability • probability distribution • expected value	• Learners may confuse the random variable with the actual outcomes or events. (ME) • Learners might incorrectly define the sample space (not listing all possible outcomes or including impossible outcomes). (ME) • Learners may assign probabilities that do not sum up to 1 or that are negative. (ME) • Learners may not ensure the total area under the probability distribution curve equals 1. (ME)

Courses
Probability and Statistics AP Statistics Integrated Mathematics for Computer Science/Information Technology Discrete Mathematics

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*
(+) 11-12.DPS.13* Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.DPS.6* 9-10.DPS.7* 9-10.DPS.8*	(+) 11-12.DPS.12* (+) 11-12.DPS.14* (+) 11-12.DPS.15* (+) 11-12.DPS.16* (+) 11-12.DPS.17* (+) 11-12.DPS.18*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Construct organized lists, tree diagrams, area models, and simulations given data. • Determine whether a discrete, continuous, or mixed random variable is used given the event. • Use the probability of an outcome to determine the probability of a random variable. • Use random variables to find the probability distribution.	• expected value • mean • probability distribution	• Learners might mistake the expected value of a random variable for the sample mean of observed data. (ME) • Learners may not understand that the expected value is a weighted average of the possible outcomes of the random variable, with each outcome weighted by its probability. (ME) • Learners may not interpret the expected value of a random variable as the mean or average outcome of the probability distribution. (ME)

Courses
Probability and Statistics Precalculus AP Statistics Integrated Mathematics for Computer Science/Information Technology Discrete Mathematics

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*
(+) 11-12.DPS.14* Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. Evaluate and compare strategies on the basis of expected values.

Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.DPS.6* 9-10.DPS.7* 9-10.DPS.8* 9-10.DPS.9* 9-10.DPS.10*	(+) 11-12.DPS.12* (+) 11-12.DPS.13* (+) 11-12.DPS.15* (+) 11-12.DPS.16* (+) 11-12.DPS.17* (+) 11-12.DPS.18*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Use the probability of an outcome to determine the probability of a random variable. • Use random variables to find the probability distribution. • Calculate the expected value for different types of random variables.	• expected payoff • expected value	• Learners might mistake the payoff value for probabilities or vice versa when calculating the expected value. (ME) • Learners may not consider all possible outcomes or overlook certain scenarios when calculating expected values. (ME) • Learners may not consider contextual factors or constraints that may influence decision-making and expected values. (ME) • Learners may not interpret the expected payoff as the average or long-term outcome of the decision. (ME)

Courses
Probability and Statistics Precalculus AP Statistics Integrated Mathematics for Computer Science/Information Technology Discrete Mathematics

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*		
(+) 11-12.DPS.15* Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities are calculated; find the expected value.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.DPS.6* 9-10.DPS.7* 9-10.DPS.8*	(+) 11-12.DPS.12* (+) 11-12.DPS.13* (+) 11-12.DPS.14* (+) 11-12.DPS.16* (+) 11-12.DPS.17* (+) 11-12.DPS.18*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish between theoretical and experimental probabilities. • Construct organized lists, tree diagrams, area models, and simulations given data. • Use the probability of an outcome to determine the probability of a random variable. • Use random variables to find the probability distribution. • Calculate the expected value for different types of random variables.	• probability distribution • theoretical probabilities • random variable • expected value	• Learners might incorrectly define the sample space. (ME) • Learners might assign probabilities to outcomes incorrectly. (ME) • Learners may confuse discrete and continuous variables. (ME) • Learners may forget the conditions (non-negativity, the sum of probabilities equals 1). (ME) • Learners might assume independence between events or outcomes when they are actually dependent. (ME) • Learners may not realize that the expected outcome is not the most likely outcome, it is the long-term average. (ME) • Learners might ignore contextual information that should be used to guide the calculation of probabilities and expected values. (ME)

Courses
Probability and Statistics Precalculus AP Statistics Integrated Mathematics for Computer Science/Information Technology Discrete Mathematics

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*		
(+) 11-12.DPS.16* Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.DPS.6* 9-10.DPS.7* 9-10.DPS.8*	(+) 11-12.DPS.12* (+) 11-12.DPS.13* (+) 11-12.DPS.14* (+) 11-12.DPS.15* (+) 11-12.DPS.17* (+) 11-12.DPS.18*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish between theoretical and experimental probabilities. • Construct organized lists, tree diagrams, area models, and simulations given data. • Use the probability of an outcome to determine the probability of a random variable. • Use random variables to find the probability distribution. • Calculate the expected value for different types of random variables. • Apply the empirical rule of normal data distribution and calculate the sample mean and standard deviation.	• probability distribution • random variable • expected value • empirical rule	• Learners may have limited or inadequate data to accurately estimate probabilities for each outcome in the sample space. (ME) • Learners may use data that is biased or not representative of the population, skewing the probability distribution. (ME) • Learners might misinterpret frequency counts as probabilities. (ME) • Learners might overfit the data by assigning probabilities based solely on observed outcomes without considering the underlying randomness or variability. (ME) • Learners may overlook outliers or extreme values in data when estimating probabilities. (ME) • Learners may not provide confidence levels or measures of uncertainty for estimated probabilities. (ME) • Learners might misinterpret the expected value as a predicted outcome rather than a long-term average. (ME) • Learners might ignore contextual information that should be used to guide the calculation of probabilities and expected values. (ME)

Courses
Probability and Statistics Precalculus AP Statistics Integrated Mathematics for Computer Science/Information Technology Discrete Mathematics

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS)

Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.

(+) 11-12.DPS.17* Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

Preceding Standard(s)	Related Standard(s)	Successive Standard(s)
9-10.DPS.6* 9-10.DPS.7* 9-10.DPS.8* 9-10.DPS.9* 9-10.DPS.10*	(+) 11-12.DPS.12* (+) 11-12.DPS.13* (+) 11-12.DPS.14* (+) 11-12.DPS.15* (+) 11-12.DPS.16* (+) 11-12.DPS.18*	

Prerequisite Skills	Key Vocabulary	Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)
• Distinguish between conditional, union, or intersection probability situations. • Construct organized lists, tree diagrams, area models, and simulations given data. • Apply the addition counting principle to determine the probability of an event. • Apply the multiplication counting principle to determine the probability of an event. • Apply conditional probability appropriately to determine the probability of an event. • Distinguish between theoretical and experimental probabilities. • Use the probability of an outcome to determine the probability of a random variable. • Use random variables to find the probability distribution. • Calculate the expected value for different types of random variables. • Apply the empirical rule of normal data distribution and calculate the sample mean and standard deviation.	• probability distribution • theoretical probabilities • random variable • expected value • probability distribution • random variable • expected value • empirical rule	• Learners might overlook the influence of sample size on the reliability of probabilities. (ME) • Learners may misjudge the nature of randomness, believing past outcomes influence future outcomes in random processes. (ME) • Learners might misinterpret probabilistic events. (ME) • Learners may not account for bias. (ME) • Learners may focus on the probabilities of different outcomes without considering the consequences of those outcomes. (ME) • Learners may not validate their probabilistic models or decision-making strategies. (ME)

Courses

Probability and Statistics
Precalculus
AP Statistics
Integrated Mathematics for Computer Science/Information Technology
Discrete Mathematics

ELEVENTH AND TWELFTH GRADE

Data, Probability, and Statistics (DPS) *Learners will ask and answer questions by collecting, organizing, and displaying relevant data, drawing inferences and conclusions, making predictions, and understanding and applying basic probability concepts.*		
(+) 11-12.DPS.18* Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).		
Preceding Standard(s)	**Related Standard(s)**	**Successive Standard(s)**
9-10.DPS.6* 9-10.DPS.7* 9-10.DPS.8* 9-10.DPS.9* 9-10.DPS.10*	(+) 11-12.DPS.12* (+) 11-12.DPS.13* (+) 11-12.DPS.14* (+) 11-12.DPS.15* (+) 11-12.DPS.16* (+) 11-12.DPS.17*	
Prerequisite Skills	**Key Vocabulary**	**Instructional Notes (IN) and Common Learner Misconceptions/ Errors (ME)**
• Distinguish between conditional, union, or intersection probability situations. • Construct organized lists, tree diagrams, area models, and simulations given data. • Apply the addition counting principle to determine the probability of an event. • Apply the multiplication counting principle to determine the probability of an event. • Apply conditional probability appropriately to determine the probability of an event. • Distinguish between theoretical and experimental probabilities. • Use the probability of an outcome to determine the probability of a random variable. • Use random variables to find the probability distribution. • Calculate the expected value for different types of random variables. • Apply the empirical rule of normal data distribution and calculate the sample mean and standard deviation.	• probability distribution • theoretical probabilities • random variable • expected value • probability distribution • random variable • expected value • empirical rule	• Learners may struggle to interpret conditional probabilities in complex decision-making scenarios with multiple factors influencing outcomes. (ME) • Learners might confuse correlation with causation. (ME) • Learners may not consider false positives and false negatives. (ME) • Learners might underestimate the importance of sample size and statistical power when analyzing probabilistic data. (ME) • Learners may overlook uncertainty and risk when evaluating decisions and strategies. (ME) • Learners might incorrectly apply decision trees or Bayes' Theorem to analyze decision-making scenarios. (ME)
Courses		
Probability and Statistics AP Statistics Integrated Mathematics for Computer Science/Information Technology Discrete Mathematics		